High potentiality and applicability of 2-D and 3-D particle imaging velocimetry

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Abstract

In a calculation of an image cross-correlation coefficient, an identification of each particle is calculated based on the similarity of the particle distribution pattern between two-consecutive images using the cross-correlation coefficient. The calculation of the cross-correlation coefficient for 8-bit image data needs long time. However, its calculation time using the binary image data takes shorter than the other method. Consequently, the binary image cross-correlation method has an advantage of the high speed algorithm for the particle tracking. Firstly the mathematical fundamentals of the binary image cross-correlation are given and generalized. Secondary applications of the method are presented, i.e., two-dimensional measurements of unsteady flows inside and outside a hollow circular cylinder, a torque converter internal flow, a flow with a bubble, a heart muscle movement, and also a three-dimensional measurement of a two-phase flow in a bubbling-jet water vessel. Finally, the high potentiality and applicability of the method are described.

1 Introduction

The techniques of particle imaging velocimetry (PIV) have been advanced quickly because they have the following merits: (1) instantaneous whole flow field measurement, (2) contact-free measurement, and (3) easy extraction and processing of physical information through the velocity information. Some reviews on PIV (e.g. Adrian [1], Hesselink [2]) describe the principles and applications of many types of PIV, which are applicable to flows from low to high Reynolds number.

Among many types of PIV, it may be said that the binary image cross-
correlation method has the highest speed algorithm to track each particle motion. The technique was developed by Uemura et al. [3] and has been applied to wide area of fluid engineering. The mathematical fundamentals and strict proof of the algorithm for computation of the cross-correlation, however, have not been given yet, although the algorithm was conceptionally understood by many users. Recently the authors have carried out the mathematically strict derivation of the final and simple form of the cross-correlation coefficient under the conditions of binary image data (Yamamoto et al. [4]).

As everyone knows, the brightness distribution cross-correlation method which was originally developed by Yano [5] and Kimura et al. [6] is one of the most popular and useful techniques. It tracks the motion of small segments consisting of fluid elements with brightness distribution patterns using the cross-correlation coefficient defined statistically in order to find the most similar patterns with 8-bit image data. On the other hand, the binary image cross-correlation method tracks the motion of each particle and it is a type of the particle tracking velocimetre (PTV). For this method, the particle image pictures are binarized through a threshold value, and then the pixels take the value of 1 inside the particle images and 0 outside them. Consequently, the equation of cross-correlation coefficient can be changed into a very simple form for the case of binary image data which is expressed using a summation of logical products. The simplicity of the cross-correlation coefficient form produces some merits such as the high speed algorithm and the cost-down of image analysis system.

In the former part of the present paper, the mathematical derivation is described and in the later part the applications of the technique to two-dimensional measurements such as unsteady flows inside and outside a hollow circular cylinder, a torque converter internal flow, a flow with a bubble, a heart muscle movement and the three-dimensional measurement of a multiphase flow in a weak bubbling-jet water vessel are shown.

Nomenclature

\( X \) : vector space, \( \beta \) : additive set family of \( X \), \( \{E_i\} \), \( \{F_i\} \) : finite disjoint classes of \( \beta \), \( \mu \) : measure on \( \beta \), \( L^2 \) : the sets of all functions on \( X \) such that \( \int f^2 \, d\mu < \infty \), where \( f \) is a function.

2 Mathematical fundamental of cross-correlation coefficient

2.1 Cross-correlation coefficient

The cross-correlation coefficient is defined as follows. The \( \int X f g d\mu \) has a finite value by the Hölder’s inequality (\( f, g \in L^2 \)). Then the cross-correlation coefficient \( C_{\epsilon} \) of \( f \) and \( g \) using the inner-product is defined as follows:
In the above equation, the fact that $|C_{f,g}| \leq 1$ is well known as the Schwarz's inequality.

### 2.2 Definition of the distribution function

The distribution function $f$ of $\{E_i\}$ is defined as follows:

$$f(x) = \sum_{i=1}^{n} \chi_{E_i}(x)$$

(2)

where, $\chi_{E_i}(x) = \begin{cases} 1 : x \in E_i \\ 0 : x \notin E_i \end{cases}$

(3)

Figure 1 shows the concept of the function $\chi_{E_i}$. In such space, the necessary and sufficient conditions for $C_{f,g} = 1$ is $f = g$. If $f$ and $g$ are distribution functions of $\{E_i\}$ and $\{F_j\}$, respectively, and then the following conditions are satisfied:

$$\int f(x)^2 d\mu(x) = \sum_{i=1}^{n} \mu(E_i)$$

(4)

$$\int g(x)^2 d\mu(x) = \sum_{j=1}^{m} \mu(F_j)$$

(5)

$$\int f(x)g(x) d\mu(x) = \sum_{i=1}^{n} \sum_{j=1}^{m} \mu(E_i \cap F_j)$$

(6)

Substitution of equations (4), (5) and (6) into equation (1) gives:

$$C_{f,g} = \frac{\sum_{i=1}^{n} \sum_{j=1}^{m} \mu(E_i \cap F_j)}{\sqrt{\sum_{i=1}^{n} \mu(E_i)} \sqrt{\sum_{j=1}^{m} \mu(F_j)}}$$

(7)

### 2.3 Definition of the measure preserving transformation $\phi : X \to X$

Let $f$ be the distribution function $\{E_i\}$, $\phi$ the transformation of $x$, and the function $f \circ \phi$ combined $f$ with $\phi$ is defined by equation (8). Here, the $\phi$ is called measure preserving transformation when equation (9) is satisfied:

$$(f \circ \phi)(x) = \sum_{i=1}^{n} \chi_{\phi(E_i)}(x)$$

$$\mu(\phi(E_i)) = \mu(E_i)$$

(8)

(9)

Furthermore, if $g$ is a distribution function, then the following equation is
obtained:

\[
C_{(f \to g)} = \frac{\sum_{i=1}^{n} \sum_{j=1}^{m} \mu(\phi(E_i) \cap F_j)}{\sqrt{\sum_{i=1}^{n} \mu(E_i)} \sqrt{\sum_{j=1}^{m} \mu(F_j)}}
\]

(10)

here, \{ \phi(E_i) \} is disjoint class of \( \beta \). If all the elements of \{ E_i \} and \{ F_j \} are congruous, then equation (10) becomes as follows:

\[
C_{(f \to g)} = \frac{\sum_{i=1}^{n} \sum_{j=1}^{m} \mu(\phi(E_i) \cap F_j)}{\sqrt{\sum_{i=1}^{n} \mu(E_i)} \sqrt{nm}}
\]

(11)

2.4 Particle Identification by the Binary Image Cross-Correlation Method

Here we assume that a constant time interval between the two-consecutive time step pictures is short enough and that velocities between the two flow fields do not change suddenly in time or space, that is, the similarity of flow patterns is preserved. Consequently, the cross-correlation coefficient is defined when the measure preserving transformation \( \phi \) is translation.

The cross-correlation coefficient between the particle \( I \) at the first time \( t \) and the particle \( J \) at the second time \( t + \Delta t \) is calculated as the following steps: (1) Let \{ \phi(E_i) \} be the neighbor of the particle \( I \), \( f \) the distribution function, and \( x_i \) the vector coordinate of the gravity center; (2) Let \{ \phi(F_j) \} be the neighbor of the particle \( J \), \( g \) the distribution function, and \( x_j \) the vector coordinate of the gravity center; (3) Define the transformation \( \phi \) as follows (Figures 2 and 3):

\[
\phi(x) = x + (x_j - x_i)
\]

(12)

Then \( \phi \) is measure preserving transformation, and equation (11) becomes as follows:

\[
C_{fg} = \frac{\sum_{i=1}^{n} \sum_{j=1}^{m} \mu((E_i + (x_j - x_i)) \cap F_j)}{\sqrt{\sum_{i=1}^{n} \mu(E_i)} \sqrt{nm}}
\]

(13)

here \( n \) and \( m \) are particle numbers in the correlation domains at times \( t \) and \( t + \Delta t \), respectively, and the particles \( I \) and \( J \) are excluded for the calculation. Since equation (13) agrees with the equation derived by Yamamoto et al. [7], it may be said that their equation has been proved to be mathematically strictly valid. Since experimental values are not
included in this discussion, equation (13) has generality. When the logical binary pixels are introduced to evaluate the value \( C_{r=g} \), equation (13) is expressed in a very simple form as follows:

\[
C_{r=g} = \frac{L}{\sqrt{nm}}
\]  

(14)

here \( L \) is the summation of logical products in the two correlation domains of \( f \) and \( g \). The candidate particle which gives the maximum value of \( C_{r=g} \) is decided as the identified particle of pair. The movement distance of the pair particle \( \Delta S' \) is calculated as the distance between the two centers of the target and candidate particles, and the particle velocity is computed as follows:

\[
\bar{v} = \frac{\Delta S'}{\Delta t}
\]  

(15)

The algorithm realizes the high speed calculation and needs only smaller memory for CPU. Thus, the binary image cross-correlation method makes inexpensive PIV system possible.

3 Applications of 2-D PIV

3.1 Unsteady flows inside and outside a hollow circular cylinder

Flows inside and outside a hollow circular cylinder were visualized and were measured simultaneously using 2D-PIV technique by the authors. Figure 4 shows the outline of an experimental apparatus. Figures 5(a) to (d) show the velocity fields inside and outside the hollow circular cylinder in a uniform flow every four seconds after the rotation of the cylinder was stopped suddenly. From the figures it is shown that the wake comes back in the center from the left side and the flow inside the hollow cylinder stops gradually. It is found that PIV measurement makes a simultaneous measurement of the unsteady whole flow fields inside and outside the cylinder possible.

3.2 Torque converter internal flow

An example of the applications of 2-D PIV based on the binary image cross-
correlation method to an internal flow of a torque converter (T/C) is presented. T/C consists of three cascades of a pump, a turbine, and a stator. Original flow field in the stator cascade of T/C was taken by a high speed video system (1,000 frames/sec) using a mirror located in the hollow of the shaft. The measured velocity vectors were rearranged at grid points. Figure 6 shows the rearranged working fluid velocity vectors in the stator cascade at the rotational speeds of the pump, the turbine and the stator of 50, 0, and 0 rpm, respectively, and the Reynolds number of about 16,000 (Wada et al. [8]). The other physical information, i.e., pressure and vorticity etc., can be extracted from the rearranged velocity field in the case of incompressible fluid. PIV measurement indicates a high possibility of developing torque converters, since the internal flow field in a torque converter is analyzed quantitatively.

3.3 Multiphase flow with a bubble
The PIV method and an image processing were applied to a multiphase flow, i.e., an upward liquid flow with a gas bubble in a rectangular narrow channel
3.4 An analysis of a heart muscle movement
An application of 2-D PIV to a heart muscle is presented. 2-D MRI (Magnetic Resonance Imaging) for a left ventricle of a heart muscle was

with a width of 80 mm and a gap size of 6 mm. Visualization of both of the bubble and the surrounding tracer particles was made simultaneously and the surrounding liquid velocity field was measured by combining the following methods. (a) The test channel was illuminated from the top besides from the sides; (b) A whole flow field were divided into five regions as shown in Figure 7 and different threshold values were allocated in each region at the conversion of original image to binary image; and (c) The binary image cross-correlation method was applied to measure the surrounding liquid velocity field. An example of results (Ohta et al. [9]) is shown in Figure 8. This method is utilized in a study on variations of a bubble shape and the surrounding flow field with respect to time.
processed by a tagging method, i.e., meshes on a heart muscle were marked by a tagging method. Figure 9 shows an original MRI taken from the side of a human body. This is a cross section in a short axis direction of the heart muscle and around it. The velocity of the heart muscle movement was measured from two consecutive pictures (MRI) using the brightness distribution pattern cross-correlation method. Figures 10(a) to (b), by Yamamoto et al., show a contraction process of the heat muscle in the center part of the original image (Figure 9) at 1/15 seconds interval, that is, velocity distributions for the muscle movement. The heart muscle, especially the upper part in the figure, contracts twisting clockwise as shown in Figure 10(a). In the next stage, the muscle in the right lower part contracts similarly (see Figure 10(b)). These results correspond to the real movement. If the calibration is made, the velocity distribution can be obtained quantitatively. This kind of analysis will be useful in a medical field.

Figure 9: Original MRI image of heart muscle.

(a) t=1/15 s  
(b) t=2/15 s

Figure 10: Velocity vectors for a heart muscle movement.
4 An application of 3-D PIV

When a flow is measured using 3-D PIV, the stereo pair matching decides the particle correspondence between the two or more camera screens. However, the more the particles, the more difficult it is for the stereo pair matching to identify the particle correspondence. Thus, a new 3-D PIV was developed by Yamamoto et al. [10] and its outline is explained in this session. The flow chart for the new three consecutive time cross-correlation 3-D PIV is shown in Figure 11. The present method first selects correct pairs from many consecutive frames obtained at a time t on each screen by 2-D binary image cross-correlation method, then correct pairs match among each image by the stereo pair matching method. Figure 12 shows 3-D velocity vectors measured by the present 3-D PIV when air bubbles are ejected into a cylindrical vessel filled with water through a nozzle installed at the center of the vessel bottom. Small circles and big circles stand for particles and bubbles, respectively. Lines started at these circles represent the velocity vectors. The results agreed with the visual observation. The present method is strongly expected for an advance in a study of flow structure of such a 3-D multiphase flow.

5 Conclusion

The present paper gives the mathematical fundamentals of the high speed algorithm for the binary image cross-correlation method. Applications of this method to 2-D PIV and 3-D PIV are illustrated. As a result, it is shown that the method has the high potentiality and high applicability.
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References