Validation analyses of advanced turbulence model approaches for stratified two-phase flows

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Abstract

The aim of this work is to develop a new numerical model for countercurrent stratified two-phase flows with wavy interface that is able to predict time averaged flow properties, such as velocity, turbulence and void profiles. Assuming an equilibrium condition between gravitational and surface energy on one hand and turbulent kinetic energy on the other hand, a statistical model for the turbulent length scale in the inner region of a two-layer turbulence approach was derived to account for the influence of the waves. The model was then compared to the originally implemented k-ϵ-model and a modified version of the k-ϵ-model of OpenFoam 2.2.0. The simulation results of the new approach were found to be in very good agreement with experimental data.
Keywords: countercurrent, stratified, two-phase flow, turbulence, waves, CFD.

1 Introduction

Stratified wavy gas/liquid-two-phase-flow can be found in many engineering applications as well as in nature. Examples might be any kind of pipe or channel flows where evaporation or condensation play an important role or wind-driven ocean currents in oceanography respectively. In both cases, the interaction between the two phases at the common interface makes the description of those flows much more complicated compared to single-phase flows. Thus, heat and mass transfer at the interface has to be considered additionally. Furthermore, shear stresses between the two phases cause flow instabilities that give rise to the development of surface waves.

WIT Transactions on Engineering Sciences, Vol 89, © 2015 WIT Press
www.witpress.com, ISSN 1743-3533 (on-line)
doi:10.2495/MPF150311

Depending on flow velocities, one can distinguish between subcritical and supercritical flow. The characteristic number, from which one can determine whether the flow is sub- or supercritical, is the Froude number. It describes the ratio of inertial to gravitational forces. For Froude numbers larger than unity, inertia dominates and the present flow is supercritical. In this regime, waves occur with only small amplitude. If the flow is subcritical, gravity forces dominate and the Froude number is less than unity. Now waves with much larger amplitude can be observed. During the transition from super- to subcritical flow, the water depth rises abruptly. This phenomenon is called a hydraulic jump.

Another phenomenon that occurs in stratified flow, when the two fluids flow counter currently, is the so-called countercurrent flow limitation (CCFL). At sufficient gas velocities, shear stresses at the interface are causing parts of the liquid flow to be reversed in the vicinity of the interface and to flow backwards in the direction of the gas flow. If the gas velocity is further increased, more and more liquid gets reversed until liquid does not reach the end of the pipe anymore.

As mentioned at the beginning, the transfer mechanisms between the two fluids at the interface are very important for the description of such stratified flows.

In principle, the wavy interface can be described with the volume-of-fluid method, resolving each wave in detail. Such methods, however, require huge CPU times, while the details of the waves are usually not of engineering interest. Instead, a statistical model describing theses waves, like a turbulence model for single-phase flow, would be desirable. According to Rashidi *et al.* [1], turbulence in the liquid phase is of significant importance for interfacial transport. This was the reason for many investigations on turbulent structures near the interface. However, turbulence of waves is still not fully understood mainly because of measuring difficulties near the wavy interface.

Nevertheless, Komori *et al.* [2] found from their experimental results in open channel flows that large eddies, generated at the wall, renew the free surface and are then reflected back into the bulk flow. Same results were also reported by Rashidi and Banerjee [3].

Rashidi *et al.* [1] performed measurement with shear-free interfaces and non-wavy interfaces on which shear was imposed. They showed that, in the case of surfaces with shear, eddies are also generated at the interface and that their main characteristics are similar to those of the eddies generated at the wall.

Later, Rashidi *et al.* [4] investigated the interaction between turbulence and waves generated by a mechanical wavemaker. It is noted that these waves do not impose any shear at the interface. They reported that the turbulence mechanisms in presence of surface waves are comparable to that in case of nonwavy interfaces. However, they found a direct impact of the wave amplitude on the eddy ejection frequency. Increasing the wave height, also increases the ejection of eddies. On the other hand, there is no direct dependency of the ejection frequency on the wave frequency and therefore on the wavelength. Furthermore, the fluid velocity and turbulence are increased under the wave crest and decreased under the wave trough causing an acceleration of the fluid from the trough to the crest.

Studies on the turbulence structure at wavy gas–liquid interfaces had also been performed by Lorencez *et al.* [5]. In contrast to the study mentioned above, interfacial waves are formed here by shear stress that is imposed by a gas flow over the liquid interface. Their results, however, showed similar effects. Turbulent fluctuations are not only ejected from the wall but also from the liquid interface which than travel into the bulk flow and increase the turbulent mixing effect. Therefore, they suggest that the generation of turbulent eddies at the interface is the main mechanism of momentum transport from the gas to the liquid flow.

Stäbler [6] used the test facility of Gargallo *et al.* [7] for his measurements in countercurrent stratified air-water flow with wavy interface in a rectangular channel. The aim of this work was to provide detailed local experimental data for the development of new modelling approaches of stratified two-phase flow. Inlet flow rates of both phases had been varied, so that measurement results are now available both in supercritical and in subcritical flow either with or without flow reversal. From this data, Stäbler found that the horizontal velocity component decreases near the interface due to high interfacial shear stresses imposed by the gas flow and the wall friction, respectively. The vertical component also shows some damping at the interface and at the wall, but it is much smaller than the horizontal one. On the other hand, turbulent fluctuations in both directions increase rapidly near the interface with the highest values for partly reversed flow. According to Stäbler, this is because of the deceleration of the liquid flow in horizontal and the interfacial wave motion in vertical direction. The time averaged phase distribution or void profiles for all flow regimes show an almost linear behaviour within the wavy interfacial. Based on these results, he derived a statistical model for the description of the void profiles from the equilibrium between turbulent kinetic energy and potential energy of the fluid particles. This model shows very good agreement with deviations less than 5% for partly reversed flow and less than 9% for subcritical flow. Within the supercritical flow regime, agreement with experimental results was not as good, but still less than 17% deviation only.

Besides these experimental studies, there has also been done some numerical works on stratified two-phase flows. Among others (e.g. Daly and Harlow [8], Akai *et al.* [9], Lorencez *et al.* [5]), the work of Berthelsen and Ytrehus [10] is quite interesting. They performed simulations of fully developed, stratified gas–liquid flows in pipes, using the level set method together with a modified two-layer-turbulence approach to include the effect of waves at the interface. The two-layer-turbulence model was first introduced by Chen and Patel [11] to overcome the shortcomings of the standard-k-ϵ-model near walls. Therefore, the flow is divided into an inner region near physical boundaries, e.g. walls or free surfaces, and an outer region that is further away from boundaries. In the latter region, the standard-k-ϵ turbulence model is applied, while a simpler one-equation k-l-model is used for the viscous-affected inner region. Berthelsen and Ytrehus consider the enhancement of turbulence at the interface due to waves as an additional displacement height in the turbulent length scale of the inner region, which is a function of an equivalent roughness parameter.

Finally, Wintterle *et al.* [12] developed a phase exchange model based on the findings of Stäbler [6]. He introduced an additional dispersion force to model the smearing of the wavy interface due to a statistical time averaging of the surface waves. To account for turbulence damping in the phase interaction zone, another production term was added to the dissipation equation of the used k-ω-turbulence model.

The motivation of this work was to develop a new turbulence model approach that is able to predict all time-averaged properties of stratified two-phase flow with wavy interface with a minimum of empirical fitting factors. For validation of the model, it was implemented into the volume-of-fluid two-phase solver interFoam of the open source CFD-simulation tool OpenFoam 2.2.0. The results were then compared with calculations with the originally implemented model and with a slightly modified version of the k-ϵ-turbulence model of OpenFoam.

2 Description of the model

2.1 Governing equations

InterFoam is based on a homogeneous two-phase model in which both fluids share the same flow field. Thus, only one single momentum equation is solved which implies that the velocities of both phases must be equal at the interface:

$$\frac{\partial \rho \vec{U}}{\partial t} + \vec{\nabla} \cdot \left(\rho\left[\vec{U} \otimes \vec{U}\right]\right) = -\vec{\nabla} p + \vec{\nabla} \cdot \tau + \rho \vec{g} + \sigma \kappa \vec{\nabla} \alpha \tag{1}$$

where $\vec{U}$ is the velocity vector, ρ is the density, p and τ are the pressure and the stress tensor respectively, $\vec{g}$ is the gravitational acceleration and $\sigma\kappa\vec{\nabla}\alpha$ the surface tension force according to Brackbill *et al.* [13].

The different phases are represented by the void fraction α (0: liquid, 1: gas) for which a separate transport equation is solved:

$$\frac{\partial \alpha}{\partial t} + \vec{U} \cdot \vec{\nabla} \alpha = 0 \tag{2}$$

The density ρ in eqn (1) is defined as a kind of density of the mixture of the two phases:

$$\rho = \alpha \rho_G + (1 - \alpha)\rho_L \tag{3}$$

A detailed description of the model can be found in Rusche [14].

2.2 Turbulence modelling

In this study, the applicability of three different turbulence models to stratified two-phase flow with wavy interface is investigated. The first one is the k-ϵ-model that is originally implemented into OpenFoam:

$$\frac{\partial k}{\partial t} + \vec{\nabla} \cdot \left(\vec{U} k\right) - \vec{\nabla} \cdot \left(\left(\nu + \frac{\nu_t}{\sigma_k}\right) \vec{\nabla} k\right) = P - \epsilon \tag{4}$$

$$\frac{\partial \epsilon}{\partial t} + \vec{\nabla} \cdot \left(\vec{U} \epsilon\right) - \vec{\nabla} \cdot \left(\left(\nu + \frac{\nu_t}{\sigma_\epsilon}\right) \vec{\nabla} \epsilon\right) = \frac{\epsilon}{k}\left(C_1 P - C_2 \epsilon\right) \tag{5}$$

where k is the turbulent kinetic energy, ϵ is the dissipation rate of k, ν is the kinematic viscosity of the form $\nu = \alpha \nu_G + (1 - \alpha)\nu_L$, ν_t is the turbulent eddy viscosity which is calculated from k and ϵ ($\nu_t = C_\mu k^2/\epsilon$) and P is the turbulence production term. $C_\mu = 0.09$, $C_1 = 1.44$, $C_2 = 1.92$, $\sigma_k = 1.0$ and $\sigma_\epsilon = 1.3$ are constants.

In a homogenous two-phase flow, eqns (4) and (5) must be modified by multiplication with the density ρ. That implies $\rho_L k_L = \rho_G k_G$ and therefore $k_L = (\rho_G/\rho_L) k_G$ at the interface, instead of $k_L = k_G$. Note that, like for the velocity, both phases also share only one common turbulence field.

The third turbulence model studied here, is a new one, which has been developed in this work. It is based on the two-layer-approach in which the turbulent eddy viscosity is calculated as follows:

$$\nu_t = C_\mu \sqrt{k} l_\mu \tag{6}$$

with different turbulent length scales l_μ for the inner and the outer region, as defined by Berthelsen and Ytrehus [10]. However, a more physical expression for turbulence length scale of the inner region is found in this study. It is a further development of the statistical model of Stäbler [6], who derived an expression for the void distribution by an equilibrium state between turbulent kinetic and potential energy of fluid particles. He reported good agreement with his experimental data especially in subcritical flow where interfacial waves are relatively high. However, for supercritical flow, where the waves are smaller, he observed some deviation between the model and the experiment, which we expect to be due to capillary waves. Therefore, surface energy was additionally included in this study.

The specific surface energy can be written as:

$$E_{surf} = \sigma \frac{\partial^2 y}{\partial x^2} \tag{7}$$

where x is the direction of travel of the wave and y the direction of its deflection. Assuming the waves to be sine-waves ($y(x) = h \sin(qx - \omega t)$) yields to:

$$\left| dE_{surf} \right| = \sigma q^2 dy \tag{8}$$

with the surface tension σ and the wave number q.

Stäbler [6] found a Gaussian distribution of the measured local velocities v with the probability:

$$P(v) = \frac{1}{\sqrt{2\pi \overline{v'}^2}} \exp\left(\frac{(v-\bar{v})^2}{2\overline{v'}^2}\right) \tag{9}$$

Extension by $\rho_L/2$ and generalization to three dimensions yields a Boltzmann distribution for the turbulent kinetic energy:

$$P(E_{kin}) = \frac{1}{\sqrt{2\pi \rho_L k_L}} \exp\left(-\frac{E_{kin}}{2\rho_L k_L}\right) \tag{10}$$

where E_{kin} is the turbulent kinetic energy of a fluid particle and k_L is the local time-averaged turbulent kinetic energy of the flow.

Each fluid particle can use its turbulent kinetic energy either to increase the potential energy by lifting a particle up from position y to $y + dy$ or to increase surface energy. The probabilities that the kinetic energy is sufficient for each of the two mechanisms are:

$$P(\Delta E_{pot}) = \frac{1}{\sqrt{2\pi \rho_L k_L}} \exp\left(-\frac{\Delta E_{pot}}{2\rho_L k_L}\right) \tag{11}$$

$$P(\Delta E_{surf}) = \frac{1}{\sqrt{2\pi \rho_L k_L}} \exp\left(-\frac{\Delta E_{surf}}{2\rho_L k_L}\right) \tag{12}$$

where $\Delta E_{pot} = (\rho_L - \rho_G) g dy$ and ΔE_{surf} can be expressed by eqn (8).

The probability that a fluid particle transfers its kinetic energy to potential energy or to surface energy, can be written as the product of the probability that liquid is at position y, the probability that an empty space is at $y + dy$ and the probability that one or both energy transfer mechanisms are activated:

$$P_1 = [1 - \alpha(y)]\alpha(y + dy)\left[P\left(\Delta E_{pot}\right) + P\left(\Delta E_{surf}\right)\right] \qquad (13)$$

Together with eqns (11) and (12) and linearization for small dy, eqn (13) yields to:

$$P_1 = [1 - \alpha(y)]\alpha(y + dy)\frac{1}{\sqrt{2\pi\rho_L k_L(y)}}\left[1 - \frac{\Delta\rho g dy}{2\rho_L k_L(y)} - \frac{\sigma q^2 dy}{2\rho_L k_L(y)}\right] \qquad (14)$$

The probability that potential and surface energies are transferred back to kinetic energy is:

$$P_2 = \alpha(y)[1 - \alpha(y + dy)]\frac{1}{\sqrt{2\pi\rho k_L(y)}} \qquad (15)$$

Assuming that both probabilities must be the same under equilibrium condition and by neglecting higher order terms, an expression for the gradient of the void fraction distribution can be found as follows:

$$\frac{d\alpha}{dy} = \alpha(y)[1 - \alpha(y)]\frac{\Delta\rho g + \sigma q^2}{2\rho_L k_L(y)} \qquad (16)$$

As reported by Stäbler [6], the region where $0 < \alpha < 1$ correlates well with the wavy region and thus with the wave height. It can easily be found that the wave height y_δ can be calculated from the reciprocal of eqn (16) evaluated for $\alpha = 0.5$.

$$y_\delta = \frac{1}{\left.\frac{d\alpha}{dy}\right|_{\alpha=0.5}} \qquad (17)$$

In the new turbulence model, the amplitude of the wave (=0.5 y_δ) is taken as turbulent length scale for the inner wavy region, whose upper and lower limits is defined as the distance $C_k y_\delta$ to both sides of the time averaged position y_M of the interface. In the outer regions, where there is only liquid or gas flow, the modified k-ϵ-model is used. Thus, the turbulent length scale in eqn (6) is calculated from:

$$l_\mu = \begin{cases} 0.5\, y_\delta & , y_M - C_L y_\delta \le y \le y_M + C_G y_\delta \\ k^{3/2}/\epsilon & , otherwise \end{cases} \qquad (18)$$

The constants C_L and C_G have been determined from a parameter study as described in the next section.

3 Results and discussion

For determination of the constants C_L and C_G of the new two-layer turbulence model and its comparison with the other turbulence models, three different measuring points from the experiments of Stäbler [6] were simulated in a two dimensional model of the channel geometry, one in subcritical, one in supercritical and one in partly reversed flow regime, respectively. The working fluids are air and water. The inlet flow rates for the different flow regimes and the wave numbers q, which were taken from the measurements, are shown in table 1. The test section investigated by Stäbler was a horizontal duct of 470mm of length and 90mm of height. From a mesh study, a mesh with 8995 cells was found to be a good compromise between accuracy and simulation time. The geometry can be seen in Fig. 1. Note that parts of the inlet and outlet zones were also included in the simulated geometry.

Table 1: Inlet conditions.

MP	Flow regime	Q_L [l/min]	Q_G [l/s]	q [1/m]
1	subcritical	16.4	39.7	51.36
2	partly reversed	16.6	89.1	106.82
3	supercritical	41.6	39.6	378.39

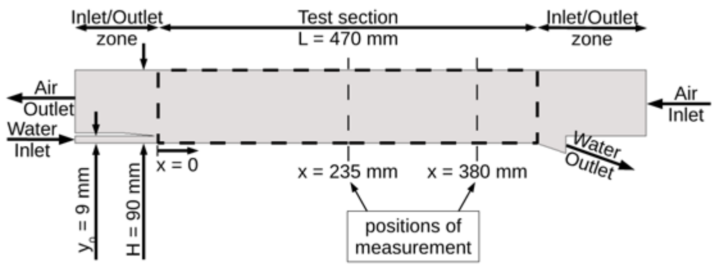

Figure 1: Geometry of experiment of Stäbler [6].

3.1 Parameter study for C_L and C_G

As mentioned before, a parameter study was done to determine the constants C_L and C_G in eqn (18), which define inner and outer region of the wavy interface. For C_L the values 0, 0.5, 1.0 and 2.0 and for C_G the values 0, 1.5, 2.0, 2.5, 3.0, 4.0, 5.0, 6.0, 7.0 and 10.0 were chosen. The results were compared with experimental data and best agreement was found for $C_L = 0.5$ and $C_G = 4.0$.

3.2 Validation analyses of the turbulence models

In the following, simulation results of the three different turbulence models are presented only for supercritical and partly reversed flow, as the simulation in subcritical flow shows similar results as in supercritical flow. The results were taken at horizontal position $x = 235mm$ downstream of the liquid inlet.

3.2.1 Supercritical flow

Fig. 2 shows the void-distribution profiles for supercritical flow. Both the modified k-ϵ-model and the two-layer approach are in good agreement with the experiment while the liquid height is overestimated by the original k-ϵ-model.

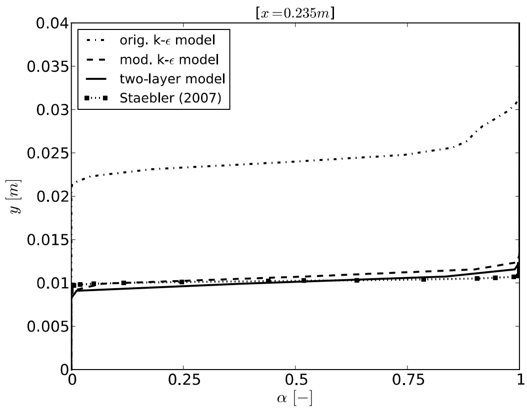

Figure 2: Void profiles in supercritical flow.

The velocity distributions are shown in Fig. 3. It can be seen that only the two-layer turbulence model is able to predict accurately velocity profiles both in the liquid and in the gas flow. While the liquid flow velocities calculated by the modified k-ϵ-model show good agreement, the model fails in the gas flow regime. The standard k-ϵ-model fails completely.

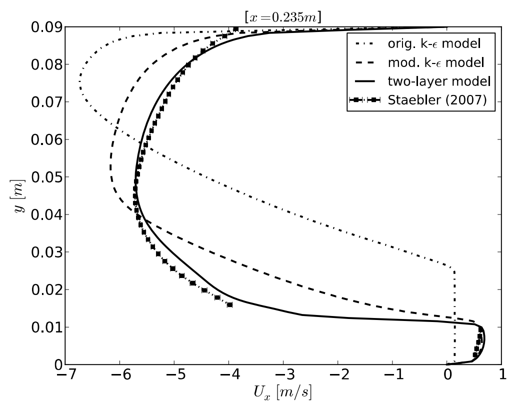

Figure 3: Velocity profiles in supercritical flow.

The same trends can also be observed for the turbulent kinetic energy (Fig. 4). Again, the two-layer approach shows the best agreement with the experiment with little overestimation in the gas phase near the interface. Unfortunately, the gas phase turbulence in the wavy region could not be measured. But from the results of Lorencez *et al.* [5], where turbulent ejections were also created at the interface due to shear, one can assume that the increase of turbulent kinetic energy at the interface, as calculated by the two-layer model, might be physical. Comparing the modified k-ϵ-model with the original model, the influence of the density in the model can clearly be seen. While the kinetic energy increases monotonically from the bottom wall to the interface in the original model, the modified model shows a sudden increase at the interface due to density ratio in the interfacial boundary condition ($k_L = \rho_G/\rho_L \cdot k_G$).

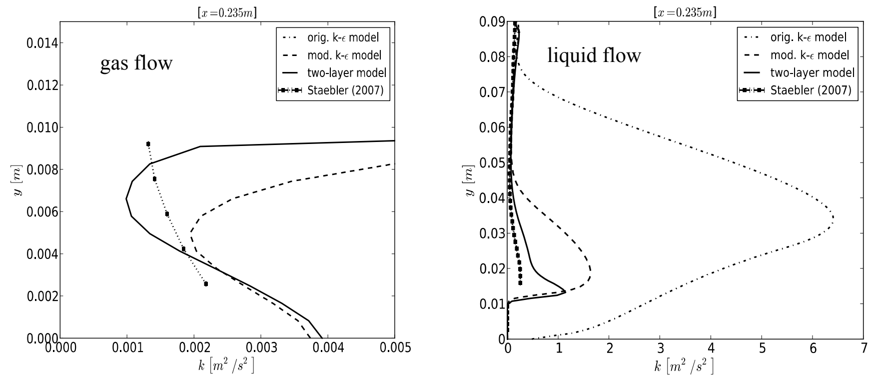

Figure 4: Turbulent kinetic energy profiles in supercritical flow.

3.2.2 Partly reversed flow

Looking at the void distribution in Fig. 5, it is apparent that only the two-layer approach is able to reflect the measured profile with little overestimation of liquid height. Even the slope and therefore the height of the waves are calculated accurately. It only underestimates the deepness of the wave troughs in the region of low void. As the slope of the profile is small, the interface predicted by the modified k-ϵ-model, seems to be rather smooth. The profile of the original k-ϵ-model cannot be seen at all because total flow reversal has been predicted, which is obviously wrong.

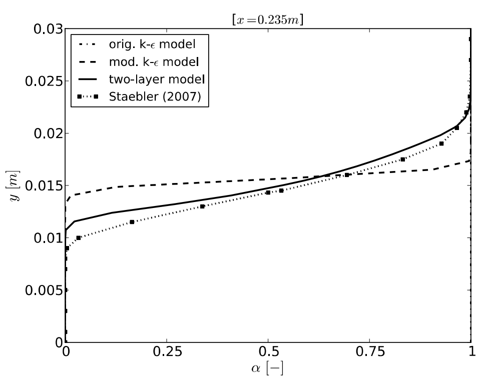

Figure 5: Void profiles in partly reversed flow.

Because of the totally reversed flow, the velocity distribution calculated by the original model is similar to the turbulent duct flow profile of one single phase. Just as for supercritical flow, the modified model is able to reflect experimental velocity data much better. However, it is once more the two-layer model that agrees best with the measurements (Fig. 6).

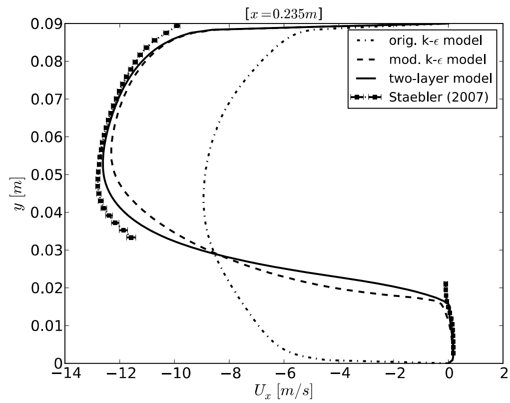

Figure 6: Velocity profiles in partly reversed flow.

The improvement due to the new turbulence model compared to the modified k-ϵ-model gets even more obvious when looking at the turbulent kinetic energy in Fig. 7. While the modified model overestimates the turbulent energy, it is predicted almost exactly by the two-layer turbulence approach especially in the region near the interface. The reason for the deviation between $y = 0.012\ m$ and $y = 0.021\ m$ is the difference in time averaging. While in the simulation the properties of the homogenous flow are averaged, only the liquid phase is measured in the experiment. Thus, results are not comparable in that region.

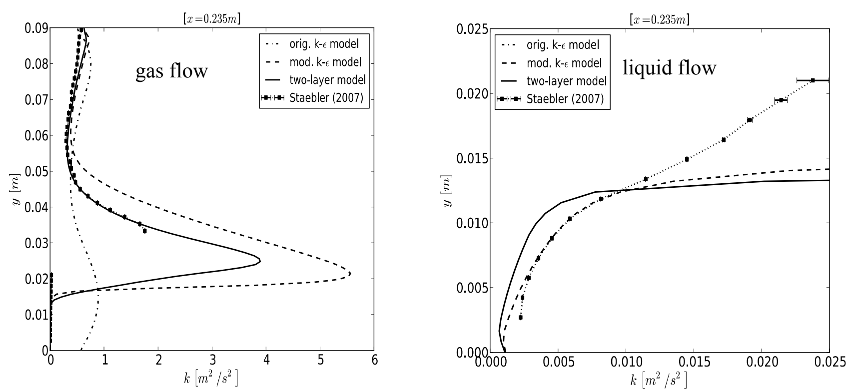

Figure 7: Turbulent kinetic energy profiles in partly reversed flow.

4 Conclusion

A new turbulence model based on a two-layer approach has been developed for countercurrent stratified two-phase flows with wavy interface. Assuming an equilibrium condition between gravitational and surface energy on one hand and turbulent kinetic energy on the other hand, a statistical model for the turbulent length scale in the inner region of the two-layer model has been derived to account for the influence of the waves. The model was then compared to the originally implemented k-ϵ-model and a modified version of the k-ϵ-model of OpenFoam. The simulation results of the new approach are in good agreement with experimental data especially in partly reversed flow regime, where the other models fail to predict a wavy interface.

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