Influence of wave steepness on extreme ship hull vertical wave bending moments

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Abstract

Sources of uncertainty in calculation of lifetime vertical wave bending moment are described concisely. Joint probability distribution of significant wave heights and mean zero crossing periods is defined. Wave data from the wave atlas Global Wave Statistics (GWS) for the North Atlantic area are analysed. Extreme significant wave heights for different return periods are calculated. Associated wave steepnesses are estimated by extrapolation procedure. It is shown in the paper that data from GWS lead to considerable overestimation of actual wave steepnesses of extreme sea states. Implications of these overestimates on lifetime vertical wave bending moments are analysed. As a result of this study, the methodology to estimate more realistic extreme vertical wave bending moments is indicated.

1 Introduction

In order to have safe and economic ship structural design, it is necessary to know the design loads given by rules of classification societies. Such rules propose extreme loads, local and global, which are necessary to maintain minimum safety standards. For the longitudinal ship strength, the most important global load component is the vertical wave bending moment at midship. The design value of that load component is given by Unified Requirement (UR) S11 of the International Association of Classification Societies (IACS). The theoretical basis of the requirement UR S11 is given by Nitta et al. [1], where it is clearly stated that the rule vertical wave bending moments correspond to the probability of exceedance of $10^{-8}$. In other words, the rule value is the most probable extreme value for the return period of 20 years, which is the ship lifetime.
It is general opinion that the lifetime vertical wave bending moment given by UR S11 is conservative, i.e. that if more sophisticated calculation methods were applied to evaluate extreme loads, the rule value would not be exceeded. However, in the recent years, there are more and more studies showing that this may not be true. Some of the very respectable researchers have shown in their studies that ship may be loaded by much more severe wave loads than those given by the rules (Faulkner [2,3], Guedes Soares [4,5]). Furthermore, some of them expressed opinion that this could be the cause of some unexplained ship losses [3].

When classification societies and other research institutions apply complex direct hydrodynamic and statistical methods for calculation of lifetime vertical wave bending moments according to their own knowledge and experience, significantly different results are obtained. These differences in results are known as the modelling uncertainty of vertical wave bending moment. Also, the results of direct analyses deviate significantly from rule vertical wave bending moments as well as from the available measured values. Significant efforts have been made in recent years to explain the differences in predicted extreme values (ISSC‘97 [4]). As a result of such studies, the main sources of modelling uncertainty have been recognised as follows:

1) The choice of wave scatter diagram,
2) Uncertainty due to calculation of transfer functions,
3) Uncertainty of the long-term prediction method,
4) Non-linearity of the response,
5) The shape of the wave spectra,
6) Uncertainty of human action.

Each of the mentioned sources of uncertainty is a rather complex problem requiring specific study [4,5]. The purpose of this paper is to analyse wave steepness of extreme sea states as the source of uncertainty of extreme vertical wave bending moment. This may be considered as part of the item 1 above.

The motivation for the analysis of wave steepness is given by ISSC‘2000 [5], where it is stated that the data from the wave atlas Global Wave Statistics (GWS) (Hogben et al. [6]) overestimate the measured steepness of extreme sea states and that there is need for further investigation of the accuracy of data obtained from GWS.

2 Joint probability distribution of significant wave heights and mean zero crossing periods

The joint probability density function of significant wave heights and mean zero crossing periods $f(H_s, T_Z)$ may be represented in the following way:

$$f(H_s, T_Z) = f(H_s) \cdot f(T_Z | H_s)$$  \hspace{1cm} (1)

where $f(H_s)$ is the marginal probability density function of significant wave heights, while $f(T_Z | H_s)$ denotes conditional probability density of mean zero crossing periods.


2.1 Marginal distribution of significant wave heights

It was found that the empirical data (wave scatter diagrams) are best approximated if the Weibull 3-parameter distribution is selected as marginal distribution of significant wave heights:

\[
F(H_S) = P(\hat{H}_S \leq H_S) = 1 - e^{-\left(\frac{H_S - \varepsilon}{\theta}\right)^\alpha}, \quad \hat{H}_S, H_S \geq \varepsilon
\]  

(2)

In the above equation, \( \hat{H}_S \) is the random variable significant wave height, while \( H_S \) is the actual value that random variable may take on. Parameter \( \varepsilon \) in the eqn (2) is known as the location parameter, \( \theta \) is the scale parameter and \( \alpha \) is the shape parameter of the Weibull distribution. The location parameter \( \varepsilon \) has important physical interpretation as small significant wave height that is always present representing permanent activity of sea waves. The procedure for approximation of empirical data by Weibull 3-parameter distribution is shown in numerous literature (e.g. Mansour and Preston [7]). For the case of the SHIPREL scatter diagram, the following parameters of the distribution are obtained: \( \varepsilon = 0.9, \theta = 2.817, \alpha = 1.4724 \). SHIPREL scatter diagram is recommended by some of the major Classification Societies and ship research institutions during the European research project “Reliability Methods for Ship Structural Design” [4]. The comparison for the probability density functions of empirical data and theoretical approximation is shown in Figure 1. As may be seen from Figure 1, good agreement is achieved between empirical data and theoretical approximation. Using Weibull distribution, extreme significant wave heights for different return periods are calculated and presented in Table 1.

![Figure 1: Comparison of Weibull probability density function and empirical significant wave heights](image-url)
Table 1: The most probable extreme significant wave heights for different return periods

<table>
<thead>
<tr>
<th>PR, years</th>
<th>1</th>
<th>5</th>
<th>10</th>
<th>20</th>
<th>50</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hs^{PR}, m</td>
<td>11.1</td>
<td>12.7</td>
<td>13.3</td>
<td>14.0</td>
<td>14.8</td>
<td>15.4</td>
</tr>
</tbody>
</table>

2.2 Conditional distribution of mean zero crossing periods

Log-normal distribution is chosen as conditional distribution of mean zero crossing periods. The distribution function is given by the following expression:

\[
F(T_Z|H_s) = \Phi \left( \frac{\ln T_Z - \mu_{\ln T_Z}}{\sigma_{\ln T_z}} \right)
\]

where \( \Phi \) is the standard Gaussian distribution with zero mean and unit variance, while \( \mu_{\ln T_Z} \) and \( \sigma_{\ln T_z} \) represent mean and standard deviation of logarithms of mean zero crossing periods, respectively. The comparison of empirical data and theoretical approximation for conditional distribution of mean zero crossing periods for significant wave height \( H_s=4.5\text{m} \) is presented in Figure 2. As may be seen from Figure 2, the log-normal distribution approximates quite satisfactorily the empirical mean wave zero crossing periods. The dependencies of mean values and standard deviations of mean zero crossing period on significant wave height are shown in Figure 3. Dependencies are presented in co-ordinate systems \((\ln H_s, \mu_{T_Z})\) and \((\ln H_s, \sigma_{T_Z})\) where these may be rather reliably approximated by straight lines. The straight lines from Figure 3 are applicable for estimation of mean zero crossing periods of extreme sea states that are not covered by wave scatter diagrams. By extrapolating straight lines from Figure 3, mean zero crossing periods of very severe sea states, with significant wave height higher than 12\text{m}, may be calculated. Thus, for significant wave height \( H_s=14\text{m} \), the calculated mean zero crossing period reads 10.25\text{s}.

The wave steepness of random sea states is defined as:

\[
s = \frac{H_s}{\lambda_Z} = 2\pi \frac{H_s}{g T_z^2}
\]

where \( \lambda_Z \) is the wave length associated to mean zero crossing period \( T_Z \), while \( g \) represents gravity constant. When the wave steepness exceeds a certain value, known as limiting wave steepness, wave breaking occurs and such sea states are not physically sustainable. Inserting \( H_s=14\text{m} \) and \( T_Z=10.25\text{s} \) in eqn (4), one obtains the wave steepness of 20-year sea state following from GWS for the North Atlantic region as \( s=1/11.7 \).
Figure 2: Comparison of log-normal distribution and empirical mean zero crossing periods

Figure 3: The mean values and standard deviations of mean zero crossing periods as functions of significant wave heights

3 Comparison with the observed extreme sea states and with other similar analyses

As may be seen from the presented analysis, the extrapolation procedure is used for the assessments of extreme significant wave height and associated wave steepness. It is a rather important and difficult question whether these extrapolations lead to physically acceptable results. To answer this question, one needs to apply subjective engineering judgement as well as to study results of
other researches and literature describing the observed and measured extreme sea states.

Sea states with significant wave heights of 14 meters are rather often encountered by sea-going merchant ships. Faulkner [3] presented evidences that bulk-carrier "Derbyshire" was lost in 1980 in sea state with significant wave height of 14m. In the same article, the following expression was recommended to calculate the significant wave height for design of ship structures in extreme conditions:

\[ H_s = 15 - (3 - L / 100)^{2.5} \]

where \( L \) is the ship length. Thus, for a ship \( L=200 \text{m} \) in length, the appropriate significant wave height from eqn (5) reads 14m. Bitner-Gregersen et al. [8] have analysed extreme significant wave heights for all wave zones in GWS for return period of 20 years. For zones Nos. 8, 9, 11, 15, 16 and 17 that are combined into SHIPREL scatter diagram, the maximum and minimum calculated significant wave heights read 15.95m (zone 16) and 14.15m (zone 11), respectively. The average value for all zones reads 15.16m, being higher than the result from the present study. This discrepancy is the consequence of different assumptions regarding the recording interval, as explained by Parunov [9]. Mansour [10] in ship reliability studies uses significant wave height of 13.7m. This value is the result of the analysis of wave data from GWS taking into account statistical correlation between significant wave heights in different wave zones [7]. From the presented literature review, conclusion may be drawn that significant wave height of 14m could be appropriate value for ship safety assessment in extreme conditions. It should be mentioned that for the design of offshore structures even higher significant wave heights may be used, depending on the location of the offshore field. Thus, for FPSO unit operating west of Shetland, significant wave height of 18m is used (Faulkner [3]). Ochi [11] has calculated significant wave height of even 20m for the return period of 20 years in the North Atlantic. Greater significant wave heights obtained for the design of offshore structures than for ship design are not surprising, since merchant ships are able to avoid the most severe sea states.

Bitner-Gregersen et al. [8] have analysed the limiting wave steepness of extreme sea states. They have compared measurements and theoretical estimates for steepnesses of very severe sea states. They recommended \( s=1/15 \) as limiting wave steepness appropriate for the analysis of ship structures. Bitner-Gregersen et al. [8] have found that the data from GWS largely overestimate the recommended wave steepness. The same conclusion was emphasised in the report of ISSC'2000. As shown in the previous section, results following from the present study are in accordance with these references. Steepnesses higher than \( s=1/15 \) are also possible, but since their frequency of occurrence is rather low, they may be appropriate for the analysis of offshore structures. According to Faulkner [3] the limiting wave steepness for ocean waves reads \( s=1/10 \). Such steep waves may occur during hurricanes and typhoons or in severe storms near the seashore i.e. when fetch is limited. From the other side, the wave steepness of fully developed sea state, described by original one-parameter Pierson-Moskowitz (P-M) spectrum reads \( s=1/19.6 \). This spectrum was frequently used
in past decades for the analysis of ship structures, despite its very low steepness. Various assumptions about steepness of extreme sea states are summarised in Figure 4. This Figure represents mean zero crossing period as function of significant wave height using:

- one parameter P-M spectrum \((s=1/19.6)\),
- recommended wave steepness \((s=1/15)\),
- analysis from global wave statistics (mean value in Figure 3),
- maximum sustainable wave steepness \((s=1/10)\).

![Figure 4: Dependence of mean zero crossing period \(T_Z\) on significant wave height \(H_s\) for different assumption about wave steepness](image)

From the analysis of the results presented in Figure 4, some interesting conclusions may be drawn:

- the data from GWS result in wave steepness of extreme sea states close to the maximum sustainable wave steepness,
- the difference between mean zero crossing periods for \(H_s=14\text{m}\) reads almost 3.5 seconds between P-M spectrum and GWS,
- recommended wave steepness for analysis of ships \(s=1/15\) gives wave period that is approximately mean value between GWS and P-M spectrum for significant wave heights greater than 13m.

It may be concluded from the presented analysis that the data from GWS overestimate the actual wave steepness of extreme sea states. The reason for overestimation could be that the extrapolation as the engineering tool is not suitable for calculation of extreme wave steepness.

### 4 Consequences on lifetime vertical wave bending moments

The consequence that different assumptions of wave steepness may have on lifetime vertical bending moments could be analysed using spectral curves. Spectral curves, also known as short-term response curves, represent double amplitude of the response calculated for range of sea spectra described by unit
significant wave height and different mean zero crossing periods. Spectral curves may be used for fairly reliable estimation of extreme short-term response. At the same time, these curves give very useful information about the sensitivity of extreme values on mean zero crossing periods. Spectral curves for the example vessel, which is a large containership, are shown in Figure 5. Spectral curves shown in Figure 5 are for two different forward speeds, of 4 knots and 24.5 knots, corresponding to Froud numbers of 0.04 and 0.24, respectively. Only head seas are assumed in calculations.

Spectral curves in Figure 5 are based on transfer functions calculated using linear strip theory and two-parameter P-M spectra with unit significant wave height and range of zero crossing periods from 5s to 15s with increment of 0.5s. It may be seen from Figure 5 that the zero mean-crossing period in the range of 10s – 14s may have quite important influence on extreme values. In this particular example, it is obvious that the mean zero crossing period based on data from GWS overestimate wave bending moments calculated for more realistic wave steepness s=1/15. If the original one-parameter P-M spectrum was used, the wave bending moment would be further reduced.

The effect of overestimation of realistic vertical wave bending moments would also affect long-term analyses based on the lifetime weighted sea method. Comparison of short-term and long-term predictions, for the case of head seas, is given in Table 2. Details of calculations are given by Parunov [9]. For the long-term prediction, SHIPREL wave scatter diagram is used. For the short-term
prediction, design sea state calculated in the previous section, described by \( H_s=14 \text{m} \) and \( T_z=10.25 \text{s} \), with duration of 3 hours, is used.

Table 2: The most probable extreme wave bending moment in 20 years, MNm

<table>
<thead>
<tr>
<th>Prediction method</th>
<th>Ship speed</th>
<th>4 knots</th>
<th>24.5 knots</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short-term</td>
<td>4743</td>
<td>5438</td>
<td></td>
</tr>
<tr>
<td>Long-term</td>
<td>4754</td>
<td>5411</td>
<td></td>
</tr>
</tbody>
</table>

It should be noted that results in Table 2 refer to linear part of wave bending moment. Non-linear contributions are outside the scope of the present paper. It may be seen from the results in Table 2 that short- and long-term prediction methods are in excellent agreement. If one takes into account more realistic steepness \( s=1/15 \), then the mean zero crossing period reads \( T_z=12 \text{s} \). In that case, the most probable extreme vertical wave bending moments in short-term sea state read 4474MNm and 5041MNm, for ship speeds of 4 knots and 24.5 knots, respectively. Therefore, the reduction of extreme wave bending moments due to the unrealistic wave steepness may be estimated to 6%. This reduction is valid only for this particular example. Generally, it has to be calculated on the case-by-case basis, according to the described procedure.

The described analysis is more accurate for smaller ships (less than 200m in length), than for the larger vessels (more than 200m in length). It is well known that the shipmasters of smaller ships would turn the ship in head seas in extreme sea states in order to avoid excessive rolling motion. For larger ships this would not be necessarily the truth. The shipmasters of very large ships feel safe, and they would not always turn the vessel in head seas, even in the most severe sea conditions. For such vessels, it would be reasonable to consider all headings as equally probable in the long-term analysis. In that case, the most probable wave bending moments of the example container ship read 4365MNm and 4777MNm for ship speeds of 4 knots and 24.5 knots, respectively. These values are lower than those in Table 2, since the ship would spend shorter time in head seas condition, where the wave bending moments achieve the highest level. Calculated values should be further reduced due to the limited wave steepness. It would be very difficult to assess exactly this reduction. However, since the greatest contribution to total vertical wave bending moment comes from head seas, it is reasonable to assume that the reduction would be the same as for the head seas. Therefore, in this case, 6% reduction of the most probable extreme vertical wave bending moment should be applied.

5 Conclusion

This paper has analysed the influence of the wave steepness on the extreme vertical wave bending moments. For that purpose, the wave steepness of extreme sea states has been studied. Joint probability distribution of significant wave heights and mean zero crossing periods for the North Atlantic has been defined. Data from the wave atlas Global Wave Statistics have been used. After that,
using extrapolation procedure, the most probable extreme sea state for different return periods, have been calculated. The sea state with return period of 20 years may be described by significant wave height $H_s=14\text{m}$ and mean zero crossing period $T_z=10.25\text{s}$. Comparison with other similar researches and with measured extreme sea states shows that the calculated significant wave height $H_s=14\text{m}$ may be appropriate for the safety analysis of large ocean-going merchant ships in extreme conditions. However, the calculated mean zero crossing period is too low, i.e. the calculated steepness is rather high and seems to be more appropriate for the analysis of the offshore structures than of the ocean-going ships. The more suitable mean zero crossing period, giving closer agreement with measured extreme sea states, reads $T_z=12\text{s}$.

The influence of the wave steepness overestimation on the lifetime vertical wave bending moments is assessed. It is presented by one example that the calculated extreme wave bending moment due to this phenomenon exceeds the more realistic value by 6%. However, this exceedance should be calculated on case-by-case basis, using spectral curves and procedure described in this paper.

It has been shown recently in numerous references that the theoretical predictions of lifetime vertical wave bending moments exceed the measured extreme values as well as wave bending moments specified in ship rules. The findings presented in this paper may be considered as one of the reasons for these overestimates.

References