Some consequences for marine simulators of statistical coherence within the phase spectrum of sea waves

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Abstract

Statistical dependence within the phase spectrum of simulated swell seas are examined in terms its effect upon the precision of short term deterministic wave prediction models. Numerical experiments show that such local coherence increases the prediction accuracy. Analytic models are developed of the prediction which explain the sensitivity of the prediction model parameters when neighbouring phase values are uncorrelated. The possibility of some degree of coherence in certain types of sea-wave are considered and sectional phase auto-correlations are presented which demonstrate such coherence in real sea data.

1 Introduction

The huge complexity of real ocean basins has meant that the stochastic view of sea-waves developed by Pierson and Neumann, [1,2,3], has provided the framework around which wave forecasting has developed. The estimation periods involved typically range from hours to days and reflect the operational planning needs of marine activities. Consequently the sea descriptions used in the design of vessels and marine simulators have been based around this view of the sea surface. In contrast to be of value in controlling real time dynamical vessel operations it is necessary to consider deterministic rather than statistical issues, [4]. The potential advantages offered by technologies able to provide predictive wave input in such real time operations, [5], have aroused interest in so called deterministic sea-wave prediction (DSWP), [6-11], and the use of such predictions in motion prediction, [4]. As the exploration of this type of technology is critically dependent upon software and wave-tank based simulations it is important in developing such simulations to be aware that certain issues neglected in statistical sea descriptions may be relevant in
deterministic work. The possibility of statistical coherence is a prime example.

The basis of DSWP is to measure the sea surface profile in a region some distance from the prediction site and use this data to build a short term prediction model to estimate the future sea profile. Estimated prediction times, $t_p$, are of the order of a few tens of seconds, [6,8], with the level of error depending upon sea type and trade offs between $t_p$, accuracy and system complexity, [7].

Under a given set of conditions for a DSWP/vessel motion prediction system there is a maximum possible value of $t_p$. From this must be deducted the sea surface measurement time, $t_m$, and the data quality assessment plus model building time $t_d$. Setting aside for the moment the issues of fundamental feasibility and difficulty, it is clearly much quicker to build a linear descriptor than its non-linear counterpart. This is well illustrated by the iterative schemes employed to separate free and bound wave systems for even a small number of components, [10,11]. Thus exploration of those circumstances which allow the use of linear, superposable, wave and vessel models is thus central to the use of DSWP/Vessel motion prediction. As will be shown, this issue goes much further than simple wave slope constraints. Of considerable importance in this respect is that many operations in the offshore oil and gas and naval sectors that will benefit from DSWP, [5], cannot take place when local wind strengths sufficient for wind waves to affect operations. Cargo lifts onto platforms, single point buoy and submerged turret based oil transfers and aircraft launch/recover are typical examples. In contrast medium and large swells can occur which restrict such operations under otherwise benign conditions. Thus it is this type of sea that is of practical importance for many DSWP/vessel motion applications.

1.1 Wave descriptions

The fundamentally non-linear nature of gravity waves has meant they have attracted considerable attention over a long period of time. The classical work prior 1960, is well described in Milne-Thomson, [12]. The key issues of directional wave-wave interaction, including multiple wave resonance and the weak growth effects were examined by Longuet-Higgins, by Lighthill and by Phillips, [13,14,15]. A major outcome of the huge body of work on gravity waves is that while the long wavelength multi-directional swells undergo intermodal energy transfer over many tens of dominant wavelengths over short distances their evolution is well approximated by linear theory. Consequently from this perspective linear wave models for DSWP, [6,8], is justifiable, for the type of seas described in section 1.0 as being important in the offshore industry and naval applications, provided the distance between the measurement region and the prediction site is only a few dominant wavelengths.

2 Statistical coherence

There is clearly a gulf between the deterministic behavior of gravity waves and the stochastic description, [1,2,3], required for long term sea state estimation, enforced by the complexity of the wave field across a real ocean basin. It would
appear that this gulf can be bridged by ignoring how the sea surface profile arose and simply measuring it, using the data as input to a local linear prediction model. This was the assumption made in early work on DSWP, [6], which employed the parsimonious sea models, [16], typically used in ship design. However there is an additional issue that has traditionally been largely ignored, not just in DSWP, but in almost all sea-wave work. This is statistical dependence.

The conventional approach in marine design or simulation work, either numerical or tank testing, has been to generate seas by combining a number of statistically independent trigonometric components. In such studies the magnitudes are sampled using a Rayleigh distribution from an appropriate spectrum and the phases are given a uniform random distribution over \(-\pi \rightarrow \pi\), [17]. Assuming that this were true for all seas, then one would expect that the auto-correlation of the phase spectra would show no statistically significantly difference from zero, except at zero correlation delay. Obtaining time series data for the swell seas of interest with which to test this has become difficult for a variety of reasons, however a accessible archive does exist for the WACSIS project, [18,19]. Given that much of the data available is for very short highly cluttered sea one might not expect to see statistical coherence present, however as figure 1 demonstrates the central portion of the phase spectrum contains highly correlated data.

Thus even for highly chaotic conditions the assumption of statistically independent trigonometric components does not hold. In order to help understand the nature of the statistical dependence in parts of the phase spectrum, simulations were employed to generate auto-correlations of the same form as that from band B. The desired structure can be produced using standard stochastic techniques, [22]. The auto-correlations of the type shown in band B of figure 1 derive from data containing both uncorrelated and coherent components. The presence of incoherent data is evidenced by the spike at zero delay while the coherent data leads to the decaying oscillatory feature. The coherent part can be formed by applying a moving average process to uncorrelated data. Figure 2a shows the auto-correlation function generated by such a technique. The moving average process extends over 100 points in the present example. The auto-correlation of purely uncorrelated data is shown in figure 2b.

2.1 The source of phase averaging

The simple moving averaging algorithm used to transform an uncorrelated data set into one with an the auto-correlation function figure 2a is a special case of a linear combination, i.e., a convolution operation. Recalling that it is phase spectra that are being correlated it should be clear that results such as those for band B in figure 1 cannot arise from purely linear operations on the wave data from which the phase spectrum was derived. Such linear processes can at most induce additive phase shifts, only nonlinear mechanisms can combine the phases, as is readily appreciated simply by considering the inter modulation processes that are produced by nonlinearities.
A simple model with which to illustrate this type of mechanism is the following iteration scheme: at the \(i_{th}\) step a weakly non-linear phase shifting spectral operator, \(\Gamma(\omega)\), maps a spectrum \(\Theta_i(\omega)\) according to:

\[
\Theta_{i+1}(\omega) = \alpha e^{-j\phi(\omega)}\Theta_i(\omega) + \beta e^{-j\phi(\omega)}\left\{\Theta_i(\omega) * \Theta_i(\omega)\right\}
\]  

(1)

Where \(\phi(\omega)\) is the phase shift induced, \(\alpha\) and \(\beta\) are constants and the operator \(*\) denotes convolution. After \(N\) iterations of \(\Gamma(\omega)\) there will be phase factors present containing linear combinations of up to \(N\) of the original phases, \(\arg\{\Theta_i(\omega)\}\), which were input to the system at the first iteration. In addition, linear phase shifts and re-scaling effects are also present.

In propagating waves such weak nonlinear effects will cumulatively intermix the phase terms, steadily increasing statistical dependence as suggested above. In contrast extreme processes, such as wave breaking, will tend to remove such coherence. Thus in chaotic seas as in the example quoted from WACSIS there would be varying degrees of coherence present at different times and over different frequency ranges as is observed in practice. When the sea is the combination of medium/large swells generated by a modest number of remote storms even stronger coherence would be expected and the acquisition of comprehensive sets of such data to examine this process is an important goal.

3 The effects of statistical dependence upon DSWP

As will be demonstrated the presence of statistical dependence within a wave system has a significant effect on DSWP. Consider a typical sea model of the type used in simulators or wave tank tests. This is trigonometric model for the sea surface elevation, \(h(x,y,t)\) at the coordinates, \(x,y,t\), which consists of the superposition of \(R\) long crested waves, each comprising \(N_i\) non-zero spectral components:

\[
h(x,y,t) = R e \left[ \sum_{i=1}^{R} \sum_{n=1}^{N_i} C_{i,n} e^{j(k_{i,n}x \cos(\theta_i)+k_{i,n}y \sin(\theta_i) - \omega_{i,n}t)} \right]
\]  

(2)

The parameters for the \(i^{th}\) wave are the complex amplitude, \(C_{i,n}\) of the nth component, whose frequency and angular velocity, \(\omega_{i,n}\) and \(k_{i,n}\) are typically linked by the classical deep water dispersion relationship. Choosing either \(\omega\) or \(k\) as a reference the angular frequency or the wavenumber values should be a noninteger set ensuring the sea model is Almost Periodic, \([20,21]\), rather than strictly periodic. The parameter, \(\theta_i\), is the angle between a normal to the \(i^{th}\) wave front and a global reference co-ordinate. Equation (2) can either be viewed mathematically as a formal Fourier decomposition or more physically as the resultant of waves propagating from \(R\) separate storm systems sufficiently remote that their individual contributing wave-fronts are locally straight at the co-ordinates of interest.

The DSWP model building process consists of using sea surface measurements to generate a Fourier Series (strictly periodic) description of the
sea over the spatial or temporal interval from which the measurements were made. Prediction is then achieved by phase shifting the Fourier Series terms in space and or time. The success of this process depends upon how well the calculated coefficients match those in equation 2. To remove algebraic clutter the situation is restricted to the case of a single remote storm, i.e. \( R = 1 \). Under such circumstances using the Fixed Point DSWP mode \([7,8,9]\), the Fourier Series estimators, \( A_m \) for the \( C_n \), can readily be shown, \([9]\), to have the form:

\[
A_m = \sum_{n=-\infty}^{\infty} C_n \frac{\sin(T\omega_n - \pi m)}{(T\omega_n - \pi m)}
\]  

where \( T \) is the measurement interval. Equation 2 shows that a cluster of values over a region of the order of a few times harmonics wide are the main contributors to \( A_m \). Now if the \( C_n \) are uncorrelated random variates then \( A_m \) provides a very poor estimator of the almost periodic function in equation 1. Conversely if the \( C_n \) are locally correlated and reasonably smooth on a local scale then the \( A_m \) constitute good estimators for the \( C_n \).

To examine this effect prediction simulations were performed using a sea generator model of the type indicated in equation 2 in which various degrees of statistical dependence were incorporated into the phases of the \( C_n \). This coherence was introduced by starting with uncorrelated data sets which were filtered with various bandwidth filters. Histograms of the errors are shown in figure 3. These histograms demonstrate the consequences of arguments made in connection with equation 2.

4 Conclusion and consequences for simulators

The issue of statistical dependence within sea wave data has been identified as a rather unexpected issue of some importance in DSWP applications and consequently in simulation tools used in its development. The conventional assumption that spectral coefficients in sea descriptions are uncorrelated has been shown not to apply in a highly chaotic example. It is shown that the form of experimentally observed phase auto-correlations is consistent with the presence of linear combinations of phase values over an extended spectral range. This situation matches sensibly with the results of the anticipated type of non-linear mechanisms. The reasons for statistical coherence affecting DSWP predictions have been considered and illustrative examples of the relationship between DSWP errors and the degree of statistical coherence have been presented. The natural approach to exploring the potential of a DSWP system in a given application is via simulations using the range of anticipated sea conditions. The differences between simulations employing statistically independent seas and those incorporating coherence as appropriate will have important consequences in assessing the economic potential offered by deterministic technologies.
Figure 1: Sectional auto-correlations taken over the phase spectrum of wave data[18,19]. The correlations are determined over the bands A, B and C, as indicated. The measurements are taken from the Wave Crest Sensor Intercomparison Study, with acknowledgement to Shell Global Solutions International, 2001. The data set designation is: EMILASER FSAMP2 Complementary 9804011600.
Figure 2a. Auto-Correlation generated by taking a 100 point moving averaging

Figure 2b: Auto-correlation of the uncorrelated data used in the generation of figure 2a. The peak at zero shift is artificially widened to distinguish it from the vertical axis.
References


