Design and analysis of a resonant gyroscope suitable for fabrication using the LIGA process
L. Yao, E. Chowanietz, M. McCormick
Department of Electronic and Electrical Engineering, School of Engineering and Manufacture, De Montfort University, Leicester, LE1 9BH, UK

Abstract
A recent surge of interest in miniature vibratory gyroscope, for applications as diverse as camera-shake compensation and automobile brake control, means that many researchers are now trying to design, simulate and fabricate these sensors. This paper presents details of a new design of miniaturised H type resonant gyroscope to be fabricated in electroformed metal using an emergent technology known as LIGA. The sensor features electrostatic drive and capacitive read out techniques.

The paper describes the simulation aided design process that was used to obtain the final sensor structure and shows how a set of equation for the electrostatic-structural coupled field problem were derived and used in conjunction with a commercial finite element analysis package.

1 Introduction
In recent years there has been a sharp growth of interest in the design and manufacture of miniature yaw-rate sensors based around the vibratory gyroscope; a device which measures angular velocity by detecting the Coriolis force which arises between rotating reference frames. At its most elementary, a vibratory gyroscope uses a resonating beam and employs the bending modes of the beam to detect yaw rate [Fig.1a ]. If the beam is excited in the x axis, then rotation about the z axis produce a Coriolois force which transfers energy to the y axis. The resulting y axis oscillation can then be detected and its amplitude gives an indication of the rate of rotation.
Vibratory gyroscope sensors thus have a large potential market in applications ranging from robotics and camera-shake compensation through to automobile navigation and chassis control systems.

Over the last ten years a number of promising designs of vibrating shells, vibrating rings, vibrating cylinders, vibrating wires, vibrating bars, and 'tuning forks' have been actively developed for practical use [1].

Among the various designs that have been put forward, a gyroscope using an 'H' form double flexural resonator appears particularly promising [Fig. 1b]. Such a structure, when supported at the zero-displacement points at the middle of the 'H' structure, has minimal energy losses and low sensitivity to spurious external vibrations. Moreover it has been suggested that such a sensor can also offer a low temperature coefficient, high quality factor and high stability [2]. Unfortunately, the sensors that have been fabricated so far have all used the piezoelectric effect for excitation and detection and therefore have been manufactured from piezoceramic material, making the finish sensors rather large (eg 45.0 × 7 × 2mm³).

In order to realise miniaturisation of the resonant gyroscope and maintain or improve its sensitivity, very small three dimensional components must be fabricated. If the components can be manufactured in metal, then electrostatic techniques can be used for the excitation and detection of vibrations [3]. One technique for manufacturing miniature three dimensional parts is the so called LIGA technique, a deep-etch X-ray lithography process which potentially offers the cost-effective mass-production of metallic microparts to sub-micron accuracy [4]. Fuller details of the LIGA technique are given elsewhere in these proceedings [5].

In order to investigate the application of LIGA to the manufacture of miniature microsensors, De Montfort University, Leicester, is collaborating with Lucas Advanced Engineering Centre, Solihull, and the Institut fur Microtechnik, Mainz, Germany, to design and fabricate an H-type resonant
gyroscope. This paper describes the design and simulation process and in particular, the analysis of the electrostatic-structural coupled field problem that arises with such a device.

2 Finite element analysis of electrostatic structural coupled fields

In the design and simulation of an electrostatically-excited H-type gyroscope, the analysis of electrostatic-structural coupled fields is particularly important because of the significant interaction of the externally applied electrostatic field with the sensor's internal mechanical field. Unfortunately, there is no commercially-available FEA software that can analyze these coupled fields directly; this section therefore shows how a set of equations for FEA of electrostatic-structural coupled fields can be derived and then used in conjunction with a commercial FEA package. Simulation and analysis of the H-type resonant gyroscope are then carried out and the final design is presented in the following sections.

In the finite element method, the equilibrium equations relating to linear elastic structural problems and electrostatic problems can be written, respectively, as:

\[
[M]\{\ddot{u}\} + [\lambda]\{\dot{u}\} + [K]\{u\} = \{F\}
\]

\[
[K_d]\{\varphi_e\} = \{Q\}
\]

where \([K]\) is the stiffness matrix, \([M]\) is the mass matrix, \([\lambda]\) is the damping matrix, \([K_d]\) is the dielectric conductivity matrix, \(\{F\}\) is the applied force vector, \(\{Q\}\) is the applied charge vector, \(\{u\}\) is the nodal displacement vector and \(\{\varphi_e\}\) is the nodal electric potential vector.

For linear materials, the constitutive equations for elastic structural fields and electrostatic fields can be written, respectively, as:

\[
\{T\} = [c]\{S\} \quad \text{and} \quad \{D\} = [e]\{E\}
\]

and the shape functions for elastic structural fields and electrostatic fields are:

\[
\{w\} = [N]\{u\} \quad \{S\} = [B]\{u\}
\]

\[
\{\varphi\} = [N_e]\{\varphi_e\} \quad \{E\} = [B_e]\{\varphi_e\}
\]

where \([c]\) is the elasticity matrix, \([e]\) is the dielectric matrix, \(\{T\}\) is the stress vector, \(\{S\}\) is the strain vector, \(\{D\}\) is the electric flux vector, \(\{E\}\) is the electric field vector, \(\{w\}\) is the displacement vector of a general point, \(\{\varphi\}\) is the electric potential vector of a general point, \([N]\) is the matrix of displacement shape function, \([B]\) is the strain-displacement matrix, \([N_e]\) is the vector of electrical potential shape function and the \([B_e]\) is the electric flux-electric field matrix.

The material behaviour of the electrostatic-structural coupled field is non-linear, therefore, in order to find the coupled finite element matrix
equations for it, the electromechanical constitutive equations must be derived first.

The electrostatic stress \( \{T_e\} \) is related to electric field \( \{E\} \) and strain \( \{S\} \) by:

\[
\begin{pmatrix}
T_{ex} \\
T_{ey} \\
T_{ez}
\end{pmatrix} = \begin{pmatrix}
\frac{1}{2} \varepsilon_x \frac{E_x^2 (1+S_2)(1+S_2)}{(1+S_2)^2} - \frac{1}{2} \varepsilon_y \frac{E_y^2 (1+S_2)}{(1+S_2)^2} - \frac{1}{2} \varepsilon_z \frac{E_z^2 (1+S_2)}{(1+S_2)^2} \\
\frac{1}{2} \varepsilon_x \frac{E_x^2 (1+S_2)}{(1+S_2)^2} + \frac{1}{2} \varepsilon_y \frac{E_y^2 (1+S_2)(1+S_2)}{(1+S_2)^2} - \frac{1}{2} \varepsilon_z \frac{E_z^2 (1+S_2)}{(1+S_2)^2} \\
- \frac{1}{2} \varepsilon_x \frac{E_x^2 (1+S_2)}{(1+S_2)^2} - \frac{1}{2} \varepsilon_y \frac{E_y^2 (1+S_2)}{(1+S_2)^2} + \frac{1}{2} \varepsilon_z \frac{E_z^2 (1+S_2)(1+S_2)}{(1+S_2)^2}
\end{pmatrix}
\]

(3)

The relationship between \( \{E\} \), \( \{S\} \) and \( \{D\} \) is

\[
\begin{pmatrix}
D_x \\
D_y \\
D_z
\end{pmatrix} = \begin{pmatrix}
\varepsilon_x E_x (1 + S_y)(1 + S_z) \\
\varepsilon_y E_y (1 + S_x)(1 + S_z) \\
\varepsilon_z E_z (1 + S_x)(1 + S_y)
\end{pmatrix}
\]

(4)

Therefore the electromechanical constitutive equations of the electrostatic-structural coupled field can then be obtained as:

\[
\{\Delta T\} = [c_c] \{\Delta S\} - [\varepsilon] \{\Delta E\}
\]

(5)

\[
\{\Delta D\} = [\varepsilon] \{\Delta S\} + [c_e] \{\Delta E\}
\]

(6)

where \([c_c] = [c] + [c_e]\) (the elasticity matrix)

\[
[c_e] = \begin{pmatrix}
\frac{\sigma_{xx}(1+S_2)}{(1+S_2)^2} & \frac{\sigma_{xy}(1+S_2)}{(1+S_2)^2} & \frac{\sigma_{xz}(1+S_2)}{(1+S_2)^2} \\
\frac{\sigma_{yx}(1+S_2)}{(1+S_2)^2} & \frac{\sigma_{yy}(1+S_2)}{(1+S_2)^2} & \frac{\sigma_{yz}(1+S_2)}{(1+S_2)^2} \\
\frac{\sigma_{zx}(1+S_2)}{(1+S_2)^2} & \frac{\sigma_{zy}(1+S_2)}{(1+S_2)^2} & \frac{\sigma_{zz}(1+S_2)}{(1+S_2)^2}
\end{pmatrix}
\]

(the electrical elasticity matrix)

\[
[c]^{-1} = \begin{pmatrix}
\frac{1}{E_x} & -\frac{V_{xy}}{E_y} & -\frac{V_{xz}}{E_z} \\
-V_{yx} & \frac{1}{E_y} & -\frac{V_{yz}}{E_z} \\
-V_{zx} & -\frac{V_{zy}}{E_y} & \frac{1}{E_z}
\end{pmatrix}
\]

(the structural elasticity matrix)

\[
[c_e] = \begin{pmatrix}
\frac{\varepsilon_x (1+S_2)(1+S_z)}{(1+S_2)} & 0 & 0 \\
0 & \frac{\varepsilon_y (1+S_2)(1+S_z)}{(1+S_2)} & 0 \\
0 & 0 & \frac{\varepsilon_z (1+S_2)(1+S_y)}{(1+S_2)}
\end{pmatrix}
\]

(the dielectric matrix)

\[
[\varepsilon] = \begin{pmatrix}
\frac{-\varepsilon_x E_x (1+S_2)(1+S_z)}{(1+S_2)^2} & \frac{\varepsilon_y E_x (1+S_2)}{(1+S_2)^2} & \frac{\varepsilon_z E_x (1+S_2)}{(1+S_2)^2} \\
\frac{\varepsilon_x E_y (1+S_2)}{(1+S_2)^2} & \frac{-\varepsilon_y E_y (1+S_2)(1+S_z)}{(1+S_2)^2} & \frac{\varepsilon_z E_y (1+S_2)}{(1+S_2)^2} \\
\frac{\varepsilon_x E_z (1+S_2)}{(1+S_2)^2} & \frac{\varepsilon_y E_z (1+S_2)}{(1+S_2)^2} & \frac{-\varepsilon_z E_z (1+S_2)(1+S_z)}{(1+S_2)^2}
\end{pmatrix}
\]

(the coupling matrix)
Referring to eqns (1) and (2) the FEA equations derived for electrostatic structural coupled fields are then:

\[
\begin{bmatrix}
    M & 0 \\
    0 & 0
\end{bmatrix}
\begin{bmatrix}
    \Delta \dot{u} \\
    \Delta \phi
\end{bmatrix}
+ \begin{bmatrix}
    \lambda & 0 \\
    0 & 0
\end{bmatrix}
\begin{bmatrix}
    \Delta \dot{u} \\
    \Delta \phi
\end{bmatrix}
+ \begin{bmatrix}
    K_c & K^2 \\
    K^{2T} & K_d
\end{bmatrix}
\begin{bmatrix}
    \Delta u \\
    \Delta \varphi
\end{bmatrix}
= \begin{bmatrix}
    \Delta F \\
    \Delta Q
\end{bmatrix}
\]  

(7)

where \([M] = \int_{vol} \rho [N][N]^T d(vol)\) is the structural mass matrix

\([K_c] = \int_{vol} [B][\epsilon_c][B]^T d(vol)\) is the modified structural stiffness matrix

\([K_d] = \int_{vol} [B_e][\epsilon][B_e]^T d(vol)\) is the dielectric conductivity matrix

\([K^2] = \int_{vol} [B][\epsilon][B]^T d(vol)\) is the coupling matrix

\([\lambda] = \alpha[M] + \beta[K_c]\) is the damping matrix

\(\{\Delta u\}\) is nodal displacement variation vector

From eqn (7), if the coupling matrix \([K^2]\), dielectric conductivity matrix \([K_d]\) and the modified structural stiffness matrix \([K]\) can be calculated and implemented in an FEA package, then the three dimensional electrostatic-structural coupled field problem can be analysed.

For a capacitively driven H-type gyroscope, driving capacitors are formed between the beam arms and fixed electrodes mounted on the substrate surface. The separation distance \(d\) of the capacitor plates is usually very small compared with the length \(l\) and width \(w\).

![A driving capacitor with small separation distance](image)

Figure 2: A driving capacitor with small separation distance

Figure 2 shows an example of the driving capacitor with a small separation distance. Within the capacitor, \(\{E\}\) and \(\{S\}\) can be approximately considered as containing only one non-zero component in the \(x\) direction. Thus a one dimensional electrostatic-structural coupled field analysis can be used to analyse the problem. If the capacitor dielectric is free space then,

\[
T_{ex} = \frac{1}{2} \varepsilon E_x^2
\]  

(8)

\[
D_x = \varepsilon E_x
\]  

(9)

\[
\Delta D_x = \frac{\varepsilon}{1+S_x} \Delta E_x - \frac{\varepsilon E_x}{(1+S_x)^2} \Delta S_x
\]  

(10)

\[
\Delta T_{ex} = \frac{\varepsilon E_x^2}{(1+S_x)^2} \Delta E_x - \frac{\varepsilon E_x^2}{(1+S_x)^3} \Delta S_x
\]  

(11)

If the initial electric field \(E_x\) within the capacitor is

\[
E_x = \frac{V}{d}
\]  

(12)

and the strain produced by the electric field in the \(x\) direction can be described as

\[
s_x = \frac{-u_x}{d}
\]  

(13)
The electrostatic structural field of the capacitor constitutes the distributed mechanical resistance at the surface $S_e$ of the sensor beam. The force per unit area at a point $p$ of the surface $S_e$ can be derived from eqns (8) to (13) and is

$$
\Delta f_x = \frac{eV^2}{(d-u_e)^2} \Delta u_x + \frac{eV}{(d-u_e)^2} \Delta V = k_e \Delta u_x + \Delta p_e
$$

(14)

where $k_e$ is the electrostatic stiffness at the point $p$ of the surface $S_e$ and $\Delta p_e$ is the electrostatic pressure at the point $p$ of the surface $S_e$.

The FEA matrix equilibrium equation for the beam is then simplified as:

$$
([K] - [K_e])\{\Delta u\} = [M]\{\Delta \dot{u}\} + [\lambda]\{\Delta \ddot{u}\} + \{\Delta F\} + \{\Delta F_{pr}\}
$$

(15)

where $\{\Delta F_{pr}\} = \int_{area_e} [N_n]^T [\Delta p_e] d(area_e)$ (the electric pressure vector)

and $[K_e] = \int_{area_e} [N_n]^T k_e [N_n] d(area_e)$ (the electrostatic stiffness matrix for the surface $S_e$).

Using the above equations, the proposed electrostatically excited H-type gyroscope can be analysed.

### 3 FEA Analysis of the H-type vibratory gyroscope

Figure 3 shows an H type double flexural vibratory gyroscope. The gyroscope is driven into a flexural vibration in the $x$-$y$ plane. Then, if an angular velocity $\Omega$ about the $y$ axis is applied to the gyro, another vibration in the $z$-$y$ plane is produced by the Coriolis force.
The equation for the vibrating gyro-motion of the H type sensor can be written as:

\[
F_x = \rho \frac{\partial^2 \xi}{\partial t^2} + \lambda \frac{\partial \xi}{\partial t} + EK \frac{\partial^4 \xi}{\partial y^4} - 2\rho \Omega \frac{\partial \eta}{\partial t} - \rho \Omega^2 \xi
\]

\[\text{(16)}\]

\[
0 = \rho \frac{\partial^2 \eta}{\partial t^2} + \lambda \frac{\partial \eta}{\partial t} + EK \frac{\partial^4 \eta}{\partial y^4} + 2\rho \Omega \frac{\partial \xi}{\partial t} - \rho \Omega^2 \eta
\]

\[\text{(17)}\]

where \(\rho\) is the density, \(\lambda\) is the damping constant, \(EK\) are the Young's modulus, \(\xi\) is the vibrating motion in the \(x\) direction, and \(\eta\) is the vibrating motion in the \(z\) direction.

When the applied angular velocity \(\Omega << \text{vibration frequency}\), the centrifugal forces \(-\rho \Omega^2 \xi\) and \(-\rho \Omega^2 \eta\) can be neglected in comparison with the Coriolis forces \(-2\rho \Omega \frac{\partial \eta}{\partial t}\) and \(2\rho \Omega \frac{\partial \xi}{\partial t}\). If the gyroscope is driven in such a way that the Coriolis force \(-2\rho \Omega \frac{\partial \eta}{\partial t}\) in the \(x\) direction is far smaller than the Coriolis force \(2\rho \Omega \frac{\partial \xi}{\partial t}\) in the \(z\) direction, the amplitude of vibrational motion is proportional to \(\Omega\). Therefore, the applied angular velocity \(\Omega\) can be estimated by detecting the vibration amplitude.

Using the FEA method, the above equations can be transformed from eqns (16) and (17):

\[
(K - K_c + K_c)\{\Delta u\} = \{M\}\{\Delta \ddot{u}\} + \{\lambda + \lambda_c\}\{\Delta \dot{u}\} + \{\Delta F\} + \{\Delta F_{\text{Fr}}\}
\]

\[\text{(18)}\]

where \(K_c = \rho \Omega^2 \int_{\text{vol}} N^T N_{\text{ce}} d(\text{vol})\) is the centrifugal stiffness matrix,

\(\lambda_c = 2\rho \Omega \int_{\text{vol}} N^T N_{\text{co}} d(\text{vol})\) is the Coriolis force matrix,

\(N\) is the matrix of displacement shape function,

\(N_{\text{ce}}\) is the matrix of shape function for centrifugal force,

\(N_{\text{co}}\) is the matrix of shape function for Coriolis force.

And

\[
[N] = \begin{bmatrix}
N_1 & 0 & 0 & N_2 & 0 & 0 & \ldots & N_n & 0 & 0 \\
0 & N_1 & 0 & 0 & N_2 & 0 & \ldots & 0 & N_n & 0 \\
0 & 0 & N_1 & 0 & 0 & N_2 & \ldots & 0 & 0 & N_n
\end{bmatrix}
\]

\[\text{(19)}\]

\[
[N_{\text{ce}}] = \begin{bmatrix}
N_1 & 0 & 0 & N_2 & 0 & 0 & \ldots & N_n & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 \\
0 & 0 & N_1 & 0 & 0 & N_2 & \ldots & 0 & 0 & N_n
\end{bmatrix}
\]

\[\text{(20)}\]

\[
[N_{\text{co}}] = \begin{bmatrix}
0 & 0 & -N_1 & 0 & 0 & -N_2 & \ldots & 0 & 0 & -N_n \\
0 & 0 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 \\
N_1 & 0 & 0 & N_2 & 0 & 0 & \ldots & N_n & 0 & 0
\end{bmatrix}
\]

\[\text{(21)}\]

Thus the FEA of the electrostatic driven resonant H type gyroscope is carried out by using eqn (18) in conjunction with a FEA software package.
4 Design and analysis of the sensor

4.1 Driving modes and the supporting position
As shown in Fig. 3, there are two modes of vibration in the xoy plane which can generate symmetrical double flexural gyro motions: the vibration mode 1 and its corresponding detection mode forms gyro motion mode 1 (Fig. 3(b)); and the vibration mode 2 and its corresponding detection mode forms gyro motion mode 2 (Fig. 3(c)).

For gyro motion mode 1 The minimum vibration points are the central points on both side surfaces of the base portion. The structure can be fixed at the small areas around these central points. Any vibration of the supporting points can be removed by adjusting the value of \( v_b \).

For gyro motion mode 2 The minimum vibration points are the central points on the top and bottom surfaces of the base portion. The structure can be held at these points. Any vibration of the supporting points can be eliminated by adjusting the value of base length \( b_l \), width \( b_w \) and \( v_b \).

4.2 The driving electric field
The electrostatic poles for driving the gyro motion can be positioned as in Fig. 3(a).

Driving electric field strength The driving strength of the electric field is limited by the breakdown field of the inter-electrode gap. The breakdown field in this region increases with decreasing separation distance (Paschen effect).

Squeeze film effect Ideally, the sensor should be operated in a vacuum, otherwise energy will be dissipated in pumping air in and out of the inter-electrode gap.

4.3 Frequency and amplitude matched H type gyroscopes
The natural frequencies and mode shapes are very important parameters in the design of a gyroscope structure. In order to increase the sensitivity of a gyroscope, the natural frequencies of the primary (excitation) and the secondary (detection) modes should be equal. Matching these two natural frequencies can be achieved by adjusting the ratio of beam width to beam thickness.

Conversely, in order to increase the precision of the sensor, the deflected mode shape of the gyro should be symmetrical. In practice, even if the structure of the gyro is symmetrical, the deflected mode shape of the gyro is sometimes still asymmetrical for each vibration mode. The deflected shape symmetrization can be obtained by adjusting the ratio of base length and width.

4.4 The final structure
(1) Model dimension parameters (referring to Fig. 3a) listed in table 1:

<table>
<thead>
<tr>
<th>( l )</th>
<th>( t )</th>
<th>( aw )</th>
<th>( bw )</th>
<th>( bl )</th>
<th>( v_b )</th>
<th>( vl )</th>
<th>( d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.5</td>
<td>0.2</td>
<td>0.1877</td>
<td>0.5</td>
<td>1.2</td>
<td>0.04</td>
<td>0.1</td>
<td>0.02</td>
</tr>
</tbody>
</table>
(2) Vibration frequencies: frequency of primary mode: 8851.4Hz
frequency of secondary mode 8850.3Hz

Nickel was selected as the material for fabricating the gyroscope. The material constants of Nickel are: Density $\rho=7860$kgm$^{-3}$; Young's modules $E = 2.0 \times 10^{11}$Nm$^{-2}$ and Poisson's ratio $\sigma=0.3$.

4.5 Simulated performance of the sensor

Based on the simulation results presented in a separate paper [6], the performance of the sensor was found to be satisfactory. Result indicate:
- Output response is linear
- Applied angular velocity range =0-70$\pi$ radian s$^{-1}$
- Sensitivity of response deflection =0.227 $\times 10^{-2}$µm per radian s$^{-1}$
- Variation range of $(\Delta C1 +\Delta C2)/2CR =0 - 0.046$
- Sensitivity of the output voltage =4.1835 mV per radian s$^{-1}$

The parameters and simulation conditions used were:
- Damping ratio=0.003
- Maximum drive deflection=5.4µm
- Initial separation distance of the detecting capacitor =10µm
- Detecting voltage=20V

5 Conclusions

An electrostatically excited vibrating gyroscope has been designed and simulated. It features a low drive voltage and a linear output response. Furthermore, because it is supported only in small areas surrounding the two zero displacement points, the resonant sensor is believed to offer a comparatively high quality factor, high stability and low sensitivity to environmental factors such as shocks and vibration. The sensor is currently being fabricated and it is hoped that experimental data will be obtained shortly.

References

4. Yao, L.F. Design and fabrication of a smart vibrating sensor, Degree transfer report, 1995, De Montfort University
5. Chowanietz, E. and Lees,A, 'Design and simulation of high-aspect ratio microstructures' MicroSim proceedings, 1995, UK,