Stochastic integer programming analysis for wastewater treatment plant design

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Abstract

It is the purpose of this paper to present a methodology of analysis which identifies optimum types and combination of unit treatment processes from the range of available alternatives. The objective is to identify the optimum combination and efficiencies of various unit processes in multistage plant; then meet design criteria. Optimality, as used herein, is defined as meeting a given treatment requirement (treatment efficiency, allowable stream loading, etc) at minimum total annual cost. Stochastic Integer Programming is the main principle used to develop the optimal model for the liquid waste treatment plant process design.

1 Introduction

There is a lot of literature related to the optimal process design of wastewater treatment plant and most of it applies Dynamic Programming (DP) to build up analysis models. In developing computer program of DP, however, individual designed programs are set up for various specific problems. These disadvantages exist in developing engineering plans or practical design. The advantages of Linear Programming (LP) are that the model establishment can be clearly understood and many popular and efficient software programs are available. As for the wastewater treatment process design, it is necessary to find the optimal flow chart on multistage and multistate treatment units to satisfy various constraints and objective functions.

This study will use Integer Programming (IP) and Mixed Integer
Programming (MIP) to develop analysis model. It will also establish Deterministic Model (DM) and Stochastic Model (SM) by addition, it will analyze and discuss the problems derived from the model establishment and provide the suitable conclusions and suggestions.

2 Defining the model

Various mathematical models will be established based on the logical procedure to analyze and discuss such derived different problems. The DM and SM are established here because the influent water quality is always not at a constant level.

2.1 Deterministic Model (DM)

2.1.1 Single waste constituent, one size.

For any type of wastewater, suppose it contains only one constituent (BOD or SS), there will be a sequence of treatment units, as shown in Figure 1, to treat the wastewater.

![Figure 1 Single Waste Constituent, One Size Flow Chart](image)

From Figure 1, a constraint can be derived:

\[
W_1 (1 - \eta_1)^{y_1} (1 - \eta_2)^{y_2} \cdots (1 - \eta_n)^{y_n} \leq \sigma_1
\]  

Where

- \( W_1 \) : the concentration of constituent in wastewater
- \( \eta_n \) : the treatment efficiency of the nth treatment unit
- \( y_n \) :
  \[\begin{cases} 
  1 & : \text{if the nth unit is selected} \\
  0 & : \text{if the nth unit is not selected}
  \end{cases}\]

By taking the logarithm form, then:

\[
y_1 \log (1 - \eta_1) + y_2 \log (1 - \eta_2) + \cdots + y_n \log (1 - \eta_n) \leq \log \left( \frac{\sigma_1}{W_1} \right)
\]

(2)

If we assume:

- \( a_n = \log (1 - \eta_n) \)
- \( b = \log \left( \frac{\sigma_1}{W_1} \right) \)

and \( C_n \) is the cost of the nth treatment unit.
Then the model can be established as:

\[
\begin{align*}
\min & \quad (c_1y_1 + c_2y_2 + \cdots + c_ny_n) \\
\text{s.t} : & \quad a_{11}y_1 + a_{21}y_2 + \cdots + a_{n1}y_n \leq b_1 \\
& \quad a_{12}y_1 + a_{22}y_2 + \cdots + a_{n2}y_n \leq b_2 \\
& \quad \vdots \\
& \quad a_{1m}y_1 + a_{2m}y_2 + \cdots + a_{nm}y_n \leq b_m \\
& \quad y_n \leq 1, \quad \forall n = 1, 2, \ldots, i, \ldots, n \\
& \quad y_n \geq 0
\end{align*}
\]

This model is of IP.

2.1.2 Multi-constituents, one size.
This can be illustrated in Figure 2.

![Flow chart](image)

Figure 2: Multi-constituents, one size flow chart

Where
- \( \eta_{nm} \): the treatment efficiency of the \( n \)th treatment unit against \( m \)th constituent.
- \( W_m \): the concentration of the \( m \)th constituent.

If we assume:

\[
\begin{align*}
a_{11} &= \log (1 - \eta_{11}), & b_1 &= \log \left( \frac{\rho_1}{W_1} \right) \\
a_{nm} &= \log (1 - \eta_{nm}), & b_m &= \log \left( \frac{\rho_m}{W_m} \right)
\end{align*}
\]

The eqn(4) can be established:

\[
\begin{align*}
\min & \quad (c_1y_1 + c_2y_2 + \cdots + c_ny_n) \\
\text{s.t} : & \quad a_{11}y_1 + a_{21}y_2 + \cdots + a_{n1}y_n \leq b_1 \\
& \quad a_{12}y_1 + a_{22}y_2 + \cdots + a_{n2}y_n \leq b_2 \\
& \quad \vdots \\
& \quad a_{1m}y_1 + a_{2m}y_2 + \cdots + a_{nm}y_n \leq b_m \\
& \quad y_n \leq 1, \quad \forall n = 1, 2, \ldots, i, \ldots, n \\
& \quad y_n \geq 0
\end{align*}
\]

Eqn(4) is also of IP.
2.1.3 Multi-constituents, multi-sizes

This can be illustrated in Figure 3

![Diagram of multi-constituents, multi-size flow chart](Image)

Where

- \( n \) represents the \( n \)th treatment unit
- \( s \) represents the \( s \)th size of the \( n \)th treatment unit
- \( \eta_{nm} \) represents the \( s \)th treatment unit
- \( \eta_{nm} \) represents the \( m \)th treatment efficiency of the \( s \)th size of the \( n \)th treatment unit
- \( Y_{ns} \) is defined as:
  - \( 1 \): if the \( s \)th size of the \( n \)th treatment unit is selected
  - \( 0 \): if it is not selected

Thus, eqn (5) can be established:

\[
\min \left[ (c_{11} y_{11} + c_{12} y_{12} + \cdots + c_{1s} y_{1s}) + (c_{21} y_{21} + \cdots + c_{2s} y_{2s}) + \cdots + (c_{n1} y_{n1} + c_{n2} y_{n2} + \cdots + c_{ns} y_{ns}) \right] \\
\text{s.t. : } \left\{ \begin{array}{l} \\
W_1 \left[ (1 - \eta_{11}^1)^y_{11} (1 - \eta_{11}^2)^y_{11} \cdots (1 - \eta_{11}^s)^y_{11} \right] \cdots \left[ (1 - \eta_{1m}^1)^y_{1m} (1 - \eta_{1m}^2)^y_{1m} \cdots (1 - \eta_{1m}^s)^y_{1m} \right] \leq \omega_1 \\
\cdots \cdots \cdots \\
W_1 \left[ (1 - \eta_{nm}^1)^y_{nm} (1 - \eta_{nm}^2)^y_{nm} \cdots (1 - \eta_{nm}^s)^y_{nm} \right] \leq \omega_m \\
y_{11} + y_{12} + \cdots + y_{1s} \leq 1, \quad y_{1s} = 1 \text{ or } 0 \\
y_{21} + y_{22} + \cdots + y_{2s} \leq 1, \quad y_{2s} = 1 \text{ or } 0 \\
\cdots \cdots \cdots \\
y_{nm} + y_{n2} + \cdots + y_{ns} \leq 1, \quad y_{ns} = 1 \text{ or } 0 \\
y_{ns} \geq 0 \quad \forall n = 1, 2, \ldots, n, \quad \forall s = 1, 2, \ldots, s 
\right. 
\]
2.2 Stochastic model

The above models assume the influent water quality is kept at a constant level, which is mostly not true. In order to reflect the actual condition, SM is developed to obtain the optimal design. There are three kinds of SM: Stochastic Linear Programming (SLP), Two-Stage Linear Programming (LPPU) and chance-constrained Programming (CCP). SLP was initially developed by Tintner[1]. Sengupta and Tintner[2] added some studies, which can be divided into expected value problems and distribution problems. The former means the expected value is included in the objective function by assuming the probability of the valuable variable "c", the right hand side "b" of the above constraints and the constant variable A is known. However, the technique to apply statistical distribution is not widely used because to analyze the sampling and testing adds the complexity of the model. Ellis[3] found that LPPU will enlarge the model, which adds more difficulties to be used by computer. In addition, it is extremely difficult to quantify the penalty function and requisite loss.

In optimal design of water treatment plant, Wu[4] also found that it is very hard to obtain the probability Pj of effluent water quality by assigning the first stage as the removed quantity of impurity before the input of water quality and the second stage as the the exceeded quantity of impurity over design standard after the input of water quality. Therefore, the most practical CCP will be used here to develop SM.

2.2.1 Single constituent, one size

Because the objective function contains no probability and is not affected, eqn(1) can be rewritten as:

\[
\Pr\{\log W_i \leq \log \alpha_i - \sum \log(1-\eta_{i1}) - \sum \log(1-\eta_{is}) \leq \alpha \}
\]

\(\alpha\) is determined by decision-maker.

Because the probability can not exist in constraints, for each constituent, it is necessary to establish the probability density function and cumulative density function based on past records to transform eqn(6) into the equivalent certainty, which makes this model becoming of LP.

2.2.2 Multi-constituents, multi-size.

Eqn(5) can be rewritten as:

\[
\begin{align*}
\min & \quad (e_1y_1 + e_2y_2 + \cdots + e_sy_s) + \cdots + (e_{1}s_1y_{11} + e_{2}s_2y_{12} + \cdots + e_{s}s_y) \\
\text{s.t.:} & \\
\Pr\{\log W_i \leq \log \alpha_i - \sum \log(1-\eta_{i1}) - \sum \log(1-\eta_{is}) \leq \alpha \}
\end{align*}
\]
3 Analysis and Discussion

3.1 Treatment efficiency

In the establishment of above models, it is assumed the treatment efficiency of each unit is constant upon treating each constituent in wastewater. In fact, it is variable and is the function of various factors.

\[ \eta = \phi (\text{Waste Composition, Strength, Volume of Waste}) \]

For DM, not considering the inconsistency of treatment efficiency is acceptable. Different capital costs are derived from different values in the range of treatment efficiency based on flexible design criteria. Higher efficiency requires higher capital cost. Lower efficiency may require lower capital cost but in other way, increases the load to the next treatment unit which will not fit the optimal design principle in entire flow-chart. Therefore, Shin[5] found that the middle value is usually selected in the rang of design criteria. Figure 4 reveals the relationship between the overflow rate and annual cost of primary settling tank and its removal efficiency.

![Figure 4: The relationship between the overflow rate and annual cost of primary settling tank and its treatment efficiency](image)

In SM, under the consideration of the random nature of influent water quality, the middle value of treatment efficiency can still be selected based on the constant flow rate.

3.2 Influent screen

There is a limit to treat each constituent for each treatment unit. Therefore, Ellis [6] found that the influent should be screened to meet this requirement. Table 1 gives screen standards for several common treatment units. This limit can be added into constraints as shown in Figure 5.

<table>
<thead>
<tr>
<th>Treatment Unit</th>
<th>Ultrafiltration</th>
<th>Ion Exchange</th>
<th>Activated Sludge</th>
<th>Reverse Osmosis</th>
<th>Carbon Adsorption</th>
</tr>
</thead>
<tbody>
<tr>
<td>DS, SS, BOD</td>
<td>&lt;50,000</td>
<td>&lt;20,000</td>
<td>&lt;50,000</td>
<td>&lt;50,000</td>
<td>&lt;50</td>
</tr>
<tr>
<td>Zn, Cr, Ni^{+6}, Pb, Phenol, CN</td>
<td>&lt;25,000</td>
<td>&lt;20,000</td>
<td>&lt;10, &lt;2.5, &lt;200, &lt;5</td>
<td>&lt;100, &lt;100, &lt;100, &lt;100</td>
<td>&lt;50</td>
</tr>
</tbody>
</table>
Thus,

\[ W_1 (1 - \eta_1)^{x_1} (1 - \eta_2)^{x_2} \cdots (1 - \eta_{i-1})^{x_{i-1}} \leq 100 \]  

In which, "100" is the influent concentration limit of S.S. when using ultrafiltration.

### 3.3 Consideration of multi-types

All of the above discussion considers only single source wastewater from municipalities. However, the wastewater treatment plants in industrial parks in Taiwan receive wastewater from many more sources. The distribution condition of probability distribution function for each constituent should be considered to establish correct cumulative density function. From the study of Ellis, the constituent concentration distribution of metal, chemical and tannery waste is a lognormal Probability Density Function (LPDF). Figure 6 shows a typical relationship between the COD concentration of metal waste and the probability in log-log paper. Ellis [7] found that the dependent relation between constituents from the same source may exist when using some treatment units. This will certainly affect the treatment efficiency. For example, a positive relation exists between oil/grease and COD concentrations in metal waste. This means the COD removal will be accompanied with oil/grease removal. Table 2 outlines the parameters in LPDF for each waste constituent.

![Figure 5: Influent screen sequence](image)

![Figure 6: The probability delineation of lognormal probability density function](image)
The eqn(10) is introduced:

\[ \text{Log(concentration)} = Xn + Sn \cdot (z) \]

Where

- \( Xn \): average logarithmic concentration
- \( Sn \): logarithmic standard deviation
- \( z \): standardized normal deviation, \( N(0,1) \) in Table 2

By using the chemical waste in Table 2 as the example:

\[ \begin{align*}
BOD &= 10 \left[ 2.9863 \times (0.9335 \cdot R(1)) \right] \\
COD &= 10 \left[ 4.1682 \times (0.4752 \cdot R(1)) \right] \\
SS &= 10 \left[ 1.6150 \times (0.6253 \cdot R(1)) \right] \\
TS &= 10 \left[ 3.2535 \times (0.6697 \cdot R(1)) \right] \\
DS &= TS - SS \\
PHEN &= 10 \left[ 2.8686 \times (0.6469 \cdot R(1)) \right] \\
Pb &= Zn = Ni = CN \times 6 = OGR = 0.0
\end{align*} \]

Where \( R(i) \), \( i=1,2,3 \) represents the normal deviation after standardization. In Table 2 BOD and COD which have different first and second expected values will have the same deviation \( R(1) \). This means they should belong to the same kind and should be positively related. In these models, there are different treatment efficiencies, \( \eta_{nm}^{*} \), for treat different constituents, which also results in different treatment costs. Therefore, the above mentioned dependent relation effect should be considered into the models.
3.4 Costs

The \( cn \) value in the model should be the annual cost converting from the initial capital cost in terms of design year and interest rate plus the annual operation and maintenance cost. From Figure 4 BOD removal rate increases with the annual cost as shown in a concave curve which make the cost function and the objective function in the models becoming nonlinear. Therefore, nonlinear programming problems exist because the objective function is nonlinear and the constraints are linear. The correct cost function should be derived from the detailed investigation and careful examination. For example, the BOD removal rate is related to the return sludge and aeration time and hence, should be separately considered and diagrammatized to be applied as shown in Figure 7 and Figure 8.

![Figure 7: The relationship between the BOD loading and annual cost and the treatment efficiency of aeration tank in activated sludge process. (Aeration Time)](image)

![Figure 8: The relationship between the BOD loading and annual cost and the treatment efficiency of aeration tank in activated sludge process. (BOD Loading)](image)

4 Conclusions and suggestions

1. Generally speaking, DP is used to establish models to obtain the optimal process design for wastewater treatment plant. However, it adds more difficulty in engineering practice because different problems require different computer programming. In this article, IP and MIP are used to establish models.
2. Stochastic models are used because of the fact that the influx water quality has random nature. By comparing SLP, CCP, and LPUU, CCP is selected to establish SM.

3. The Treatment efficiency is affected by constituent, quantity and temperature of waste influent and therefore, is not constant. The suitability of the model should not be affected if the middle value of the design criteria range is selected to apply.

4. It should not be overlooked that each treatment unit has its limit for each constituent in wastewater. Therefore, a constraint in structural considerations.

5. The probability distribution of influent constituents should be investigated, analyzed and tested for find out the distribution type. It is generally log normal distribution, CDF and PDF established based on this distribution can be accurate and therefore, become the transform basis of SM.

6. Due to the nonlinear character of the cost functions, the Model will have nonlinear programming problems because the objective function is nonlinear and the constraints are linear.

7. Generally speaking, the optimal design for process and treatment units from cost versus efficiency relation can not be achieved based on conventional design procedure, In addition, the optimal design to each treatment unit can not guarantee the optimal design for entire process.

8. This study is to systematically develop a methodology for practical application under different conditions.

References


