Mobile and Rapidly Assembled Structures III, C.A. Brebbia & F.P. Escrig (Editors)
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# A new type of hardening structure

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## Abstract

This paper presents a new type of structure whose original geometry changes under external load in such a way as to provide an elastic increasing resistance to the applied load - a hardening system.
The structure is composed mainly of "basic units" each of which consists of a number of bars and a cable.
The bars are connected to one another by pinned joints, though they are not necessarily triangulated. The cable, common to all the units, is attached to the end supports and extends continuously over some of the internal joints.
Changes in the original form of the structure under external load is mainly due to displacement of these joints along the cable. Those changes originate an internal tensioning force which acts on the structure as a partial counterbalance to the effects of the external loads.
The general configuration of the structure resembles that of a truss.

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# 1 Introduction

This paper presents a particular type of structure; its main characteristic is that the initial geometry changes under external load in such a way as to provide an increasing elastic resistance to an increasing applied load; the type of a hardening system. In Figure 1 an example of this type of structure.

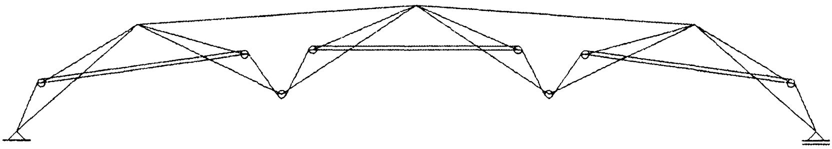

Figure 1

In Figure 2 the structure of Figure 1 was unfolded to show its main components: 2a, 2b, represents two sets of ordinary, articulated bars that when superimposed one into the other produces the original structure of Figure 1.

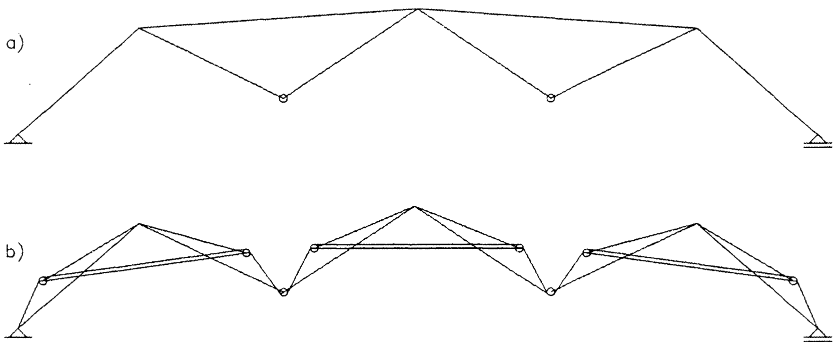


Figure 2

In Figure 3 the set of bars from Figure 2b where schematically represented by dotted lines; with this schematic representation the structure resembles a truss; the reasons for this representation will be come apparent furtheron.

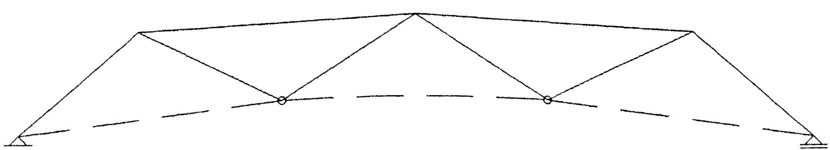

Figure 3

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## 2 A spring structure

The structure from Figure 2b behaves under load like a chain of springs. Such a structure will be referred to further on as the spring structure. This spring structure consists of a number of bars and a cable. The bars are connected to one another by pinned joints, though they are not necessarily triangulated. The cable is attached to the end supports and is passed over continuously over some of the internal joints. Changes in the original form of the structure under external load, is mainly due to displacement of these joints along the cable.
The basic spring structure Figure 4a., is composed of two identical basic units: ABCDE and CFGHI Figures 4b, 4c.
The basic unit is symmetrical and consists of two separate bars AB and BC, Figure 4d, with articulated connections, a triangle DBE hinged at B, Figure 4e and a cable ADEC, Figure 4b.
The basic spring structure has two fixed supports A, G and a sliding support at joint C which can move on the direction AG only, Figure 4a.
A continuous cable attached to supports A, G, passes over joints D, E, C, H, I, so as to allow displacement of these joints along the cable, Figure 4a.

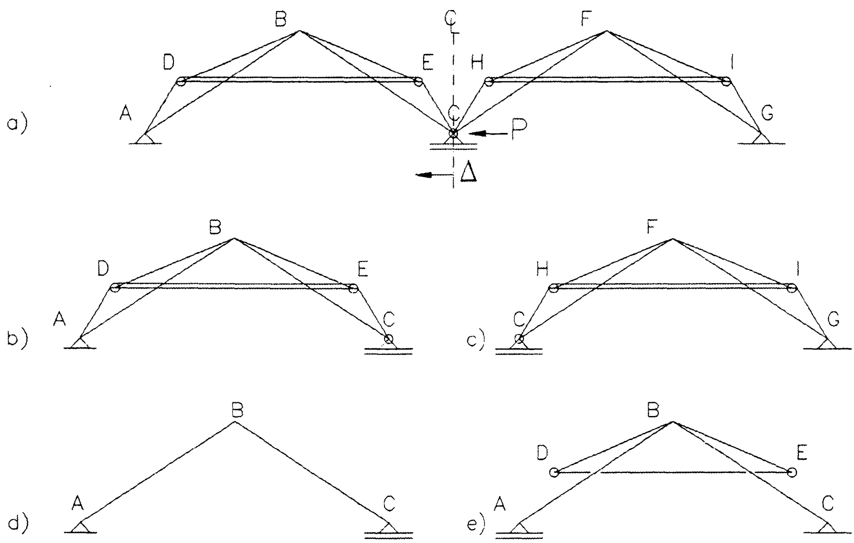


Figure 4

Any displacement Δ, see Figure 4a, of sliding support C under an external horizontal load P, will cause a change ΔL of the original length L of the cable; it is possible to assure ΔL > 0 for a geometry ABC by an appropriate selection of triangle BDE; in this case the basic spring structure resists the action of P and the movement of support C will stop at the point of equilibrium.

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The mathematical relation P(Δ) between the external load P and the displacement Δ is a continuous, differentiable and non-linear function of Δ for a given system (the system is characterized by the length and cross-section of the bars and the cable, the angle between bars and the modulus of elasticity of the material).

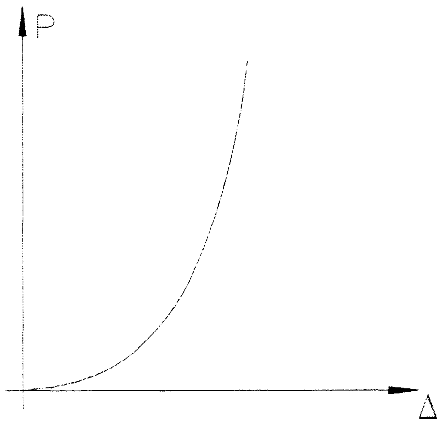


Figure 5 "load-deformation" curve for the basic spring structure, typical of a hardening system.

Figure 6 shows the equilibrium position of the basic spring structure. Displacement of joint C will stop when the two initial identical basic units will differ in their respective geometries so as to satisfy the equilibrium conditions for a tensioning force all along the cable.

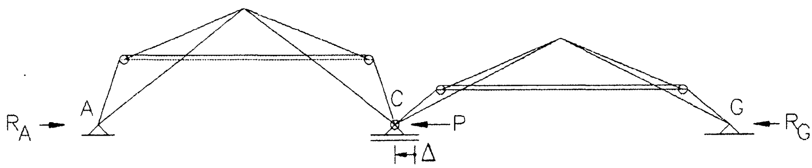


Figure 6

It can be proved that the external reactions $R_A$, $R_G$, in the fixed supports A, G, will act as indicated in Figure 6 and $R_A > P$.
The displacement Δ of joint C is the result of translation of joints D, E, C, H, I, along the cable together with elastic changes in the lengths of all the basic units components. Δ is an elastic displacement, greater than the result of elastic changes in the units components only.

$$R_G = R_A - P = \text{support reaction}$$

represents an internal tensioning force the structure creates while joints translates along the cable.
Let us call $P_{SS}$ (self-stressing force) the external resultant of the internal tensioning force; then:

$$R_A = P + P_{SS} \qquad R_G = P_{SS}$$

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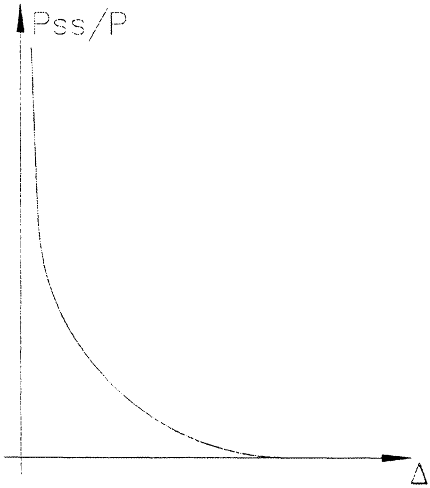


Figure 7

Figure 7 shows the relation of the self-stressing force Pss and the external load P as a function of displacement Δ.

# 3 Imaginary compression bar - An auxiliary concept.

Since the basic unit is designed to react only to a force P, acting in the direction AC Figure 8a., it can be represented - schematically - by an imaginary compression bar AC connecting its supports, Figure 8b.

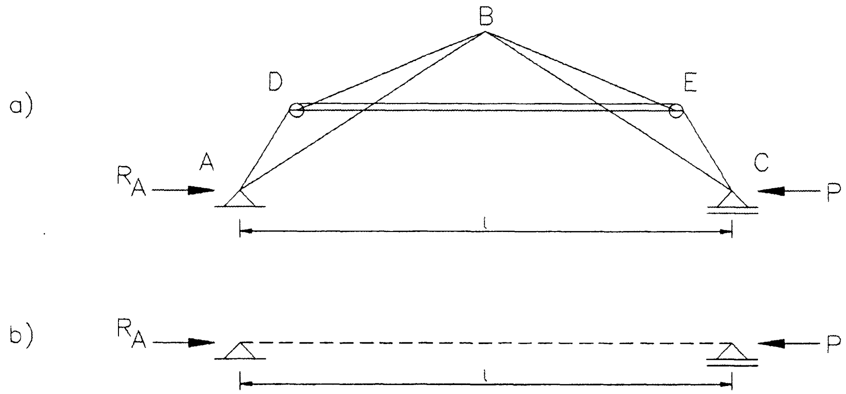


Figure 8

The basic spring structure can be represented by two compression bars, bar 1 and bar 2, Figure 9.
Figure 9b represents the initial state, before loading. Figure 9c represents the final state, under action of external load P.

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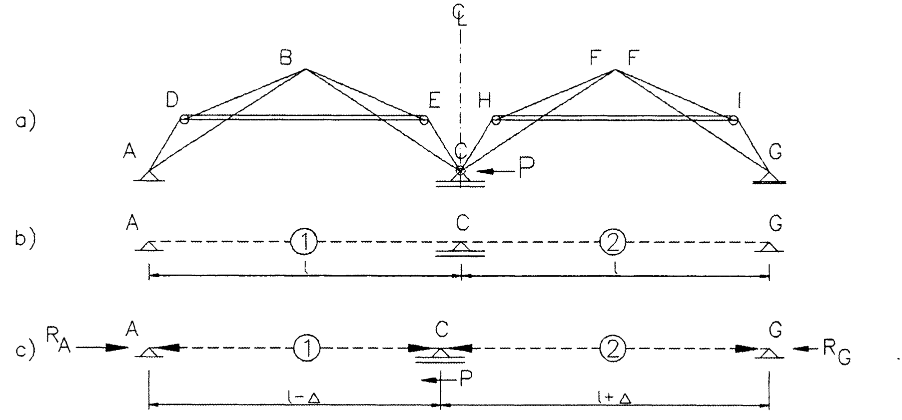


Figure 9

It should be pointed out that both imaginary bars, are stressed in compression after displacement Δ; see acting directions of external reactions $R_A$, $R_G$ Figure 9c.
It can be proved that the axis of symmetry of the hinged triangles BDE and FHI Figure 9a will continuously coincide with the axes of symmetry of the basic units ABC, CFG.

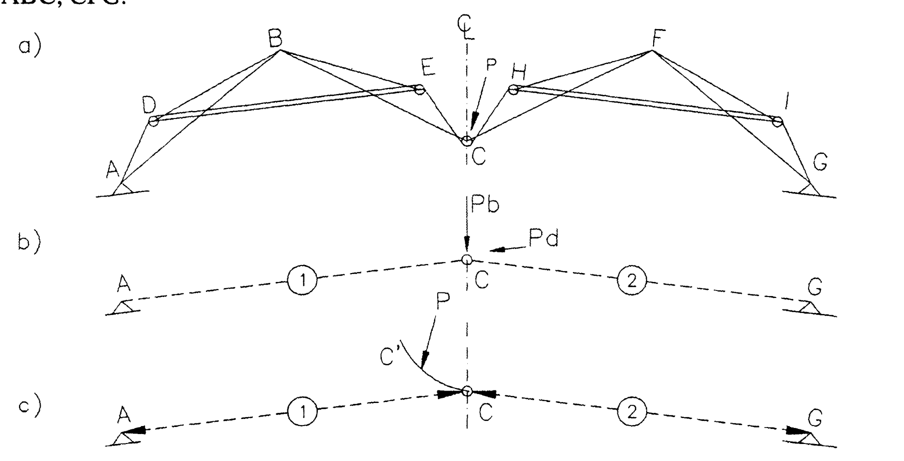


Figure 10

In Figure "10a", two basic units are not aligned; for weightless components the structure is in a static, but labile situation.
An external load P, Figure 10a, represented by two components $P_b$ (in the bisector of the two basic units) and $P_d$ (in the direction of one of the basic units), Figure 10b, will cause joint C to move along a pre-determined path, Figure 10c, until the internal tensioning force created by the spring structure disappears; the structure arrives to C, again a static labile situation.

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# 4 Spring frame structure

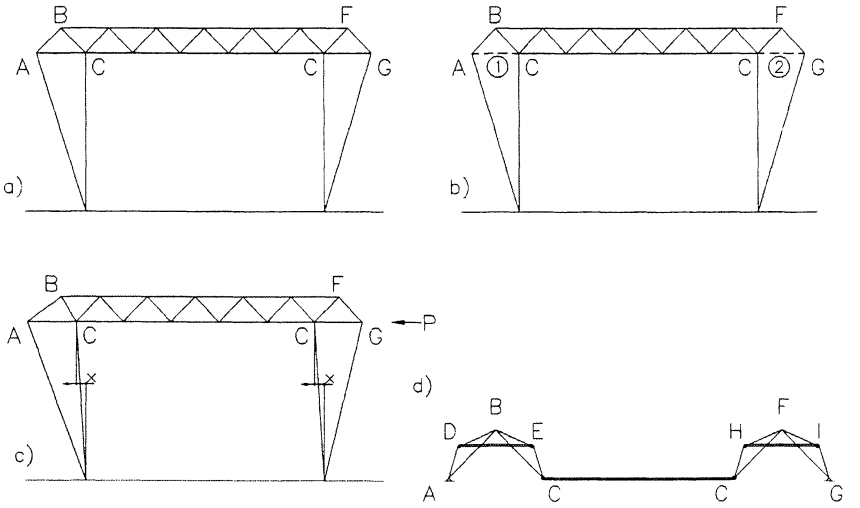


Figure 11

Figure 11a shows an ordinary frame structure.
Figure 11b shows the spring frame structure with two basic units represented by imaginary compression bars (numbered 1 and 2); joint C is noted twice to indicate that it is the sliding joint common to both basic units with one continuous common cable attached to A, G, Figure 11d.
Figure 11c, the equilibrium position of the spring frame structure under external load P.
The deformation "x" of the spring frame structure under external load P, for instance, is:

$$x = x\ (s) + x\ (\Delta)$$

x(s) = the deformation as a result of elastic changes in the length of the bars.
x ($\Delta$) = the deformation as a result of the displacement $\Delta$ of joint C along the cable.
Hence the "load-deformation" curve for the spring frame structure is similar to that of the basic spring structure, see Figure 5.
Figure 12 shows, for comparison, a number of "load-deformation" curves:
a = "load-deformation" curve for a structure with linear response.
b= "load-deformation"curve for a straight wire supported between two fix points, where the load is applied perpendicularly to the wire, at midway between supports.
c = "load-deformation" curve for a structure with a bilinear hardening response.
d = "load-deformation" curve for a "hardening spring structure".

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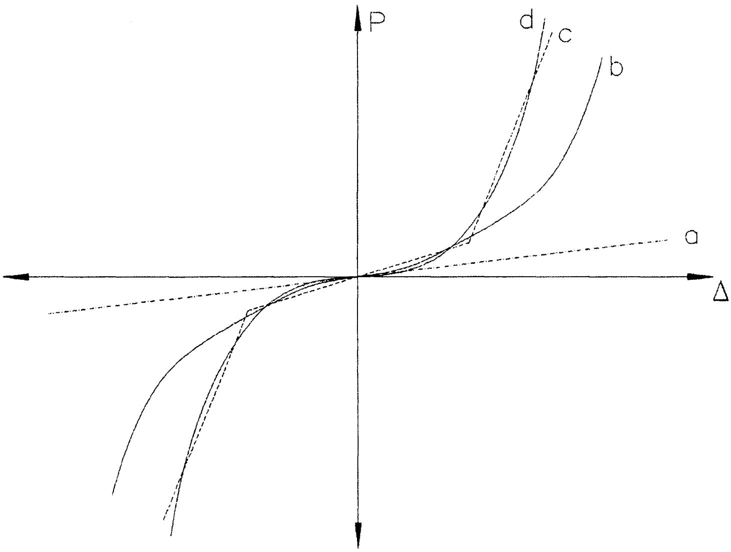


Figure 12

From experiments that have been done with "hardening systems" under earthquakes and by comparison of the "load-deformation" curves in Figure 12, it can be expected a very efficient behavior of the spring structure under earthquakes.

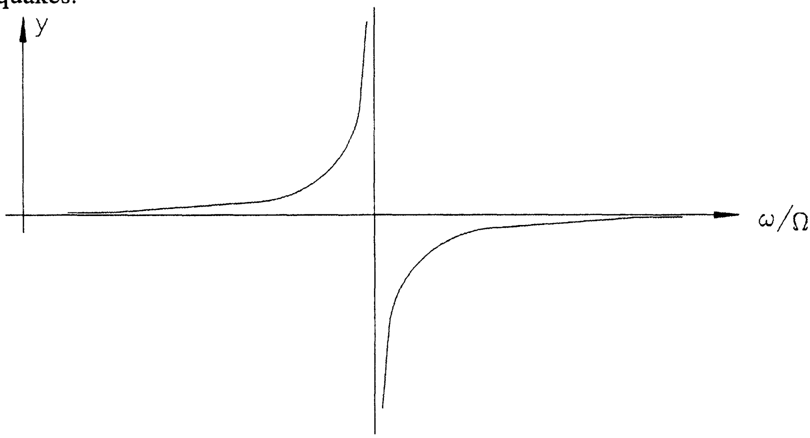


Figure 13 The response curve of the spring structure, a "jump and drop function"

## 5 A self adaptable structure

The structure of Figure 14a changes the internal lever arm under an external load. The lever arm increases where the moments induced by the load increases.

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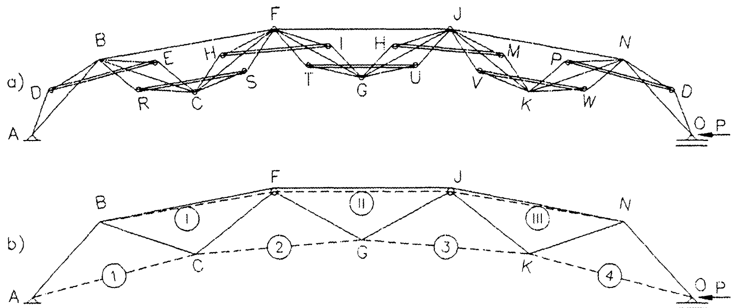


Figure 14

In Figure 14b a schematic representation.
The structure is composed of:

- a continuous cable BFJN, attached to joints BN; joints F, J can move along that cable,
- four basic units, 1,2,3,4 with a continuous cable A,D,E,C,H,I,G,L,M,K,P,Q,O, attached to joints A,O ; the internal joints can move along that cable,
- three basic units, I, II, III, with a continuous cable B,R,S,F,T,U,J,V,W,N, attached to joints B,N ; the internal joints can move along that cable.

We assume weightless components and friction free movable joints.
An external load P acts at joint O, moveable support, in the direction A,O; at A the structure has a fixed articulated support.
If the movable joints of the structure are locked to prevent their displacement along the cable, it is clear that bar FJ will be in tension and those joints will have the tendency to move outwards, that is joint F towards B and joint J towards N.
If only joints F and J are locked the spring structure behaves as we already know: an internal tensioning force will be self-produced in the chain 1,2,3,4, while support O moves towards A. At the equilibrium position we can decomposed the external load P Figure 15: $P = P_1 + P_2$
$P_1$ is in equilibrium with the internal forces of an ordinary truss,
$P_2$ equilibrates the horizontal component of $P_{SS}$.
The internal tensioning force on the chain 1,2,3,4 produces a tendency of joints F,J to move inwards. This tendency dominates since the ration $P_{SS}/P \rightarrow$ oo for the initial displacements $\Delta$ of the joints; see Figure 7.

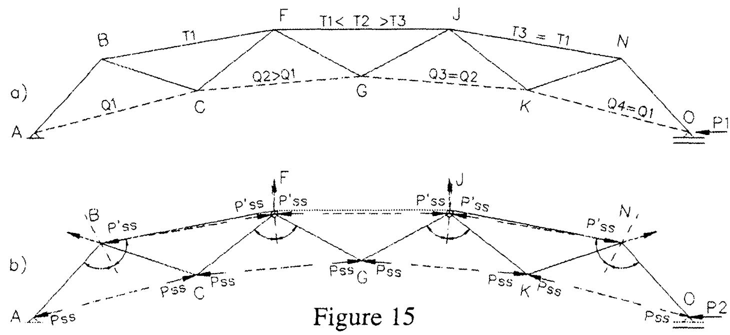


Figure 15

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Since the real structure has unlocked joints, movement of joints F and J, one toward the other, affects the chain I, II, III, and produces a second self-stressing force, PSS, which is included in the final equation of equilibrium; see Figure 15b.

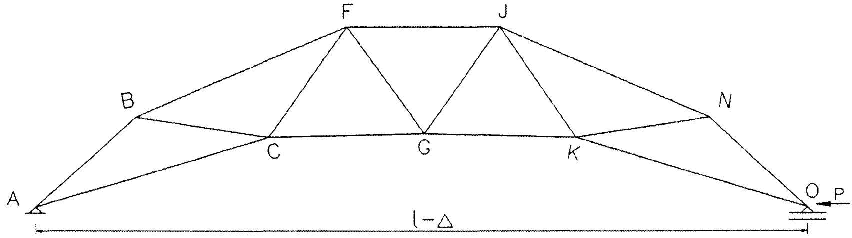


Figure 16

Figure 16 shows the self adapted geometry of the structure under load P. At position of equilibrium the internal lever arm of the structure is self adjusted to the moments induced along the structure by the external load.

## 6 Assembling - erection - demounting - applications

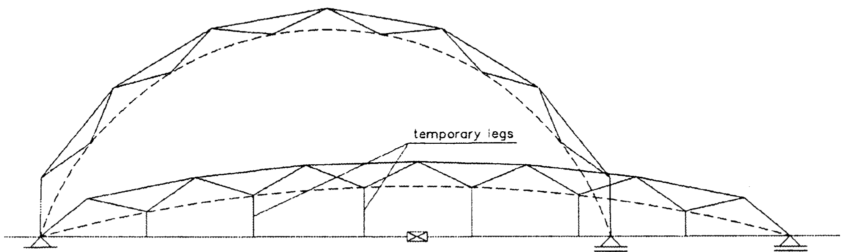


Figure 17

The structure is suitable for rapid assembling, erection and demounting with the construction technique developed by the author, O. Sircovich Saar [1]. The structure can be fully prefabricated and assembled at ground level on temporary legs, as a very flat arch.
With a provisory cable and a ratchet the flat arch is forced into the post buckling response region and rises smoothly to the designed hight.
Demounting is achieved by allowing one of the extreme supports to move outward progressively until all components reach ground level.
The structure is particularly appropriate for multiple demounting and erections; for example as in a temporary lightweight covering , multipurpose building.

## 7 Pss , self stressing force ; numerical example

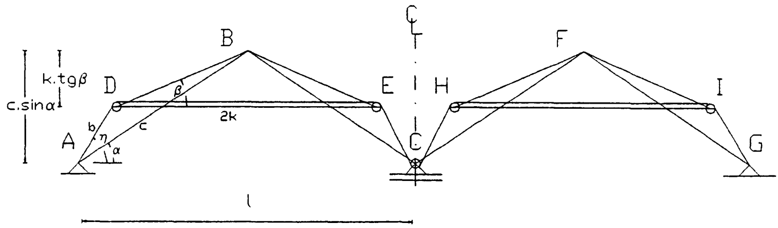


Figure 18

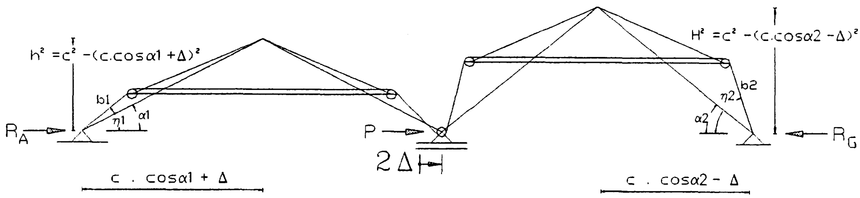


Figure 19

In Figure 18 a steel structure in its initial position.
In Figure 19 the position of equilibrium of the structure under an external horizontal load P at support C.
The structure is consider weightless, which is equivalent to $\Sigma G / P \rightarrow 0$ ;
$\Sigma G$ = structure's self weight; therefore no vertical reactions.
Let's define :

$L = 4b + 4k$ = initial length of continuous cable ADECHIG with a section Ac.
$L' = 2b_1 + 2b_2 + 4k$ = length of continuous cable at position of equilibrium.
In the following calculations the structure is considered frictionless and "small values" are omitted in order to clarify a mathematical way to calculate Pss
The omitted values are :

$$\Delta k / k \;;\quad \Delta c / c \;;\quad \Delta c / k \;;\quad \Delta k / c$$
$$\Delta c^2 \;;\quad \Delta k^2 \;;\quad \Delta c \cdot \Delta k$$
$$\Delta c / \Delta \quad ( \; L \gg b, c, \text{ or } a \;;\quad \sigma_T \gg \sigma_{b,c,k} \; )$$

where : $\sigma_T$ = tensile stress in the continuous cable
$\sigma_{b,c,k}$ = compression stresses in bars b,c,k.

$$b^2 = c^2 + k^2 - 2k \cdot c \cdot \cos\alpha - 2k \cdot c \cdot \sin\alpha \cdot \tan\beta + k^2 \cdot \tan^2\beta \tag{1}$$

$$b_1^2 = c^2 + k^2 - 2k \cdot c(\cos\alpha + \Delta/c) - 2k \cdot c[1 - (\cos\alpha + \Delta/c)^2]^{1/2} \tan\beta + k^2 \tan^2\beta \tag{2}$$

$$b_2^2 = c^2 + k^2 - 2k \cdot c(\cos\alpha - \Delta/c) - 2k \cdot c[1 - (\cos\alpha - \Delta/c)^2]^{1/2} \tan\beta + k^2 \tan^2\beta \tag{3}$$

$$(\Delta L) = 2(b_1 + b_2) - 4b \tag{4}$$

$$P = T \frac{\partial(\Delta L)}{\partial(2\Delta)} \quad , \quad (\text{ principle of virtual work }) \tag{5}$$

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$$T = \frac{A \cdot E}{L} (\Delta L) \quad , \qquad \text{( tensile force at continuous cable )} \tag{6}$$

$$\frac{\partial (b_1+b_2)}{\partial \Delta} = - k\{1 - \tan\beta[1 - (\cos\alpha + \Delta/c)^2]^{-1/2}(\cos\alpha + \Delta/c)\} / b_1 + k\{1 - \tan\beta[1 - (\cos\alpha - \Delta/c)^2]^{-1/2} (\cos\alpha + \Delta/c)\} / b_2 \tag{7}$$

$$P = \tfrac{1}{2}A \cdot E \; \frac{(b_1 + b_2 - 2b)}{K + b} \cdot \frac{\partial (b_1 + b_2)}{\partial \Delta} \tag{8}$$

Forces and reactions at supports A,G, must fulfill conditions of equilibrium $\Sigma H = 0$ , $\Sigma V = 0$ ; then

$$Pss = R_A = T \left( \frac{\sin\eta_1 \cdot \cos\alpha_1}{\sin\alpha_1} - \cos\eta_1 \right) \tag{9}$$

Table 1

| $2\,\Delta$ [cm] | 2 | 20 | 24 |
|---|---|---|---|
| b1[m] | 0.735 | 0.674 | 0.666 |
| b2[m] | 0.752 | 0.822 | 0.839 |
| $\partial$(b1+b2)/$\partial\Delta$ | 0.009 | 0.188 | 0.273 |
| T[kN] | 45.Ac | 89.Ac | 118.Ac |
| P[kN] | 0.41.Ac | 17.Ac | 32.Ac |
| Pss[kN] | 34.Ac | 52.Ac | 57.Ac |
| Pss/P | 83 | 3.1 | 1.78 |

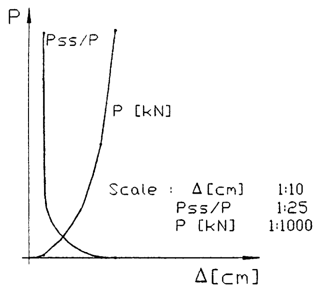

Figure 20

Table 1 and Figure 20 present results for parameters values as follows :
b = 0.739 m' ; c = 1.74 m' ; k = 1.053 m' ; l = 3.0 m'
$\alpha = 30°$ ; $\beta = 15°$

## References

[1] O.Sircovich Saar, Self Erecting Two-Layer Steel Prefab. Arch.; Proc. of the International Conf. On Space Str ; Elsevier ; Guilford ; UK ; pp. 823-827; 1984