Ship waiting time in a river port with priority servicing and limited anchorage area

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Abstract

Arrivals and loading/unloading operations of ships and barges in river ports are frequently considered to be of a stochastic nature and are therefore convenient to be modelled with queueing theory. In this paper, we considered a hypothetical port in which ship arrivals are assumed to follow the Poisson distribution and their servicing, that is, loading and/or unloading rates are assumed to have a deterministic distribution with constant service rate. In addition to this, we assumed that the vessels are served on a “first come – first served” basis with non-preemptive priorities and that the anchorage area is limited, being situated in the restricted inland waterway. As usual in real-life situations, self-propelled (motor) barges and motor ships are given the first class priority and towed or pushed barges are given the second class priority. Due to waiting area (anchorage) restrictions, arriving ships which find the anchorage full are not allowed to join the system, regardless of the priority class they belong to. The whole port system is designed as an M/D/c/N queueing system in Kendall’s notation and approximate solutions were obtained with numerical examples which are intended to be helpful to the port engineer or port planner in designing the process in the best possible, time saving way.

Keywords: river port, queueing system, priority servicing, ship arrivals, limited anchorage, ship waiting time.

1 Introduction

Real operations with inland ships are considered dependant on the transport technologies or, more precisely, on the kind of ships and tows used on inland
waterways which require port services. Each inland port can be considered as a network consisting of a set of sequential or parallel links and nodes. The port link is a micro-technological element representing a set of port operations or activities between port nodes. The port node is a place of current stay of cargo flow such as storages, sheds, quay apron, shed truck spots, custom spots etc.

Ships move from anchorage to berth, from berth to berth, from berth to anchorage, port reloading facilities move from storage area to storage area (ship hold-apron, apron-port warehouse, warehouse-truck spots etc.), rail trains and lorries move from port to inland destinations and vice versa. The sum of all vehicle movements between nodes will indicate the intensity of interdependence between them. Every inland port subsystem may in fact be represented as a matrix and transition probabilities can be computed.

This paper discusses the anchorage-ship-berth link at the river port as a queueing system with priority servicing at the berths. The stochastic characteristics and components of the link operation are as follows:

- Times of arrival of single ships (self-propelled barge or motor cargo ship) or in bulk (pushed and pulled barge tows) in the port cannot be precisely given.

- Service time (loading/unloading time) is a random variable depending on handling capacities of berths, carriage of barges, hydrometeorological conditions, size of an arriving group, etc.

- Berths are not always occupied; in some periods there are no barges (the capacities underutilized); and there are the time intervals of high utilization when the queues are formed.

Operations of the anchorage-ship-berth link include the following:

- Waiting in the anchorage areas; if all anchorage positions are occupied, ships or barge tows are rejected and have to go to another waiting area in that or some other adjacent port.

- Vessels move from anchorage to berth by port tug or without it (self-propelled barges).

- Loading and/or unloading at berth.

- Towing of barges and the departure of self-propelled barge after loading/unloading to the anchorage area or leaving the port.

This cycle is called the turnaround time for the ships and the barge tows in the river ports.

Due to the influence of a great number of factors (water level, technological functioning of devices, meteorological conditions, number of barges in tows, distance between the anchorage area the berth line, position of
ships at the anchorage, etc.), these processes cannot ideally follow one another or have constant time intervals, but they are subject to permanent changes. There are longer or shorter waiting time intervals due to the aforementioned reasons that extend the turnaround time and unsuitably influence the port and fleet operations.

It is natural that barge operators look with favor on a port having sufficient berthing space to accommodate every barge on arrival, thereby eliminating costly waiting time. Berths in such a port may be vacant much of the time. It is also natural to expect that the barge operators of river port terminal facilities generally would not want to have ship berths standing idle. Somewhere between these opposing objectives each port must reach a compromise - the number of berths that will achieve the most economical transfer of cargo between ships and inland destinations.

An ideal situation is one in which all berths are occupied all of the available time. This situation is impossible to achieve in practice because of the random times of arrival of ships and variations in size of tonnage discharged and loaded in barges or ships. Arrivals of ships can be single or in groups, and the size of an arriving group is a random variable and thus are more general being also more difficult to handle. A systematic study of queueing theory provides a base of knowledge that can be applied to improve the efficiency of many queueing systems in the real world. The method can be used to give explicit solutions to many particular problems. Furthermore, the paper discusses queueing systems with head-of-the-line and non-preemptive priority servicing with limited anchorage area.

The application of queueing theory to shipping and port problems has been made by Plumlee [6], Nicolaou (1967), Wanhill [8], Agerschou et al. [1], Berg-Andreassen and Prokopowicz [2], Radmilović [7], and Zrnić et al. [9].

2 Statement of the problem

The anchorage-ship-berth link is considered as the $M/D/c/N$ queue model with nonpreemptive priorities in servicing at berths. There are two classes of priorities among arriving ships in river port. Barges of pushed and pulled barge tows have the lower priority class in servicing, whilst self-propelled barges and motor cargo ships have higher priority class in servicing. Obviously, customers are single ships and barge tows, whereas the service channels are berths operating on cargo loading/unloading.

In the port under analysis, the anchorage-ship-berth link is assumed as follows:
- Applied queueing model is stationary, with finite waiting area at anchorage.
- Sources of the arriving pattern are not integral parts of the anchorage-ship-berth link.
- Arriving units can be single ships and barge tows. The arrival process of service-seeking entities follows the Poisson distribution.
- Service channels are berths with similar or identical independent cargo-handling capacities.
- Service is assumed to be deterministic and provided to ships on a first-come-first-served basis. It is assumed that the priority discipline is of nonpreemptive head-of-the-line type.
- Number of priority classes is adopted to be "1" and "2", where a lower number indicates a higher priority class (self-propelled ship has priority class "1" which is higher in priority than class "2" - barges from pushed and pulled barge tows).
- Anchorage is a buffer of finite capacity before the berth service area. Whenever the anchorage capacity is exceeded the ships are rejected from the system irrespective of the priority class they belong to.
- Ship queue length, or the number of ships waiting at the anchorage area is finite and given.

The river port terminal has the c berths for the service. The mean cargo handling rate per berth is \( R \). Apart from the possible arrival of \( c \) ships for service, there are \( B \) spaces in the anchorage. Ships arrive according to a time-homogeneous Poisson process with mean arrival rate \( \lambda \). Using mathematical derivations, Bose and Pal [3], here, we can begin with the derivation of ship average waiting time.

The average waiting time of a ship belonging to priority class \( n \) will consist of three components. The first component is the time that the ship on an average has to wait before any of the berths become free. The second component is the waiting time due to ships of priority classes \( n \) or higher, who have arrived before the new ship and are waiting to be served. The last component is the waiting time due to ships belonging to priority classes higher than \( n \) who arrive when the ship belonging to priority class \( n \) waits to be served.

Kleinrock [4] proposed the solution for average waiting time in queue for a customer belonging to priority class \( n \) as

\[
\overline{W}_{qn} = \frac{\overline{W}_0}{(1 - \sigma_{n-1})(1 - \sigma_n)}
\]  

(1)

where \( \overline{W}_0 \) is the average waiting time of a new arrival until one of the \( c \) berths becomes free.

2.1 At this point, it is convenient to define

\[
\sigma_n = \sum_{i=1}^{n} \rho_i \quad (i = 1, 2, \ldots, n - \text{priority classes})
\]  

(2)

where:

\[
\rho_i = \frac{\lambda_i (1 - P_N)}{cR}
\]  

(3)

where
\( \lambda \) - arrival rate for ships belonging to priority class \( i \) (units/time period).

\( P_N \) - probability that there are \( N \) ships in the port (i.e. the probability that the anchorage is full and any new ships to the port are lost).

\( \lambda_i (1-P_N) \) – effective arrival rate due to the presence of the anchorage area with finite capacity in front of the berth service area.

Using derivations of Bose and Pal [3], the average waiting time for the new ship, until one of the berths becomes free is given by:

\[
\overline{W}_0 = \frac{T}{c + 1} \sum_{s=c}^{N-1} P_s
\]

where \( T = 1/R \) and \( P_s \) is the probability that there are \( s \) ships in the port.

Now, substituting the value of \( \overline{W}_0 \) from (4) into (1) we yield

\[
\overline{W}_{qn} = \frac{(T/(c + 1))\sum_{s=c}^{n-1} P_s}{(1-\sigma_{n-1})(1-\sigma_n)} \tag{5}
\]

Calculation of the steady-state probability is independent of priority considerations as service time is not dependent on priority class and so the result of the non-priority queue can be used here. Therefore,

\[
P_s = \begin{cases} 
\frac{(c \rho)^s P_0}{s!} & \text{for } s = 0, 1, \ldots, c-1, \\
\frac{(c \rho)^c 1 - \xi}{c!(1 - \rho)} \xi^{s-c} P_0 & \text{for } s = c, c+1, \ldots, c+B-1, \\
\frac{(c \rho)^c}{c!} \xi^B P_0 & \text{for } s = c + B
\end{cases} \tag{6}
\]

\( s = 0, 1, \ldots, c-1, \)

\( s = c, c+1, \ldots, c+B-1, \)

\( s = c + B \)

where
\[ \rho = \frac{\lambda}{cR} \]  

(7)

\[ \lambda = \sum_{i=1}^{n} \lambda_i \]  

(8)

\[ P_0 = \left\{ \sum_{s=0}^{c-1} \frac{(c\rho)^s}{s!} + \frac{(c\rho)^c}{c!} \cdot \frac{1 - \rho \xi^B}{1 - \rho} \right\}^{-1} \]  

(9)

\[ \xi = \frac{\rho R_D}{1 - \rho + \rho R_D} \]  

(10)

where \( R_D \) is an approximation factor for high traffic intensity, as follows:

\[ R_D = \frac{\text{Expected waiting time for deterministic service system}}{\text{Expected waiting time for exponential service system}} \]  

(11)

or,

\[ R_D = \frac{\rho}{2} + \frac{(1 - \rho)c}{c + 1} \]  

(12)

Using Eqs. (1) – (12) we can analyze the river port as \( M/D/c/N \) queuing system with non-preemptive head-of-the-line priority discipline, which is shown in the following section.

3 Numerical results

In this section we will present the dependence of the average waiting time of a ship or barge belonging to a particular priority class with various parameters associated with the river port. The parameters under consideration are: ship arrival rate (\( \lambda \)), service or loading/unloading rate (\( R \)), number of berths (\( c \)) and the anchorage capacity (\( B \)).

Tables (1) and (2) show the variation of the average waiting time with arrival rate for the high and low priority class (self-propelled barges and motor ships with high priority class “1” and barges from pulled or pushed tows with low priority class “2”), respectively.

In Table (1) we presented the numerical results for:
- the steady-state probability that the river port is idle ($P_0$);
- the steady-state probability that $s$ barges and/or motor ships are waiting at the port anchorage ($P_s$);
- the steady-state probability that there are $N$ ships in the river port or the probability that the anchorage is full and any new ship arrivals to the port are rejected ($P_N$);
- the average waiting time of a new ship until one of the $c$ berths becomes free ($\bar{W}_0$);
- the average waiting time at the anchorage for a self-propelled barge or a motor ship of priority class “1” ($\bar{W}_{q1}$), and
- the average waiting time at the anchorage for a barge from a barge tow of priority class “2” ($\bar{W}_{q2}$)

All above values are calculated in function of the ship arrival rate ($\lambda = 1, 3$ and $7$), the number of berths ($c = 4$), the constant service rate for ships belonging to any of the priority classes ($R = 2$) and the capacity of the anchorage ($B = 5$ ships).

For the $M/D/c/N$ queue, the service rate is assumed to be the same for all priority classes ($R_1 = R_2 = \ldots = R_n = R$).

Table 1: River port performance parameters for $c = 4$, $R = 2$ and $B = 5$.

<table>
<thead>
<tr>
<th>Ship arrival rate ($\lambda$)</th>
<th>$P_0$</th>
<th>$P_s$</th>
<th>$P_N$</th>
<th>$\bar{W}_0$</th>
<th>$\bar{W}_{q1}$</th>
<th>$\bar{W}_{q2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.6065</td>
<td>0.0018</td>
<td>1.44E-07</td>
<td>0.0002</td>
<td>0.000193</td>
<td>0.00022</td>
</tr>
<tr>
<td>3</td>
<td>0.2376</td>
<td>0.0665</td>
<td>0.0025</td>
<td>0.0066</td>
<td>0.0082</td>
<td>0.0131</td>
</tr>
<tr>
<td>7</td>
<td>0.0191</td>
<td>0.6614</td>
<td>0.0367</td>
<td>0.0661</td>
<td>0.1143</td>
<td>0.7276</td>
</tr>
</tbody>
</table>

Table 2: River port performance parameters for $c = 2$, $R = 3$ and $B = 8$.

<table>
<thead>
<tr>
<th>Ship arrival rate ($\lambda$)</th>
<th>$P_0$</th>
<th>$P_s$</th>
<th>$P_N$</th>
<th>$\bar{W}_0$</th>
<th>$\bar{W}_{q1}$</th>
<th>$\bar{W}_{q2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.7144</td>
<td>0.0476</td>
<td>1.04E-09</td>
<td>0.0053</td>
<td>0.00578</td>
<td>0.00693</td>
</tr>
<tr>
<td>3</td>
<td>0.2858</td>
<td>0.2852</td>
<td>0.000247</td>
<td>0.0317</td>
<td>0.0423</td>
<td>0.0844</td>
</tr>
</tbody>
</table>
Figure 1: Waiting time of the 1st class priority ships for $c = 4$, $R = 2$ and $B = 5$.

Figure 2: Waiting time of the 2nd class priority ships for $c = 4$, $R = 2$ and $B = 5$.

Table (2) contains the numerical results for $P_0$, $P_s$, $P_N$, $W_0$, $W_{q1}$ and $W_{q2}$ in function of ships arrival rate of $\lambda = 1$ and 3 ships per day, $c = 2$, $R = 2$ and $B = 8$. Also, the total arrival rate of ships to the river port is equally distributed between the two priority classes.

Comparison of Tables 1 and 2 shows that the average ship waiting time increases for both high and low priority ships with increase in the anchorage capacity and with decrease in the number of berths. This leads us to the
conclusion that a trade-off between number of berths and anchorage capacity must take place in order to balance the waiting time within acceptable rates for both ship and port operators.

4 Conclusion

In this paper, the river port is analyzed as the M/D/c/N queuing system with non-preemptive priorities. From the application of the queuing model used in this analysis we can see how various operational parameters and their variations such as the number of berths, capacity of the anchorage, loss probability, service rate and average ship waiting times can affect the operational conditions of a river port under consideration.

These results can serve as a useful first approximation and as the base knowledge for the analysis of port operations in both existing and ports in the phase of initial design, where the space limit is an issue to be taken into account.

References