Membrane deflection under pneumatic and electrostatic actuation without FEM simulation: application to electrostatic micropump

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Abstract

In this paper, a static model of a circular or square membrane under both pneumatic and electrostatic actuation is presented. This model provides an understanding of some of the essential characteristics (deflection, strike volume, capacity) for numerous applications (accelerometer, pressure sensor, (thermo)pneumatic or electrostatic micropump, etc.). Using these results along with some particular phenomena of the electrostatic actuation (sticking and hysteresis), the global dynamic simulation of an electrostatic micropump is then performed.

1. Introduction

Because electrostatic actuation is governed by non-linear equations, the study of the membrane deflection under electrostatic actuation is a somewhat delicate undertaking. So, in order to identify the static characteristics of such a system (deflection, capacity, strike volume, etc.), this study is often performed with a Finite Element Model (FEM) simulation. However, many geometrical and physical parameters (membrane size, material characteristics, applied voltage, backdraft pressure, etc.) exist and many numerical FEM simulations must be conducted in order to estimate the influence of each parameter. In this paper, an analytical parametric method for establishing the membrane deflection under both simultaneous pneumatic and electrostatic actuation is proposed. From this membrane deflection equation, the static characteristics (capacity, displaced volume, etc.) can be easily deduced. Then, with this physical phenomenon-based approach, the influence of each parameter can be quickly known.

During the dynamic functioning, the non-linearity leads to instability phenomena which cause hysteresis due to the sticking of some membrane points...
on the electrode. Usually, classical dynamic simulations do not take into account these phenomena and are then constrained by a collapse voltage limit. In the second part of this paper, a dynamic simulation of an electrostatic micropump is presented; it is based on the static results obtained in the first part of the paper. The advantages of this dynamic simulation are the absence of the use of heavy FEM simulations as well as the large domain of application due to its performance capabilities at higher voltages than the collapse voltage.

2. Static characteristics of membrane deflection under pneumatic and electrostatic actuation

2.1 Pneumatic actuation

Based on the assumption of a thin elastic isotrope membrane under small deflections, the law governing the membrane deflection is commonly presented by the Lagrange equation: \( \frac{\partial^2 W(x,y)}{\partial h^2} = \frac{P(x,y)}{D h^3} \), where \( W(x,y) \) is the deflection at point \((x,y)\), \( P(x,y) \) the applied pressure, \( h \) the membrane thickness and \( D \) the membrane stiffness.

Two geometrical cases can be studied: circular or rectangular membrane.

2.1.1 Circular membrane

In this case, due to symmetry, the Lagrange equation leads to a direct analytical solution: \( W(r) = \frac{P}{64 D h^3} \left( R^2 - r^2 \right)^2 \), where \( R \) is the radius of the membrane and \( r \) the point radius value.

2.1.2 Rectangular membrane

In this case, there is no direct solution to the Lagrange equation, which can be rewritten as follows: \( \frac{P(x,y)}{D h^3} = \frac{\partial^4 W(x,y)}{\partial x^4} + 2 \frac{\partial^4 W(x,y)}{\partial x^2 \partial y^2} + \frac{\partial^4 W(x,y)}{\partial y^4} \). This is a fourth-degree equation for \( W \) and no direct analytical solution for \( W(x,y) \) exists. Some authors have presented an approximated solution of the center membrane deflection \( W(0,0) \) [1]. But should the entire membrane deflection be needed, this would prove insufficient. A simple solution would be to choose the FEM calculation, yet in doing so, modelling control is lost, and dependence on the time calculation occurs (especially in optimizing a system with variations in all parameters).

In fact, an approximated solution of the entire membrane deflection can be derived from the strain energy method [2]. In the case of a uniformly applied pressure, the approximated membrane deflection is a polynomial function series \( \phi(x,y) \) that satisfies the boundary conditions (the four edges being built in). In
order to generalise this calculation, a change in variable is used [3]:

$$X = \frac{2x}{a} \quad \text{and} \quad Y = \frac{2y}{b} \quad \text{then} \quad \phi(X, Y, \lambda) = \left[ (1 - X^2)(1 - Y^2) \right]^2 \sum_{i=0}^{n} \sum_{j=0}^{m} c_{ij} X^i Y^j$$

where \(a\) and \(b\) are the dimensions of the rectangular membrane and \(\lambda = \frac{b}{a}\).

Due to symmetry, the coefficients \(i\) and \(j\) are even. Thus, the deflection becomes:

$$W(x, y, P) = W_0(0, 0, P) \* \phi(X, Y, \lambda) \quad \text{with} \quad W_0(0, 0, P) = \frac{P \cdot a^2 b^2}{16 D h^3}.$$  

To estimate the polynomial coefficient \(c_{ij}\), the total energy \(E_i\) of the system (the energy of the deflection in addition to the energy of the pressure applied) is calculated and minimized with respect to \(c_{ij}\) during a virtual deflection.

$$E_i \propto \int_{-1}^{1} \int_{-1}^{1} \left( \lambda^2 \frac{\partial^4 \phi}{\partial X^4} + 2 \frac{\partial^4 \phi}{\partial X^2 \partial Y^2} + \frac{1}{\lambda^2} \frac{\partial^4 \phi}{\partial Y^4} + 2 \right) \phi \, dX \, dY$$

The calculation of \(c_{ij}\) is performed by solving the \(c_{ij}\) equations: \(\frac{\partial E_i}{\partial c_{ij}} = 0\). Knowing the values of coefficients \(c_{ij}\), we obtain an estimation of the entire membrane deflection in the case of a uniform pneumatic pressure \(P\) being applied: \(W(x, y, P)\).

### 2.1.3 Results

Given these expressions for the membrane deflection under pneumatic actuation, various static characteristics can be calculated. For example, the capacity between membrane and electrode and the volume strike for an actuation are:

$$C(P) = \frac{\varepsilon}{e} \int_{-a/2}^{a/2} \int_{-b/2}^{b/2} \frac{1}{1 - \frac{W(x, y, P)}{e}} \, dx \, dy \quad \text{or} \quad C(P) = \frac{\varepsilon 2\pi}{e} \int_{0}^{R} \frac{r}{1 - \frac{W(r, P)}{e}} \, dr$$

where \(e\) is the membrane-electrode distance (with \(P=0\)), and

$$V_m(P) = \int_{-a/2}^{a/2} \int_{-b/2}^{b/2} W(x, y, P) \, dx \, dy \quad \text{or} \quad V_m(P) = 2\pi \int_{0}^{R} r \, W(r) \, dr$$

### 2.2 Pneumatic and electrostatic actuation

All of the previous results presuppose that the applied pressure is constant and uniform on the membrane. However, in the case of an electrostatic pressure \(P_e\), its value is modified by the deflection:

$$P_e(x, y) = \frac{\varepsilon U^2}{2 e^2 \left(1 - \frac{W(x, y)}{e}\right)^2}$$

As an initial approximation, we can consider this pressure as a constant
and equal to the electrostatic pressure with no deflection \( P_e(x,y) = \varepsilon U^2 / 2e^2 \). In fact, this approximation is acceptable only for a very small deflection \( W \). The actual electrostatic force is greater than the estimated force and with this approximation, the error is greater than 10% if \( W > \left( 1 - \sqrt{1/11} \right) e \) (i.e. \( W > 0.0465e \)). For example, if \( e = 5 \mu m \), the error on the applied pressure \( P_e(x,y) \) is greater than 10% if \( W(x,y) > 0.23 \mu m \). Therefore, in most instances, the variation in electrostatic pressure with the deflection must be taken into account.

In order to avoid FEM simulations, we have built a model of the membrane deflection under electrostatic pressure by cutting it into an infinite number of simple models in which the electrostatic pressure is constant yet dependent on the deflection. It can be noted that for each point, the deflection value is linear with respect to the pressure. It acts like a spring representing the internal strains, with the surfacic spring constant being (Fig. 1):

\[
K(x,y) = \frac{16 D h^3}{(a.b)^2 \varphi(x,y,\lambda)} \quad \text{or} \quad K(r) = \frac{64Dh^3}{(R^2 - r^2)}.
\]

![Figure 1: Modelling of the membrane](image)

In the case of both simultaneous electrostatic and pneumatic pressure, each membrane point deflection is thus obtained by solving the equation:

\[
K(x,y) W(x,y) + P + \varepsilon U^2 \frac{1}{2e^2 (1 - W(x,y) e)^2} = 0
\]

This is a third-order equation for \( W \) which yields an analytical solution [4]. This result enables demonstrating once again the well-known instability of such a system [5], but it remains more general by taking into account the pneumatic backdraft \( P \). If \( W < (e + 2P K)/3 \), the solution is stable; else, if \( W > (e + 2P K)/3 \), the membrane point is sticking on the electrode and then \( W(x,y) = e - d \) where \( d \) is the insulator thickness. Then, for each membrane point,
we can define a collapse voltage limit: \( U_c(x,y) = \sqrt{\frac{8K(x,y)}{27\epsilon}} \left( e - \frac{P}{K(x,y)} \right)^3 \).

The analytical solution of the membrane deflection under both pneumatic and electrostatic actuation allows evaluating the capacity \( C(P,U) \) or the volume \( V_m(P,U) \) (Figs. 2-3). These results highlight the hysteresis phenomenon which appears when some membrane points are sticking (Fig. 4). The results presented have been obtained with a silicon circular membrane whose characteristics are depicted in Section 3.

![Figure 2a: Volume estimation for an increasing voltage (0→120V)](image1)

![Figure 2b: Volume estimation for a decreasing voltage (120V→0)](image2)

![Figure 3a: Capacity estimation for an increasing voltage (0→120V)](image3)

![Figure 3b: Capacity estimation for a decreasing voltage (120V→0)](image4)

All of these static results (membrane deflection, strike volume and capacity for electrostatic and/or pneumatic actuation) can be applied to modelling accelerometers, pressure sensors or micropumps. In the following sections, the dynamic simulation of an electrostatic micropump operating under extreme conditions (with sticking and hysteresis effects) will be presented.
3. Electrostatic micropump

Most micropumps produced by silicon bulk micromachining are of the reciprocating type [6]: a flexible membrane driven by an actuator (piezoelectric [7], electrostatic [8], thermopneumatic [9] or electromagnetic [10]) induces a chamber pressure variation. This pressure variation serves to displace a fluid via two anti-return passive microvalves. A schematic diagram of an electrostatic micropump is presented in Fig. 5. The geometrical characteristics of the studied electrostatic micropump are as follows: a silicon membrane surface of 12.5 mm$^2$, a membrane thickness of 25 μm, an electrode-membrane distance of 5 μm, the insulator placed on the electrode has a thickness of 0.1 μm and a relative permittivity of 8, the anti-return passive valves have a surfacic spring constant of 9.6e8 N/m3, and the perimeters of the inlet and outlet channels are 6.3 mm.

4. Micropump dynamic simulation

For the dynamic simulation based on the assumption of an incompressible liquid,
the law governing the exchange between the inlet and the outlet is the mass conservation, which can be expressed by: \( \frac{dVol}{dt} = \Phi_{in} - \Phi_{out} \), where \( Vol \) is the chamber volume, \( \Phi_{in} \) the fluid flow between inlet and chamber, and \( \Phi_{out} \) the fluid flow between chamber and outlet. All these parameters have the pressure chamber \( P \) in common. Therefore to simulate the pump, \( P \) must be calculated.

As an initial approximation, the chamber volume variation is taken as equal to the variation of the deflected membrane volume \( (V_m) \). As shown in Section 2, the chamber volume can be expressed as a function of both the chamber pressure \( (P) \) and the voltage \( (U) \) acting on the membrane. Accordingly

\[
\frac{dVol(P,U)}{dt} = \frac{\partial V_m}{\partial P} \left( \frac{dP}{dt} \right) + \frac{\partial V_m}{\partial U} \left( \frac{dU}{dt} \right).
\]

The term \( dU/dt \) is generated by the actuation imposed. The two parameters \( \partial V_m/\partial P \) and \( \partial V_m/\partial U \) are derived from the development of the static model described in Section 2. Due to the hysteresis phenomenon of the electrostatic actuation, these parameters are a function not only of the current voltage and pressure but also of their previous values. Thus, in our subsequent simulations, we integrated it into the model in order to evaluate its influence on the micropump’s behavior. In fact, for increasing voltage, we have considered the strike volume presented in Fig. 2a, whereas for decreasing voltage, we have taken the strike volume presented in Fig. 2b.

To evaluate the flow characteristic, we applied the Torricelli formula, which can be deduced from the Bernoulli equation: the fluid velocity \( v \) is \( v = \sqrt{2\Delta P/\rho} \), where \( \rho \) is the fluid density and \( \Delta P \) the pressure difference between the two faces of the valves. The flow is the product of the fluid velocity and the equivalent section of its passage through the valve. Then, like for the membrane, the valve spring constant \( K_v \) is evaluated, and the static flow characteristic becomes: \( \Phi = c \left( \frac{\Delta P}{K_v} \right) \left( \sqrt{2\Delta P/\rho} \right) \), where \( c \) is the perimeter of the channel. \( \Delta P/K_v \) represents the equivalent deflection of the valve.

It is useful to know the electrical current needed by the micropump. Because the capacity between the membrane and the electrode depends on both the pressure \( P \) and the voltage \( U \), the expression for the current is:

\[
i = \left( \frac{\partial C}{\partial P} \left( \frac{dP}{dt} \right) + \frac{\partial C}{\partial U} \left( \frac{dU}{dt} \right) \right) U + C \left( \frac{dU}{dt} \right).
\]

In fact, similar to the approach for strike volume, for increasing voltage, we have considered the capacity presented in Fig. 3a, whereas for decreasing voltage, we have taken the capacity presented in Fig. 3b.
In order to run this simulation, we used a straightforward software called ‘Simulink’, which is part of the Matlab product series. The simulation structure is presented in Fig. 6. It’s a simulation by blocks, with each block symbolizing the characteristics of a micropump element. The structure is evolutionary, thereby enabling the model to become progressively more complex.

5. Results

In order to highlight the sticking and hysteresis effects, the simulations have been presented with a sinusoidal voltage of 120 V and 75Hz (Fig. 7).

Two kinds of simulation are depicted ((a) and (b)). They both reflect the sticking effect. Some authors have still approached this non-linearity phenomenon due to the electrostatic pressure behavior [12], but they were limited by instability and did not use higher voltages than the collapse voltage limit. Our simulations have been performed with voltages to provoke the sticking. Moreover, the hysteresis is taken into account in simulation (b); simulation (a) only uses Figs. 2a and 3a, whereas simulation (b) uses Figs. 2a and 3a for rising voltage and Figs. 2b and 3b for decreasing voltage.

Simulation results are presented in Figs. 8-11. It can be noted that for increasing voltage, curves (a) and (b) are identical and the hysteresis phenomenon appears only for decreasing voltage (b). On the various figures, we have indicated the characteristic points of sticking and unsticking. With these two kinds of simulation, the benefit of working with voltage causing the sticking of some membrane points can be remarked: both the strike volume and the back pressure are higher. Moreover, it
is to be pointed out that if the membrane were stuck, the hysteresis would modify the micropump’s performance: the hysteresis maintains the membrane at a stable position with almost no volume modification (Fig. 9) during the voltage decrease, once the voltage falls under the unsticking limit, the membrane tends to return to its stable position and causes a high variation in the chamber pressure (Fig. 8). According to these results, the simulation of an electrostatic micropump needs to integrate both the evolution of the particular electrostatic pressure with respect to the membrane deflection and the hysteresis being generated by the membrane sticking on the electrode. In reality, as opposed to simulation (b), simulation (a) does not reflect the actual workings of the micropump. Fig. 10 allows estimating the electrostatic micropump performance in terms of flow. In order to estimate power consumption, the current evolution is presented in Fig. 11.

6. Conclusion

A static model of a circular or square membrane under both pneumatic and electrostatic pressure has been developed. The originality of this model is that the evolution of the electrostatic pressure with respect to the membrane’s deflection is taken into account, and the electrostatic pressure is not assumed to
be constant over the entire membrane surface. On the basis of this static modelling approach, an original dynamic simulation of an electrostatic micropump has been presented. This simulation is more general than the classical ones because it can be used with any kind of voltage even those above the collapse voltage. In addition, we have shown that a sticking simulation must include the hysteresis phenomenon.

References