Visibility and viewshed algorithms in an information system for environmental management

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Abstract

Viewshed calculation is one of the basic procedures existing in an Information System, which has the ability to manage elevation data. Visibility calculation used to be a very time consuming procedure. The rapid development of computer technology propagated the demand for faster and better algorithms, of higher quality and of a more sophisticated logic. Viewshed depends on the height of the viewing point. The alteration of this height brings alteration of the viewshed. Different techniques have been developed concerning the different ways according to which: (a) data are collected from the Digital Elevation Model (DEM) for the viewshed calculation, (b) Visual Point and Target Point is defined horizontally and vertically, (c) the decision VISIBLE or NOT VISIBLE is taken for a point. An answer "VISIBLE" or "NOT" is not enough information to define if a point is really visible or not. Visibility depends on a series of factors and conditions that must be taken into account. This article attempts to collect all necessary information about visibility and viewshed analysis concerning conventional approaches. Basic viewshed algorithms are described, alternative methods of viewshed analysis are examined and different techniques used in viewshed calculation are referred. All this analysis of algorithms and techniques is accompanied by certain environmental application examples. Further on, factors that influence the accuracy of a viewshed and also possible control procedures are referred.
Keywords: visibility, viewshed, GIS, environment, accuracy.

1   Introduction

Viewshed calculation from a given observation post is nowadays one of the basic processes accomplished with the use of GIS [1]. The concept of visibility was initially examined with the appearance of Digital Terrain Models (DTM) and GIS. The development of DTMs and GIS has solved many problems related to difficult and repetitive calculations, giving the scientists the opportunity to better manage and utilise their time. A definition of visibility is [2]:

"Two points in a DTM (P, P') are intervisible, only when there is a straight line which connects point P to point P' without intersecting the DTM at a point between points P, P'."

With regard to the viewshed, V table is formulated, whose elements are a) $V_{ij}=1$ when P_{ij} is visible from P; and b) $V_{ij}=0$ in the opposite case. The viewshed contains the areas of table V, where $V_{ij}=1$. Fisher in 1994 [1, 3, 4, 5] and De Floriani and Mangillo in 1993 [6, 7] have proposed other visibility analysis types, which are horizons (primary and secondary) and deviations from the line of sight. Horizons are lines of viewshed that separate the visible relief from the sky (skyline). The viewshed does not extend beyond the primary horizon [1]. Deviations are defined as the elevation differences between the line of sight and the points of the relief.

Alternative visibility analysis types may be useful in landscape analyses and in monitoring any visual impact due to the spatial arrangement of activities, which must or must not be visible. Of particular use is the notion of deviation, which defines the maximum or minimum permissible elevation limits of the activities.

Viewshed depends directly on the observation elevation. Usually an increase of the observation elevation results in an increase of the surface area of the part that is visible. Calculating a viewshed is usually a time-consuming procedure. Developments in IT favour only a partial solution to the problem, which still remains unsolved. Various algorithms have been developed in order to address the problem of time-consuming calculations by using "intelligent" processing paths.

Various types of viewsheds have been devised and developed, in order to meet different needs. Therefore, the typical viewshed, where each point / pixel / triangle is determined as visible or not visible, is a binary representation of the problem. In other viewsheds, each point is codified in proportion to the number of times it is seen from the observation posts [4], or in proportion to the surface area of the wider region where it has visibility within a particular radius. One could easily identify from such maps the observation posts that provide a long range of visibility and towards a particular direction.

New types of viewsheds are the probabilistic viewsheds [3, 5, 8, 9, 10] and the fuzzy viewsheds [3] which do not determine their viewpoints in the conventional manner, as "Visible" or "Not Visible", but provide either the possibility for a viewpoint to be probably seen, or the value of the membership function in the viewshed (fuzzy).

This paper presents the notions of visibility and viewsheds with references from the literature, trying to cover the main aspects of this matter. The usefulness of these algorithms in a management information system is obvious, especially when researching and addressing environmental issues and problems.

Visibility, viewsheds and the various calculation algorithms are presented within the framework of conventional approaches. This paper does not address non-conventional approaches, such as probabilistic approaches that have been developed and are researched during the last years [3, 5, 8, 9, 10].

2   Viewshed calculation algorithm

A general conventional algorithm for calculating visibility between two points is the following [11]:

- Start from a view point A
- Specify the line of sight which connects view point A to a certain target point B
- "Move along" the line of sight from view point A to target point B
- If one of the points of the relief, along the line of sight, is higher than the line of sight, then define target B as "Not Visible"
- If upon reaching target B, no point of the relief is encountered along the line of sight, which is higher than the line of sight, then define target B as "Visible"

In its generic form, this conventional algorithm is the basic logic in all visibility calculation, viewshed and visibility analysis algorithms [12, 13, 14]. What differentiates the various algorithms is the technique for obtaining the data (viewpoint location, target point location, elevations) and the decision-making technique for "Visible" or "Not Visible" targets.

In the case of viewshed calculation, the algorithms are differentiated as to the process flow, the techniques used for reducing the calculation time and the techniques applied for optimizing the quality of the results.

2.1 Traditional viewshed calculation algorithms

This section refers to traditional algorithms used for calculating viewsheds. The term "traditional" distinguishes these algorithms from other algorithms that are also conventional, but more "intelligent", more flexible and certainly more appropriate in several and, perhaps, more specialized cases. In addition to the standard algorithm, two known "traditional" algorithms are the following:

- Algorithm Weighted on the Visible Surface Area from the Observation Post
- Algorithm Weighted on the Distance from the Observation Post

In the standard algorithm, which was described in section 2, lines of sight are implemented from the viewpoint to each target point in the area of interest (usually in an area of specific radius). Each target point is examined as "Visible" or "Not Visible" and is codified accordingly using binary representation (1 or 0) [1, 4, 6, 8, 9, 11, 15]. This algorithm is the most time-consuming one.

Another form of recording visibility, which does not examine whether the pixels' location is "Visible" or "Not Visible", but it examines the percentage of their surface area, which is visible from the observation post, is developed in the algorithm weighted on the visible surface area from the observation post. Pixels are not viewed as horizontal, flat or square forms, but as an inclined plane with specific slope and aspect. The observer therefore deals with the possibility not to see the pixel's surface at all or to see a part of it or its entire surface. What should be examined is the angle formed by the line of sight with the vertical line in the center of the pixel [15]. This algorithm is very useful in cases where binary information (Visible", "Not Visible") is not adequate.

It is normal for the visibility range in a viewshed to decrease when the target point is displaced away from the viewpoint. Therefore, the most distant target points, although characterized as "Visible", like the proximal target points, provide lower viewshed due to the distance. The third traditional algorithm makes use of this property and codifies the pixels, not with a binary value (1, 0), but with a weight that varies according to the distance [15]. This is accomplished, on the condition that the pixel is visible on the basis of the standard algorithm.

2.2 New enhanced viewshed calculation algorithms

Certain new algorithms are based on the basic logic of the previous methods and are differentiated as to the technique, which they try to improve and accelerate. A technique for accelerating the visibility analysis process is the main ray technique. The Main Ray Technique makes use of the property of a "Visible" point to present a wider vertical angle calculated from the Nadir, from the not visible points that follow it in the track of the line of sight over the relief (Figure 1) [16]. This technique includes the following procedure:

- Point 1 is not examined, because it is always regarded as "Visible". The ray angle of point 1 is measured and determined as the main ray
- The angle of point 2 is measured and compared to the angle of the main ray
- If it is narrower, then point 2 is "Not Visible" and the procedure moves to point 3
- If it is wider, then point 2 is "Visible" and its angle is determined to be the angle of the main ray. The procedure moves to point 3
- All points are examined along the track of the line of sight, without the need to examine each point separately with its own line of sight. Therefore, the examination of e.g. Pixel 5 does not require the examination of all the previous pixels, but only of pixel 4 (i.e. the first pixel upstream)

The three known algorithms that are presented below are the "Spiral" Examination Algorithm, the Perimetric Examination Algorithm and the "Pseudo-target" Algorithm.

Based on the "Spiral" Examination Algorithm, the visibility analysis and the viewshed calculation develop spirally outwards from the viewpoint to the

perimeter of the ray defined as the visibility barrier (Figure 2) [16]. Each point of the spiral is examined with the use of the main ray technique. After examining the point, the elements of its main ray are registered, and are therefore available for use in the next spiral cycle in the corresponding direction. The elements of the main directions are then interpolated at the intermediate points beyond the 8 main rays (i.e. the majority of the points), in order to proceed with the examination of the points' visibility (Figure 2). For example, in order to examine point B03, points A01 and A02 are interpolated at the elements of their main rays.

   One of the advantages of this algorithm is that each point, in order to be examined in terms of its "Visible" or "Not Visible" property, needs only the elements of the main ray (or of the main rays, if interpolation is required) of the point (or the points) of the immediately preceding spiral cycle. This accelerates significantly the algorithm in comparison to the speed of traditional algorithms mentioned above.
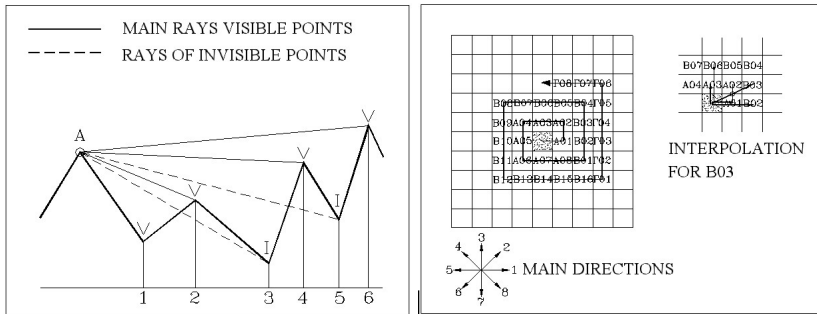

Figure 1: Main ray technique.   Figure 2: Spiral examination algorithm.

   The perimetric examination algorithm makes also use of the main ray technique. Its difference with the "spiral" examination algorithm is the logic on which the viewshed calculation develops [16]. The pixels that constitute the perimeter of the area whose viewshed is required are identified based on the radius within which the viewshed is calculated. The lines of sight towards each of these, resolved based on the main ray technique, are implemented from the viewpoint. The result of this resolution is that each pixel in contact with the track of the line of sight on the relief is codified as "Visible" or "Not Visible" (Figure 3). This algorithm is definitely faster than the traditional ones already mentioned. However, we couldn't say whether it is faster or slower than the first algorithm, that of the "spiral" examination. This is due to the fact that speed difference depends on various parameters in each case (pixel size, hypsometric data range, etc.).

   In order to determine whether a target point is visible, the "pseudo-target" algorithm makes use of the property: "a target point is definitely visible from the intersection point of its line of sight with the horizontal plane at the highest elevation of the track of the line of sight on the relief" (Figure 4) [11].

Based on this property, the algorithm substitutes this target point with a "pseudo-target" point, which is placed at the intersection point. Therefore, the visibility examination between the viewpoint and the target point is a matter of examining the existence of visibility between the viewpoint and the "pseudo-target" point. The examination is limited within spacing D', instead of D, which under suitable conditions accelerates significantly the calculation of visibility. This algorithm may prove useful in the visibility calculation, in cases of forest and fire protection network monitoring, scheduling of flights of identification aircraft, specification of air-defence locations (anti-aircraft weapon systems), etc.
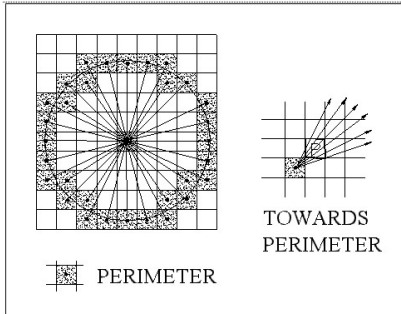

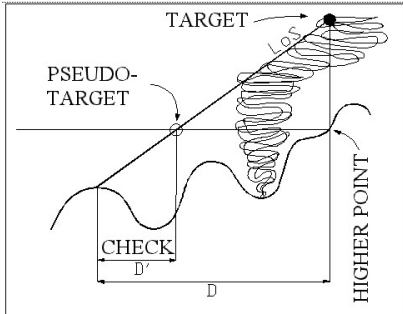Figure 3: Perimetric examination algorithm.

Figure 4: "Pseudo-Target" algorithm (the case of a target well above the relief).

3   Obtaining data for viewshed calculation

Algorithms are often compared and their results are also compared, without paying much attention to the quality of the elements used and to the way these are measured. With regard to viewsheds, three aspects are considered to be important and must be presented:

- How hypsometric information is obtained from a DTM.
- How the orientation of a viewpoint and a target point is carried out at a horizontal tracing position (and following this hypsometrically).
- How the decision if a target point is "Visible" or "Not Visible" is made.

3.1 Obtaining hypsometric information

A DTM is a hypsometric information registration in various points in space. A line of sight, when crosses a DTM, is nearly impossible to pass over only the points that have hypsometric information and are located either on the center of the pattern or on one of its edges. What is needed is to define the hypsometric location of the track of the line of sight on the DTM. There are many related techniques suggested and used by various scientists during the last years.

The gridded model technique was proposed by Yoeli in 1985 [3]. In this technique, the elevation of the DTM characterizes the centre of each pattern. The

centers are interconnected horizontally and vertically (not diagonally) to a grid of sloped lines. The elevations of the track of the line of sight on the DTM are calculated at the intersections of the track line with the grid of the sloped lines (Figure 5). An interpolation is applied at the points of intersection in the sloped segments.

The Digital Elevation Model Triangulation Technique follows the logic of the gridded model technique, by extending the use in diagonal connections [3]. Diagonal connections are used, which intersect the line of sight laterally (Figure 6). The elevation values along the track of the line of sight are calculated at the intersection of the track line with the grid of these sloped segments.

The densified gridded model technique was proposed by Tomlin in 1990 [3]. She suggested the overlaying of a secondary grid of horizontal and vertical lines on the initial main grid, at the limits of the patterns (Figure 5). The elevation values in the connections of the secondary grid are calculated with interpolation in the elements of the main grid. The elevation values for the track of the line of sight over the DEM are calculated at its intersections with the main and secondary grid of straight - line segments.
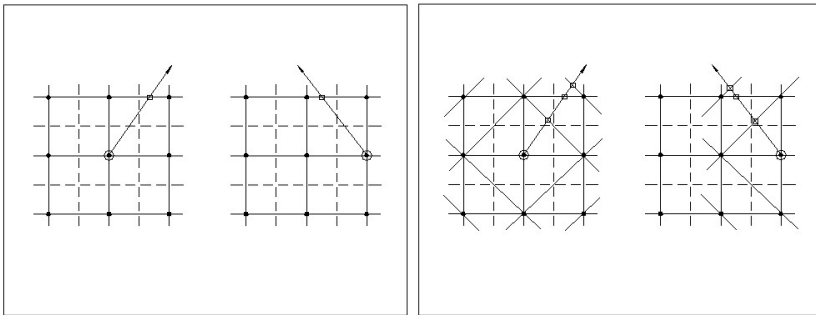

Figure 5: Gridded model technique.   Figure 6: DEM triangulation technique of normal structure.

This last technique addresses DEM as horizontal quadrangular surfaces, of the size of a pixel, which are located at the elevation that characterises them. This approach results in the DEM presenting "stairs-steps" at the limits of the pixels. The elevation values for the track of the line of sight are calculated at the points where the track meets and intersects these "stairs-steps". Due to the discontinuity of the "stairs-steps", the elevation of the "stairs" is recorded to be the higher of the two (Figure 8) [4]. This technique has been used by many scientists (Felleman, Travis / 1990, Teng, Davis / 1992, Burrough / 1986, see [3]) not only for visibility analysis purposes, but also for any kind of DEM management (discharge drainage, intersections, drainage networks, etc.).

In the first three techniques, progressing gradually from the first to the third, the extent of the viewshed is reduced. This is due to the fact that when progressing from the first to the third technique, the number of the points with an elevation, which should be compared to the elevations of the line of sight at the

corresponding locations, increases. The increase of these points increases also the possibility for the target point to be determined as "Not Visible", resulting to the fact that the total amount of "Not Visible" targets in a viewshed increases at the expense of the "Visible" targets.

The problem of the limited viewshed is even more intense in the fourth technique, that of the "stairs-steps". The projections created by the "stairs" increase the possibility of visibility obstruction, and therefore, of characterizing more target points as "Not Visible". A relevant examination by Fisher 1993 has shown that when using the fourth technique the extent of the viewshed is reduced by 75-95% compared to the first technique [3]. Therefore, the possible reduction is indeed significant and depends on the relief form, on which the viewshed calculation takes place.
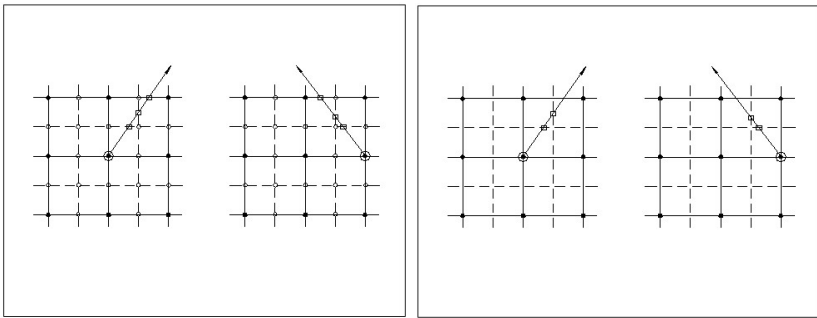

Figure 7: Densified gridded model technique.   Figure 8: "Stair-steps" technique.

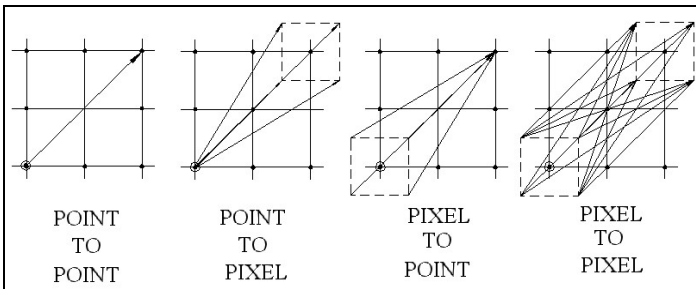POINT TO POINT   POINT TO PIXEL   PIXEL TO POINT   PIXEL TO PIXEL

Figure 9:   View and target point orientation.

3.2 View point and target point orientation

Since a normally structured DEM contains the notion of the pattern, we deal with the dilemma which location x, y, z on the pattern to define as the observation post. Therefore the following question arises: the observation post and the target are seen as points or as patterns occupying a certain surface area? And if they are seen as patterns, which of their elements shall be examined, given that patterns as surfaces have an indefinite number of points?

Fisher assumes that the observation posts and the targets locations, if seen as patterns, are then represented by their four angles [3]. Therefore, there are four possible combinations to address the problem, which are presented in Figure 9.

Furthermore, the following rules / assumptions apply, in order to decide whether a target shall be included in the viewshed:

1. A Target point is defined as "Visible" from an Observation Post, if it fulfills the definition of visibility
2. A Target pattern is defined as "Visible" from an Observation Post, if at least one of its angles fulfils the definition of visibility
3. A Target point is defined as "Visible" from an Observation Pattern, if it fulfils the definition of visibility from at least one angle of the Observation Pattern
4. A Target Pattern is defined as "Visible" from the Observation Pattern, if at least one angle fulfils the definition of visibility from at least one angle of the Observation Pattern

It can be readily concluded that the first case will present much narrower viewshed than the fourth case, because in the fourth case the examination of visibility for the same pair of observation post – target post, is performed 4x4 times, i.e. 16 times, and one time is adequate for defining it as "Visible" so as to determine the target as "Visible".

3.3 Decision-making for "Visible" or "Not Visible" target

After having determined the techniques to obtain the previous data in the visibility examination, we must combine this data and compare the line of sight to its track over the DEM, so as to decide if the target is "Visible" or "Not Visible". This comparison can be carried out in many ways, which are summarized below, in Figure 10 [3].
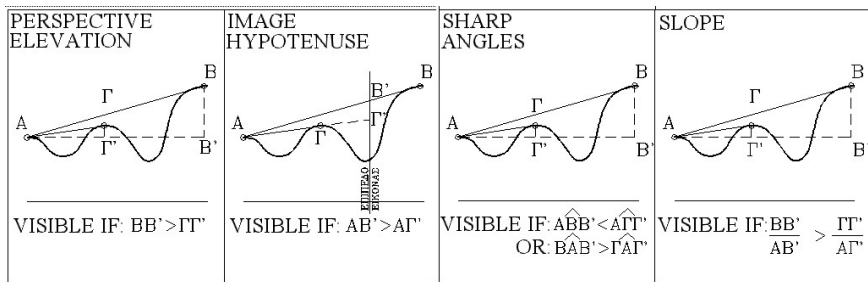

Figure 10:   Decision making techniques for "Visible" or "Not Visible" target.

It should be stressed that these comparison methods constitute a purely mathematical process treatment of exactly the same data and they should give exactly the same result. This, however, is not really true, because the operations performed by the computer depend on the accuracy and the decimals that can be processed by the computer in question.

When comparing the viewsheds that derived from these four comparison techniques, the first one with a comparison of the perspective elevation and the

second one with a comparison of the inclination of the connection line to the observation post [3], it appeared that there is a difference between the two viewsheds as to their size. This is due to the fact that the inclination of the connection line involves more operations with "sensitive" numbers which affect the accuracy of the final results in the comparison.

4   Viewshed examination

The key factors that affect the accuracy of a viewshed are [5]:
- Hypsometric Information Accuracy
- Viewshed Calculation Algorithm
- Hypsometric Information Structure (DEM structure)
- Accuracy of the Horizontal tracing and Hypsometric Location of the View point and the Target point

Each of these factors affects, more or less, the accuracy of the viewshed and alters its form and shape. The most important factor is the influence of the accuracy of the information used [2, 17, 18, 19, 20], which is not usually known, and then the influence of the structure of the hypsometric information (in normally structured DEMs, the pattern's size) [21]. These effects are not always linear and proportional. Randolph and Ray [16] found out in one of their studies that the larger the viewshed of a point, the smaller the influence of the DEM's accuracy on its size.

One of the main problems regarding the viewshed calculation, in contrast to many other functions of GIS, is the ex-post examination of its quality, due to various factors [3]. These factors are summarized in two categories:
- Physical obstacles and physical deformities
- Error factors in the hypsometric information and in the simulation of the physical ground relief

The only way to control the viewshed calculated in relation to the actual viewshed is its perspective representation (3D) [21] and its comparison to what is seen in the viewshed in the specified observation post. This examination can be carried out either with an on-site visit in the physical relief, or with the use of photographs taken from the view point and the shooting data taken from the perspective representation.

It will be concluded that there are deviations between the two viewsheds, and this is due to the two categories of factors mentioned above.

The first category includes factors such as the vegetation covering the relief, the effect of the Earth's curvature, the effect of the atmosphere on the visibility, the effect of the solar light reflection and the shading of the relief. The factors of the plant coverage and the Earth's curvature may alter the viewshed upon its examination. The first factor may cause great alteration, while the second one a smaller one, as it affects only long distances from the view point. These effects are static and permanent. On the contrary, the factors of the atmosphere, of the solar light reflection and shading may affect the viewshed examination, to a smaller or larger extent. This effect is variable (dynamic), because it is a function

of time. These factors are not static and cause certain vagueness in the image the observer sees or shoots in the viewshed [1, 5, 10].

The second category of factors, which affect the calculated viewshed making it not comparable to the corresponding actual viewshed, includes the hypsometric data and physical relief description model (DEM, TIN, etc.).

The factor of the quality of the hypsometric information was presented in the previous paragraphs. Deviations of the hypsometric information from reality, of the order of 10m, 20m, 30m, [17] cause a variation in the viewshed and difficulties in its examination. The approach of the physical relief by a distinct model is another factor that makes the viewshed examination harder. However, we cannot avoid this factor.

The great influence of all the above-mentioned factors in the quality of the viewshed examination, in combination to the fact that these factors do exist at a large extent, leads us to the conclusion that viewsheds include a great deal of uncertainty [1, 8, 9]. This uncertainty guides research into new probabilistic approaches, such as the probabilistic viewsheds and the fuzzy viewsheds [3, 5, 8, 9, 10]. These approaches reject the use of a clear binary viewshed form (VISIBLE, NOT VISIBLE) and adopt milder forms (PROBABLY SEEN, SEEN WITH A p% PROBABILITY).

5   Comments – discussion

This paper attempts to present an aspect of the complexity and the significance of viewshed calculation and of the various approaches that have been developed and to some extent remain to be acknowledged. An attempt was made to present to the audience and the user of various viewshed calculation algorithms the fact that the viewshed is not just an examination of the hypsometric information of the type "**HIGHER" or "LOWER"** having direct result into the definition of "VISIBLE" or "NOT VISIBLE TARGET"; it is a more serious process requiring knowledge and skepticism. It is a process that does not allow the evaluation of its results, without prior knowledge of the parameters involved.

The presence of these algorithms in management information systems and, especially, in systems that manage environmental issues and problems constitute a significant and integral part of the approach. However, it involves risks in cases of ignorance, and given also the speed in obtaining the results using such systems, the results produced are either inadequate or false. Decision-making based on questionable results may complicate the problems and lead to their false management.

This paper attempted also to provide the reader with the dynamic of such processes. The word dynamic means that these processes cannot be regarded as static and therefore not susceptible of modification and improvement. On the contrary, they evolve and develop with the aim to optimize their result.

The concept of the accuracy of the result is included in all the forms of the results that derive from data processing. It is now a common place that each registration, processing and transformation of information is accompanied by an

error margin and a percentage of uncertainty. This error is transmitted to the information products, through the error transmission law.

Uncertainty leads to a probabilistic approach of the problem. In this way, one could imagine dealing with the perspective representation of a calculated viewshed; this representation is accompanied by other probable perspective representations of the same viewshed. Probabilistic and fuzzy approaches of the viewshed are still under research and will continue to be a matter of study for scientists in the forthcoming years.

What is of concern is the utilization of the results of these new approaches. How a probabilistic viewshed or a fuzzy viewshed will be utilized, given that the existing GIS software cannot manipulate probabilistic or fuzzy information. Furthermore, it is difficult to convince software development and marketing companies to integrate the results of the research in their systems, given the time-consuming and cost-burdening procedures.

Moreover, there is a great problem with all relative bodies that provide digital (or sometimes analogue) information to the public. These bodies are either unable to deliver digital information with its relative accuracy (at a spatial or not spatial level, as the case may be) or sometimes avoid doing so in order to prevent doubts as to the credibility and the quality of their research results.

Issues such as the one mentioned above should be specified and regulated by pertinent bodies and organizations, in order to allow users of the systems as well as of the information to have access to information about the accuracy of the final product, using sounded and widely acceptable processes.

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