A three-dimensional finite element method for simulating gas and water two-phase flow induced by excavation in sedimentary rock

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Abstract

This paper describes the developed software MGF2 (Mine Gas Flow 2-Phase)-3D and its applications. Based on the finite element method on two-phase seepage flow like methane gas and underground water, MGF2 has been mainly applied to simulate the gas gushing problems for disaster prevention during coal mining and excavation processes in sedimentary rock for nuclear waste disposal. The partial differential simultaneous equations for two-phase permeable flow, sorption/diffusion formulations and their finite element discretization as well as two examples of practical application using MFG2 are described in this paper.

Keywords: two-phase flow, finite element method, three-dimensional, diffusion/adsorption.

1 Introduction

Nowadays, environmental and disaster-prevention problems are becoming more and more outstanding in the engineering of energy resources, coastal construction and nuclear waste underground disposal. Multiphase flow analysis [1] is also becoming more and more important in the environmental remediation of soil and subsurface water pollutions, coal seam gas extraction, carbon dioxide geo-sequestration, etc. The projects to simulate and predict gas coupled with water permeable flow phenomena induced by excavation in sedimentary rock are increasing. In China, coal, the mining and consumption of which are bringing large quantities of greenhouse gas to the atmosphere, is unfortunately by far the main primary energy resource, which makes it an urgent need to enhance the coal bed methane production as the most reliable means to reform current energy
structure in this largest developing country. In Japan, prominent budgets for the research and development of carbon dioxide geological and coal-seam sequestration are known. So far, although there are a few simulators on multiphase flow within the above fields, e.g., TOUGH2 [2], GEM [3], COMET [4], they are all based on the finite difference or finite volume method. It is not only scholarly significant but also of commercial potential from the point of domestic technical support and simulation consultancy to develop a simulator based on the finite element method.

2 Fundamental theory on the two-phase flow

2.1 The domination simultaneous equations of water and gas two-phase flow

In order to grasp the behaviour of subsurface co-existing gas and water in the gas phase and liquid phase respectively, the simulation should be carried out on the basis of not separately and singly dominant equations, but simultaneously coupled dominant equations with consideration of mechanical and physical interaction between the two gas and water phases.

\[
\nabla \cdot \left[ b_g \frac{k_g}{\mu_g} \kappa_g (\nabla p_g + \gamma_g \nabla Z) + R_g - b_w \frac{k_w}{\mu_w} \kappa_w (\nabla p_w + \gamma_w \nabla Z) \right] + q_g + (q_g + R_g \cdot q_g)
\]

\[
\nabla \cdot \left[ b_w \frac{k_w}{\mu_w} \kappa_w (\nabla p_w + \gamma_w \nabla Z) \right] + q_w = \frac{d}{dt} (\Phi b_w s_w)
\]

where

- \( \nabla \cdot \) is divergence operator of vector field;
- \( \nabla \) is gradient operator of scalar field;
- \( b_n \) (n = g or w, i.e., gas or water) is flow shrinkage factor (-); \( b = 1/\beta \), where \( \beta \) is formation volume factor (-), which is the ratio of the volume \((m^3)\) at a certain state \( V \) to that at the standard state \( V_s \), i.e., \( \beta = V/V_s \).
- Since \( V \) is the function of pressure, \( \beta_n = \beta_n(p_n) \). So is \( b \).
- \( M_n = k_n K/\mu_n \) is phase mobility; \( k_n \in [0,1] \) is phase relative permeability (-), which is the function of phase saturation;
- \( k_n \in [0,1] \) is phase relative permeability (-), which is the function of phase saturation;
- \( K \) is abstract permeability tensor \((m^2)\);
- \( \mu_n \) is phase viscosity \((kg/m/sec)\);
- \( p_g \) is pressure of gas \((Pa)\).
$p_w$ is pressure of water (Pa); the difference of pressure in the gas and water is called the capillary pressure dependent on the saturation of water or gas; 
\[
\gamma_n = \rho_n \cdot g, \quad \text{where} \quad \rho_n = \rho_w(p_w) \quad \text{is phase mass density (kg/m}^3) \quad \text{and} \quad g \quad \text{is gravitational acceleration (9.8 m/sec}^2); 
\]
$Z$ is elevation (m); 
$R_s = R_s(p_w)$ is gas solubility (-) in water; 
$q$ is positive for a source or negative for a sink term with regard to gas (sm$^3$/m$^3$/sec); particularly in this paper, it will represent the diffusive flow rate of the desorbed methane from the microscopic primary porosity in the coal matrix to the macroscopic permeable natural cleat network [5]; 
\( \Phi \) is the porosity effective for water or gas permeable flow and if necessary 
\( \Phi = \Phi(p_w) \); 
The phase saturation (-) must of course satisfy 
\[
S_g + S_w = 1. 
\]

### 2.2 Finite element discretization

Discretizing Equation (1) and (2) by the Galerkin finite element method yields 

\[
\begin{align*}
[A_g]_{mk} \{ p_g \}_k + [F_g]_{mk} \frac{\partial \{ p_g \}_k}{\partial t} + [H_g]_{mk} \frac{\partial \{ S_w \}_k}{\partial t} &= \{ Q_g \}_m - \{ G_g \}_m + \{ D_g \}_m + \{ C_g \}_m \\
[A_w]_{mk} \{ p_g \}_k + [F_w]_{mk} \frac{\partial \{ p_g \}_k}{\partial t} + [H_w]_{mk} \frac{\partial \{ S_w \}_k}{\partial t} &= \{ Q_w \}_m - \{ G_w \}_m + \{ D_w \}_m 
\end{align*}
\]

where 

\[
\begin{align*}
[A_g]_{mk} &= \sum_{e=1}^{N} \int_{V_e} \frac{\partial N_{n_e}}{\partial x_i} \left( \frac{k_{rg}}{\mu_g} + R_0 \frac{k_{rw}}{\mu_w} \right) K_{ij} \left( \frac{p_g^{v,n+1}}{P_0} \right) \frac{\partial N_{n_k}}{\partial x_j} dV_e \\
[F_g]_{mk} &= \sum_{e=1}^{N} \int_{V_e} N_{n_e} \left( \phi + p_g^{v,n+1} \beta_r \right) \left[ (R_0 - 1) S_w^{v,n+1} \right] \frac{1}{P_0} N_k dV_e \\
[H_g]_{mk} &= \sum_{e=1}^{N} \int_{V_e} N_{n_e} \phi (R_0 - 1) \frac{P_g^{v,n+1}}{P_0} N_k dV_e \\
\{ Q_g \}_m &= \sum_{e=1}^{N} \int_{S_e} N_{n_e} (-u_g - R_0 u_w) \left( \frac{p_g^{v,n+1}}{P_0} \right) dS_e \\
\{ G_g \}_m &= \sum_{e=1}^{N} \int_{V_e} \frac{\partial N_{n_e}}{\partial x_i} \left( \frac{k_{rg}}{\mu_g} \gamma_g + R_0 \frac{k_{rw}}{\mu_w} \gamma_w \right) K_{ij} \left( \frac{p_g^{v,n+1}}{P_0} \right) dV_e 
\end{align*}
\]
Additionally, the coefficient matrix of each time differential term needs to be diagonally concentrated.

Using the addition and subtraction method, the time differential terms with regard to saturation can be eliminated from (3) and (4) so that the pressure will be sought. The saturation can then be renewed by substituting the sought pressure into either (3) or (4).

### 2.3 Formulations of sorption/diffusion

In the case of coal bed gas extraction, e.g., methane adsorbed on the internal surface of the primary porosity will at first be desorbed and diffuse into the secondary fractures, then permeable flow occurs coupled with water [6, 7].

The source or sink term of gas adsorption/diffusion is

\[
q_s^m = \rho_0 \cdot q_s = \rho_0 (1 - \phi) \left(- \frac{\partial C}{\partial t}\right) \equiv \rho_0 (1 - \phi) \frac{C^n - C^{n+1}}{\Delta t} = \rho_0 \frac{1 - \phi}{\tau} \left(C^n - C^{n+1}\right) \tag{5}
\]

where

\[
C^{n+1} = C^n \cdot \exp\left(-\frac{1}{\tau} \Delta t\right) + C_e \left[1 - \exp\left(-\frac{1}{\tau} \Delta t\right)\right] = C_e + (C^n - C_e)\exp\left(-\frac{1}{\tau} \Delta t\right) \tag{6}
\]
$q_s^m$ is massive source/sink of gas sorption/diffusion (kg/m$^3$/sec);

$\rho_0$ is gas density under the standard status (gas pressure = 1 atm) (kg/m$^3$);

$\tau$ is sorption time (sec), which is decided by the diffusion coefficient and the Warren/Root shape factor [5];

$L_C$ is Langmuir concentration (-) or (St. m$^3$/m$^3$)

$L_P$ is Langmuir pressure (Pa)

$C$ is instant concentration (-), i.e., gas volume under standard status contained in unit volume of coal matrix at present time (St. m$^3$/m$^3$);

$P_d$ is desorption pressure (Pa)

$C_e$ is equilibrium concentration decided by Langmuir formula, which is the boundary condition of fracture for diffusion (St m$^3$/m$^3$);

$$C_e = \begin{cases} 
\frac{C_L \cdot P_d}{P_L + P_d}, & \text{if } p_g \geq P_d \\
\frac{C_L \cdot p_g}{P_L + p_g}, & \text{if } p_g < P_d 
\end{cases}$$

(7)

3 Application to the practical projects

After MGF2-3D was developed, it was verified by some simple and self-evident computation examples, where special attention was paid to the mass conservation from the change of the contents to the temporal accumulative flow rates. Then the following two projects were carried out.

3.1 Gashing gas predictive simulation contributing to the ventilation design for underground excavation of a nuclear waste disposal site

Figure 1: Model appearance.
Figure 2: Excavation system.

Figure 3: Change of gas content with time.
3.2 Simulation to predict gas flow induced by both boring and excavation in a mine

Figure 4: Gas gushing rate results.

Figure 5: Model appearance.
Figure 6: Boring distribution.

Figure 7: Result example one.
Figure 8: Result example two.

4 Conclusion

The solver of MGF-3D2, which is based on finite element method, has been integrated into developed pre-post processor, a user friend interface at the level of commercial simulator. In addition, the manual in Japanese, English and Chinese has also been made. It is expected that MGF-3D2 would play an active role in the engineering over the related fields.

References