A contribution to the problem of the continuous dewatering process

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Abstract

Because of its energetic efficiency, gravity thickening is one of the most convenient processes of lowering the volume of suspensions. One of the problems that has to be solved when designing equipment for, say, a digestion tank for the continuous process of suspension dewatering, is to determine the optimum height of the compression zone of the given suspension. This paper presents a solution for this problem. The suspension is considered as a two-phase continuum where both the phases are independently movable. The behaviour of such a system can be described by a set of partial differential equations of general Darcian mechanics. Making use of the known hydromechanical characteristics of the investigated two-phase system, a problem with a set of two ordinary differential equations and related boundary conditions was formulated and numerically solved. The unknown functions of the problem are the liquid-phase pressure and the solid-phase concentration. The method of solution is described and the achieved results are visualized and presented.

1 Introduction

The process of compression of a suspension has to be studied in detail when various industrial technologies are designed, particularly when the optimum height of the compression zone of sedimentation tanks has to be determined, e.g. Tuček and Koníček [6]. The previous reduction of the suspension volume minimizes, for example, the space of digestion tanks and improves the efficiency of filtration or centrifugation. The gravity thickening is, because of its low energetic demands, one of the most convenient processes of lowering volume of suspensions and significantly increasing the concentration of their solid phase.
During the sedimentation process of a suspension, a sharp interface develops separating the upper zone of free sedimentation from the lower zone of compression. The position of this interface results from the balance of mass exchange between both the zones. It is also affected by the behaviour of the suspension under compression. The sedimentation zone is characterized by zero value of the effective stress of the solid phase. On the other hand the effective stress of the solid phase is negative in the zone of compression. In the case of compressible suspensions, the negative solid-phase stress may increase the liquid-phase pressure which pushes the liquid phase out of the zone of compression and continues the required solid-phase thickening.

There are several possible approaches to describe the mechanism of such processes in suspensions. Mls et al. [4] suggested to solve the problem by means of the theory of water seepage through porous media, particularly applying the Darcy law. Toorman [5] presented a discussion of two different approaches and suggested a possible unification.

2 The applied theory

Under certain assumptions imposed on the flow velocities and the hydraulic conductivity, Mls [3] formulated the following system of partial differential equations governing the Darcian mechanics of two-phase media.

\[
\frac{\partial}{\partial t} \left( \rho_w(x, t) \ n(x, t) \right) + \frac{\partial}{\partial x_j} \left( \rho_w(x, t) \ w_j(x, t) \right) = 0, \tag{1}
\]

\[
\frac{\partial}{\partial t} \left( \rho_s(x, t) \ (1 - n(x, t)) \right) + \frac{\partial}{\partial x_j} \left( \rho_s(x, t) \ v_j(x, t) \right) = 0, \tag{2}
\]

\[
\frac{\partial}{\partial t} \left( \rho_w(x, t) \ w_i(x, t) \right) + g \frac{\partial x_3}{\partial x_i} \ \rho_w(x, t) \ n(x, t) \\
+ n(x, t) \ \frac{\partial p}{\partial x_i}(x, t) + g \ \rho_w(x, t) \ n(x, t) \ K^{-1}_{ij}(x, t) \ u_j(x, t) = 0, \tag{3}
\]

\[
\frac{\partial}{\partial t} \left( \rho_s(x, t) \ v_i(x, t) \right) + g \ \rho_s(x, t) \ (1 - n(x, t)) \ \frac{\partial x_3}{\partial x_i} - \frac{\partial \tau_{ij}}{\partial x_j}(x, t) \\
+ (1 - n(x, t)) \ \frac{\partial p}{\partial x_i}(x, t) - g \ \rho_w(x, t) \ n(x, t) \ K^{-1}_{ij}(x, t) \ u_j(x, t) = 0, \tag{4}
\]

\(i = 1, 2, 3\) and the summation rule has been used. In these Equations, \(t\) is time, \(x = (x_1, x_2, x_3)\) are space coordinates with \(x_3\) axis oriented vertically upwards, \(\rho_w\) and \(\rho_s\) are densities of the liquid phase and the solid phase respectively, \(n\) is porosity of the medium, \(w\) and \(v\) are the volumetric flux-density vectors of the liquid phase and the solid phase respectively, \(g\) is the gravitational acceleration, \(p\) is the liquid-phase pressure, \(K\) is the hydraulic-conductivity tensor, \(u\) is the relative flux-density vector of the liquid phase satisfying

\[
u = w - \frac{n}{1 - n} \ v, \tag{5}\]
and $\tau$ is the effective solid-phase stress. To simplify expressions concerning the solid phase, it is convenient to introduce the volume fraction of the solid phase. It will be denoted $s$. According to its definition, it holds

$$ s(x, t) = 1 - n(x, t). \quad (6) $$

In the theory, both the phases are supposed to be independently movable. The first two Equations are expressions of the continuity axiom, and the Equations (3) and (4) are equations of motion of the liquid phase and the solid phase respectively.

### 3 The constitutive equations

Consider following process. A suspension of given volume fraction of the solid phase is continuously charged to the upper part of a vertical cylinder. Due to its higher density, the solid phase moves downwards at a higher rate than the liquid phase. Consequently, the relative flux density of the liquid phase is positive (i.e. oriented upwards) everywhere in the suspension column which makes the volume fraction of the solid phase to increase downwards. The discharge of the suspension at the bottom of the cylinder reflects exactly the total charge of the solid phase and the bottom value of the solid-phase volume fraction. More exactly, denote $s_i$ and $Q_i$ the initial value of the solid-phase volume fraction and the total charge of the suspension at the input, and $s_b$ the value of the solid-phase volume fraction at the bottom of the cylinder. Then it holds for the total discharge $Q_b$ of the suspension at the bottom

$$ Q_b = \frac{s_i}{s_b} Q_i. $$

The mass balance of the liquid phase gives

$$ (1 - s_i) Q_i = (1 - s_b) Q_b + Q_t, $$

where $Q_t$ is the discharge of the liquid phase over the top of the cylinder.

Making use of Equations (1) to (4), the described process can be studied in detail. Generally this system consists of eight equations containing 14 unknown functions: $\tau$, $w$, $v$, $p$ and $s$, hydraulic conductivity $K$ is supposed to be a known constitutive function. Densities $\rho_w$ and $\rho_s$ of the liquid phase and the solid phase are known constants. As the considered problem is one-dimensional, the system of governing equations reduces to four equations with five unknown functions: $\tau$, $w$, $v$, $p$ and $s$. Consequently, it is sufficient to add two constitutive equations in order to complete the solved system to a closed one. These two equations will characterize the particular suspension under consideration, e.g. Handová and Sladká [1].

The first constitutive equation is the hydraulic conductivity which will be defined as a function of the solid-phase volume fraction $s$. The described process
evidently satisfies the condition

$$\frac{\partial s}{\partial t}(m, t) \geq 0$$

for every value of a material coordinate $m$ and for every time $t$. Hence, it is a monotonous process defined by Mls [2] as a process satisfying for every two values $m_1$ and $m_2$ and every two times $t_1$ and $t_2$ the inequality

$$\frac{\partial s}{\partial t}(m_1, t_1) \frac{\partial s}{\partial t}(m_2, t_2) \geq 0.$$ 

The monotonous process enables us to exclude the effect of hysteresis and allows for the assumption, that the effective solid-phase stress is a function of the solid-phase volume fraction.

If we further suppose that the investigated suspension is a mixture of kaolin and water, the results obtained by Mls [2] can be used to define the required constitutive equations in the following form

$$K(s) = A_1 s^{A_2},$$  \hspace{1cm} (7)

and

$$\tau(s) = \begin{cases} 0 & \text{for } s \leq B_2, \\ g(\rho_s - \rho_w) \left( B_1 \ln \left( \frac{s + B_1}{B_2 + B_1} \right) - s + B_2 \right) & \text{for } s > B_2, \end{cases}$$  \hspace{1cm} (8)

where $A_1 = 1.048 \times 10^{-7}$, $A_2 = -2.468$, $B_1 = 3.055 \times 10^{-2}$ and $B_2 = 3.834 \times 10^{-2}$. In this case, the units are $[k] = \text{m/s}$ and $[\tau] = \text{Pa}$.

### 4 The solved problem

Consider the above described process of continuous suspension thickening in a vertical cylinder allowing for the control of the suspension discharge. A visible interface develops in the suspension column separating the lower part of the column, the zone of compression, from the overlying zone of free sedimentation. At the top of the column, there is a layer of outflowing water, more precisely a layer of suspension with negligible solid-phase concentration. Suppose that the values of $s_i$, $Q_i$ and $s_b$ are given. Denote $S$ the cross section of the column. This set of data together with the above constitutive equations determine the uniquely the thickening process.

The problem being one-dimensional, we denote $x$ the coordinate oriented vertically upwards. The continuous process with the prescribed constant data determines steady state values of all the involved functions; we solve a steady-state problem. It is also evident that it is sufficient to restrict our consideration to the zone of compression. Under the above conditions, the height of this zone is an unknown value. On the other hand, the value of the solid-phase volume fraction
Figure 1: The dependence of the height of compression zone on the volume fraction of the solid phase.

at the unknown upper boundary is determined by the constitutive equations; this defines a free-boundary problem.

In the case of one-dimensional steady-state flow, functions \( w \) and \( v \) can be found solving the first two equations of the system (1) to (4) and the given conditions. Their expression is

\[
v = \frac{Q_i}{S} s_i
\]

and

\[
w = \frac{Q_i}{S} \frac{s_i}{s_b} (1 - s_b).
\]

The one-dimensional steady-state form of Equations (3) and (4) reads

\[
g \rho_w n(x) + n(x) \frac{dp}{dx}(x) + g \rho_w \frac{n(x)}{K(s(x))} u(x) = 0.
\]
\[ g \rho_s (1-n(x)) - \frac{d \tau}{dx}(s(x)) + (1-n(x)) \frac{dp}{dx}(x) - g \rho_w \frac{n(x)}{K(s(x))} u(x) = 0. \] (12)

Two unknown functions, the solid-phase volume fraction and the liquid-phase pressure, remain in the system. Making use of Equation (6) and constitutive equations (7) and (8), Equations (11) and (12) build a system of two nonlinear ordinary differential equations with the above unknown functions \( s \) and \( p \).

## 5 Numerical solution

Several problems for the system (11) and (12) have been formulated and numerically solved. The required degree of dewatering, expressed by \( s_b \), is the initial value of function \( s \). The value of liquid-phase pressure \( p \) is known at the top.
of the compression zone. In order to find the unknown height of the compression zone, the given value \( B_2 \) of the solid-phase volume fraction can be utilized. Consequently, the problem can be solved using the shooting method and iterating for the height \( x_H \) of the compression zone satisfying

\[
s(x_H) = B_2
\]

and for the prescribed value of the liquid-phase pressure at \( x_H \).

The computed dependence of the height of compression zone \( x_H \) on the prescribed solid-phase flux density is shown in Figure 1. Triangles denote the values for \( s_b = 0.1 \) and squares denote the values for \( s_b = 0.15 \).

The height of the compression zone necessary to reach a prescribed degree of dewatering is firmly dependent on the given flux density of the solid phase. Figure 2 shows the dependence of the height of compression zone \( x_H \) on the solid-phase volume fraction \( s_b \) for two different values of solid-phase flux density. Triangles belong to \( v = 3.5 \times 10^{-7} \text{ m s}^{-1} \) and squares belong to \( v = 5.0 \times 10^{-7} \text{ m s}^{-1} \).

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**References**