Calculation of a Forced Plume in a Coastal Area by a Three-Dimensional k-ε-E Model
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Abstract

The purpose of this study is to investigate the diffusion mechanism of thermal water, behaving a forced plume, discharged from a power plant with in-situ observation and numerical approach. A three-dimensional k-ε-E model was developed to predict the diffusion process and to apply in practical uses. The field observation was conducted to investigate the dispersion process of a forced plume in a sea where a semidiurnal tidal current was dominant and a large amount of thermal water ~50m^3/s was discharged from a thermal power plant with multiple pipes. The horizontal eddy diffusivity in the area where each plume overlapped and formed one large plume was well estimated by the width of a plume. The observed vertical eddy viscosity at this area was ~ 1 x 10^2cm^2/s which was in good agreement with the result calculated by the numerical model. The results of the calculations also showed that the direction of the plume and the horizontal distribution of the velocity and the temperature were in good accordance with the observation results, however there was a little discrepancy between both results concerning the diffusion width of a plume at the surface.

1 Introduction

Recently, some large power plants have been adjacently constructed in coastal areas in Japan and have discharged a large amount of thermal water heated up ~ 7°C against the ambient sea temperature. In these cases, the thermal water is generally released from submerged multiple outlets into the sea with high speed and behaves as a forced plume with positive buoyancy.
The diffusion process of the thermal water discharged from submerged outlets is governed initially by the discharge momentum where the turbulent mixing entrains the ambient water. After the thermal water ascended to the sea surface by the buoyancy, the diffusion process is governed by the advection and the dispersion of the coastal current in the far field. The prediction of these complicated dispersion process of the thermal water is important from the view points of the hydraulic design of the outlet and the environmental assessment of the discharge. A three-dimensional turbulence model is useful for the prediction, however it requires large computational time and memory, and also three-dimensional observation data for the evaluation. Owing to these constraints, there are few studies concerning the development of the turbulence model to predict the diffusion process in coastal areas. Recent progress of computational ability encourages us to calculate the complicated diffusion process by a three-dimensional turbulence model in practical uses. Many experimental studies have been carried out to investigate the mean characteristics of the diffusion process of a horizontal forced plume (e.g. Katano et al.). However, the turbulent mixing process of a forced plume have not been sufficiently investigated yet. Wada proposed a three-dimensional k-ε model for the prediction of the thermal diffusion of a surface buoyant jet in the area near an outlet. He conducted the basic experiments in a simple flume and discussed the effectiveness of the model through a comparative study between the results of hydraulic experiments and calculations. The prediction models regarding the thermal diffusion discharged from submerged outlets however have not been developed in practical uses.

The purpose of this study is to investigate the diffusion mechanism of a forced plume discharged from submerged multiple pipes both observationally and numerically, and to improve a numerical prediction model of thermal water diffusion in practical uses. Nakashiki et al. studied the diffusion process of a horizontal forced plume discharged from a single pipe through a series of hydraulic experiments in a simple vertical flume and they calculated the diffusion process by using a three-dimensional k-ε-E model. They showed that the calculated distribution of the mean velocity and temperature, the ascending path of a plume and the turbulent intensity were in good agreement with the experimental results. Nakashiki et al. also investigated the dispersion process of a forced plume horizontally discharged from the submerged multiple pipes (two or four pipes) using the same turbulence model. They showed that the calculated results such as the trajectory of a plume, the profiles of the mean velocity and the mean temperature were in good agreement with the experimental results in the area where each plume overlapped and interacted. However, the calculated width of temperature diffusion at the surface was a little smaller than that by the experiments. In this study, the field observation was conducted to investigated the phenomenon of a forced plume in a coastal area where a
large amount of thermal water ~50m^3/s was discharged from a thermal power plant with multiple pipes. A semidiurnal tidal current M2 was dominant in this area. And the numerical calculation using the same turbulence model as used in the studies by Nakashiki et al\(^4\)\(^5\) was conducted and its applicability was estimated through comparative studies with the observation results.

2 Numerical Model

Figure 1 shows the computational domain divided into two areas to save the calculation time. The small area for the calculation in the near field of an outlet is defined in the size of 2.7km × 2.6km where a three-dimensional k-ε-E model is used for the calculation of the dispersion process of a forced plume with variable fine meshes (see Table 1). The large area for the calculation in the far field is defined in the size of 8.0km × 10.0km where a quasi three-dimensional model is used for the calculation of the coastal currents with coarse constant meshes. A one-way nesting method was adopted to connect with both areas in the calculations, which sets up the inflow boundary condition in the small area by interpolating the computational results in the large area and sets up the outflow boundaries by the Neumann condition as an open sea. The porosity method\(^6\) is used to simulate the sea bottom topography with good accuracy.

![Figure 1: Calculation domain.](image)

Table 1: Calculation meshes.

<table>
<thead>
<tr>
<th>Mesh Size (Large Area)</th>
<th>Mesh Size (Small Area)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Δx = 200(m)</td>
<td>Δx = 1.86(m) ~ 198(m)</td>
</tr>
<tr>
<td>Δy = 200(m)</td>
<td>Δy = 1.86(m) ~ 234(m)</td>
</tr>
<tr>
<td>Δz = 1(m) ~ 2.0(m)</td>
<td>Δz = 1m</td>
</tr>
</tbody>
</table>

2.1 Small Area

2.1.1 Governing Equations

\[
\begin{align*}
\frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j} & = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j}(\nu \frac{\partial U_i}{\partial x_j} - u_i u_j) + \beta(T - T_e) g_i, \\
\frac{\partial U_j}{\partial x_j} & = 0,
\end{align*}
\]
\[
\frac{\partial T}{\partial t} + U_i \frac{\partial T}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \alpha \frac{\partial T}{\partial x_j} - \bar{u}_j \theta \right) + \frac{Q_i (T_e - T_s)}{c \rho H_w},
\]
(3)

\[
\frac{\partial k}{\partial t} + U_i \frac{\partial k}{\partial x_j} = \frac{\partial}{\partial x_j} \left\{ \left( \frac{k^2}{\varepsilon} + \nu \right) \frac{\partial k}{\partial x_j} \right\} + P_k + P_b - \varepsilon,
\]
(4)

\[
\frac{\partial \varepsilon}{\partial t} + U_i \frac{\partial \varepsilon}{\partial x_j} = \frac{\partial}{\partial x_j} \left\{ \left( \frac{C_e k^2}{\varepsilon} + \nu \right) \frac{\partial \varepsilon}{\partial x_j} \right\} + \frac{\varepsilon}{k} \left( C_{e1} P_k - C_{e2} \varepsilon + C_{e3} P_b \right),
\]
(5)

\[
P_k = -\bar{u}_i \bar{u}_j \frac{\partial U_i}{\partial x_j}, \quad P_b = -\beta g_i \bar{u}_i \theta,
\]
(6)

where \( U_i \): mean velocity (cm/s), \( u_i \): fluctuation of velocity (cm/s), \( p \): total pressure (g/cm\(^2\)), \( \nu \): molecular viscosity (cm\(^2\)/s), \( \rho \): density (g/cm\(^3\)), \( \beta \): coefficient of thermal expansion (\(^{\circ}C\)), \( T \): mean temperature (\(^{\circ}C\)), \( T_s \): sea surface temperature (\(^{\circ}C\)), \( T_e \): ambient temperature (\(^{\circ}C\)), \( T^\circ \): thickness of surface layer (m), \( Q_i \): coefficient of radiation (cal/cm\(^2\)s\(^{\circ}C\)), \( c \): specific heat of water (kcal/kg\(^{\circ}C\)), \( k \): turbulence energy (cm\(^2\)/s\(^2\)), \( \varepsilon \): energy dissipation rate (cm\(^2\)/s\(^3\)), \( C_{e1} = 0.09 \), \( C_{e2} = 0.09 \), \( P_{rt} = 0.7 \), \( C_{e1} = 1.44 \), \( C_{e2} = 1.92 \), \( C_{e3} = 1.4 \sim 2.0 \), \( C_{\theta} = 0.13 \), \( C_{\theta 1} = 0.62 \).

2.1.2 Boundary Conditions

At the inflow boundary, the velocity was fixed as the values interpolated from the computational results in the large area. The temperature was set equal to the ambient temperature and \( k \) and \( \varepsilon \) were set as follows,

\[
k = \alpha U^2 \left( \alpha = 0.002 \right), \quad \varepsilon = C_{\mu} k^{3/2} / h,
\]
(7)

where \( h \): sea depth. The values of \( k \) and \( \varepsilon \) at an outlet were also set by the equation (7) with defining \( h \) as the height of an outlet. At the wall boundary, the velocity was set as a free-slip condition and the other variables were set as the Neumann condition. The Neumann condition was used for all the variables along the other lateral boundaries. At the sea surface, the velocity was defined as a free-slip condition and the temperature as adiabatic in summer and radiative in winter, and the other valuables as the Neumann condition.

2.1.3 Calculation of Eddy Viscosity and Diffusivity

The eddy viscosity of a forced plume was defined as follows,

\[
\nu_t = C_{\mu} \frac{k^2}{\varepsilon}.
\]
(8)

On the other hand, the eddy viscosity of the coastal current \( \nu_A \) was defined on the basis of the observation data, and the Richardson's four-thirds power law near the coast,

\[
\nu_A = \min(\nu_{ob}, \nu_L),
\]
(9)
where \( \nu_{\text{obs}} \) : observation data, \( \nu_L : 0.01 \times L^{4/3}(cm^2/s) \). The eddy viscosity \( \nu_T \) for the calculation from an outlet to the far field was finally defined as follows,

\[
\nu_T = \max(\nu_A, \nu + \nu_i).
\]

The eddy diffusivity was also defined by the value of the eddy viscosity with the Prandtl number of 0.7.

2.2 Large Area

2.2.1 Governing Equations

\[
\frac{\partial \overline{U}_i}{\partial t} + \overline{U}_j \frac{\partial \overline{U}_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} (A_j \frac{\partial \overline{U}_i}{\partial x_j}),
\]

\[
\frac{\partial \overline{U}_j}{\partial x_j} = 0,
\]

\[
\frac{\partial \eta}{\partial t} + \overline{U}_i \frac{\partial \eta}{\partial x_i} - \overline{W}_s = 0,
\]

where \( \overline{U} \) : mean velocity (cm/s), \( \eta \) : sea level (cm), \( \overline{W}_s \) : vertical mean velocity at surface (cm/s). The horizontal eddy viscosity \( (A_x, A_y) \) was defined as \( 5 \times 10^4 \text{cm}^2/\text{s} \) from the result of the field observation and the vertical eddy viscosity \( A_z \) was assumed as \( 1 \times 10^0 \text{cm}^2/\text{s} \).

2.2.2 Boundary Conditions

The sea level was set in the open boundaries based on the tidal data. The velocity was defined as a free-slip condition at the wall and as the Neumann condition at the other boundaries.

3 Computational Results and Discussions

3.1 Coastal Current

The amount of \( \sim 50 \text{m}^3/\text{s} \) of thermal water was discharged from the nine pipes with speed of \( 3.0 \text{m/s} \) at the depth of \( 4 \text{m} \) under the surface (see Table 2 and table 3). The calculation was conducted in the two weather conditions of summer and winter. The radiative coefficient was inferred as \( Q_1 = 8.17 \times 10^{-4} \text{cal/cm}^2\text{s}^\circ \text{C} \) based upon the study by Katano.\(^8\)

Figure 2 shows the tidal ellipse comparing the both calculated and observed results at four stations (see Figure 3). Although there are a little discrepancy in the results at ST10\((\sim 1 \text{mand} - 3 \text{m})\) and ST25 \((-3 \text{m})\), the both current patterns are approximately in good agreement. The ebb current was dominant during the observation time. Figure 4 shows the ebb
current pattern in the large area, which is used for the calculation of the thermal water diffusion.

Table 2: Numerical conditions.

<table>
<thead>
<tr>
<th>Environmental Temperature(°C)</th>
<th>Discharged Temperature(°C)</th>
<th>AT(°C)</th>
<th>Salinity</th>
<th>Discharge(m³/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>August, 1994</td>
<td>December, 1982</td>
<td>28.6</td>
<td>34.4</td>
<td>5.8</td>
</tr>
<tr>
<td>22.0</td>
<td>14.3</td>
<td>5.8</td>
<td>30.0</td>
<td>48.9</td>
</tr>
<tr>
<td>14.3</td>
<td>19.3</td>
<td>5.0</td>
<td>31.0</td>
<td>50.0</td>
</tr>
</tbody>
</table>

Table 3: Characteristics of the outlet.

<table>
<thead>
<tr>
<th>Number of Pipes</th>
<th>Diameter of Pipe</th>
<th>Discharged Velocity</th>
<th>Height of Outlet</th>
<th>Water Depth</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>1.54(m)</td>
<td>3.0(m/s)</td>
<td>D.L. - 4(m)</td>
<td>D.L. - 6(m)</td>
</tr>
</tbody>
</table>

Figure 2: Comparison of the tidal ellipse of both the observations (solid line) and the calculations (dotted line).

3.2 Eddy Viscosity and Eddy Diffusivity

The eddy viscosity and diffusivity near the outlet was observed at the point ~100m off the outlet (see Figure 5) by the velocimeters at three layers of 2.0m, 4.0m and 5.5m above the sea bottom. The horizontal velocity distribution of a forced plume at the surface was also measured by ADCP towed by a ship. The energy spectrum of the observed velocity showed that the peak of the energy appeared on the period of 100 seconds. This
peak seems to be the period due to a forced plume, because the significant wave period is 2.6 seconds and the period of the seiche of the surrounding bay are 350 and 700 seconds. Then the eddy viscosity and diffusivity were calculated from the fluctuations of the velocity shorter than 100 seconds.

Figure 5 : Surface velocity distribution Figure 6 : Relation between the length of a forced plume by the ADCP. scale of a plume and the eddy diffusivity.

3.2.1 Horizontal Eddy Diffusivity

Brooks proposed the length scale to estimate the horizontal eddy viscosity of an effluent plume in a coastal area as follows,

\[ L = 2\sqrt{3}\sigma, \quad \sigma = \frac{1}{1.88} b_{1/2}, \]  

(14)

where \( L \) : length scale, \( \sigma \) : standard deviation of velocity distribution, \( b_{1/2} \) : half width of velocity distribution. Based on the width at the point of \( b_0 \), the value of \( b_{1/2} \) at the point of \( l_1 \) where the velocimeter was set was derived and the length scale of \( L \) was estimated. The horizontal eddy diffusivity at the point of \( l_1 \) was calculated from the observation data according to the Taylor’s theorem as follows,

\[ A_H = \beta u^2 \int_0^\infty \gamma_u(\tau) d\tau, \]  

(15)

where \( \beta \) : Euler-Lagrange transform coefficient(=1), \( \gamma_u \) : auto-correlation of velocity, \( \tau \) : phase. Both values of the horizontal eddy diffusivity and the length scale were subject to the Richardson’s four-thirds power law (see Figure 6). This result indicates that the horizontal eddy diffusivity can be well estimated by the length scale in the area where each plume forms one large plume. However, a turbulence model is useful to estimate the eddy diffusivity in the area close to the outlets, where the turbulent mixing of each plume is highly excellent and it is difficult to estimate the eddy diffusivity by an empirical method.
3.2.2 Vertical Eddy Viscosity

The vertical eddy viscosity was estimated from the observation data at the point of $l_1$ by the following three methods based on the Taylor's theorem $A_{z1}$, the Reynolds stress $A_{z2}$, and the logarithmic law $A_{z3}$ (see Table 4). According to the calculated overall Richardson numbers, the vertical eddy viscosity is expected small in Period1 and Period2, and large in Period3 and Period4 by the effect of the density stratification. The table indicates that the only $A_{z1}$ is consistent with this tendency. The vertical eddy viscosity $A_{z1}$ at the point of $l_1$ is $\sim 1 \times 10^2 \text{cm}^2/\text{s}$. Figure 7 is the distribution of the calculated vertical eddy viscosity at the depth of 4m below the surface at which the observation data is measured. The calculated vertical eddy viscosity is in the same order $O(100)$ of the observation data.

Table 4: Results of the vertical eddy viscosity by the observation data.

<table>
<thead>
<tr>
<th>Period</th>
<th>Height above Bed</th>
<th>$A_{z1}$</th>
<th>$A_{z2}$</th>
<th>$A_{z3}$</th>
<th>$R_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period1</td>
<td>5.5(m)</td>
<td>158.6</td>
<td>202.3</td>
<td>952.2</td>
<td>0.76</td>
</tr>
<tr>
<td>August 2</td>
<td>4.0(m)</td>
<td>138.5</td>
<td>272.2</td>
<td>790.6</td>
<td></td>
</tr>
<tr>
<td>(15:00-18:00)</td>
<td></td>
<td>122.7</td>
<td>164.8</td>
<td>302.5</td>
<td></td>
</tr>
<tr>
<td>Period2</td>
<td>5.5(m)</td>
<td>133.7</td>
<td>196.8</td>
<td>644.9</td>
<td></td>
</tr>
<tr>
<td>August 2</td>
<td>4.0(m)</td>
<td>106.0</td>
<td>284.8</td>
<td>492.2</td>
<td>1.96</td>
</tr>
<tr>
<td>(21:00-24:00)</td>
<td></td>
<td>71.2</td>
<td>292.0</td>
<td>202.5</td>
<td></td>
</tr>
<tr>
<td>Period3</td>
<td>5.5(m)</td>
<td>85.9</td>
<td>826.0</td>
<td>812.2</td>
<td>4.26</td>
</tr>
<tr>
<td>August 3</td>
<td>4.0(m)</td>
<td>108.3</td>
<td>448.9</td>
<td>649.1</td>
<td></td>
</tr>
<tr>
<td>(12:00-15:00)</td>
<td></td>
<td>88.0</td>
<td>65.9</td>
<td>154.8</td>
<td></td>
</tr>
<tr>
<td>Period4</td>
<td>5.5(m)</td>
<td>63.8</td>
<td>228.7</td>
<td>481.4</td>
<td>4.15</td>
</tr>
<tr>
<td>August 3</td>
<td>4.0(m)</td>
<td>77.6</td>
<td>167.7</td>
<td>464.2</td>
<td></td>
</tr>
<tr>
<td>(17:00-20:00)</td>
<td></td>
<td>67.3</td>
<td>52.3</td>
<td>163.1</td>
<td></td>
</tr>
</tbody>
</table>

3.2.3 Mean Profile of Velocity and Temperature

Figure 8 shows the velocity distribution at the surface. The profile of the velocity and the trajectory of the plume bending eastward affected by the coastal current are well consistent in both results. The maximum value of the surface velocity $\sim 60 \text{cm/s}$ and its position to appear at the surface also agreed well in both results, however the trajectory of the calculation near lands is much bent compared with the observation result. One of the reason of this discrepancy is the underestimation of the velocity near a land because of the coarse mesh.

Figure 9 shows the comparison of the horizontal temperature distribution of both observational and computational result at the surface, where the temperature is shown as the relative value against the ambient temperature. The both offshore extent of the value of 1°C approximately agreed well, however there is a little discrepant as to the width of the diffusion extent between both results. The assumption of isotropic turbulence in a
plume might be the reasons of this discrepancy.

Figure 8: Surface velocity distribution of the observation (left) and the calculation (right).

Figure 9: Surface temperature distribution of a forced plume against the ambient sea temperature.

4 Conclusion

The diffusion mechanism of a forced plume discharged from submerged multiple pipes in a coastal area was investigated by in-situ observation and computation by a three-dimensional k-ε-E model. In the area where each plume overlapped and formed one large plume, the observed vertical eddy viscosity was $\sim 1 \times 10^2 \ (cm^2/s)$ and consistent with the calculation result as order of the value. The calculated results also showed that the direction of the plume, the horizontal distribution of the velocity, and the offshore distance of temperature diffusion at the surface were in good accordance with the observation results. There were some discrepancy in the results between observation and calculation owing to the difference of the mesh size in two computational domain and the assumption of the isotropic turbulence. The result, however, suggested that the three-dimensional k-ε-E model was effective for the prediction of the diffusion process of the thermal water in a coastal area in practical use.
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References


