Characterization of dislocation in underground mass using coupled modeling

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Abstract

This paper deals with an application of coupled modeling in the identification of dislocation occurring in coal mines, threatening workers in the underground structures. The bumps can be induced by different circumstances. One of them is an accumulation of energy in unpredictable dislocations. The measurement on site for learning the position of bumps is very expensive and unreliable. One of the most reliable approaches is physical modeling, which enables one to carry out parametric studies and after certain results from these models one can assess the most probable concentration of stresses. On the other hand, the stresses are measured in a very difficult way, so that numerical analysis should be prepared. In numerical analysis the contact problem in limit state estimation is based on the data from physical modeling. Physical modeling seems to be the best for linear analysis. This is not the case in our study and great efforts are needed to estimate the real behavior of the material. The support of physical modeling mathematical formulation and numerical treatment can lead us to the location of bumps. In general, a large iteration should be used to solve the strongly nonlinear problem in all subdomains ranged in boundaries given by possible dislocations, and on the interfacial boundaries (dislocations). In order to eliminate some principal directions of iteration the physical modeling is used and the numerical processes become bearable. On the other hand, more dislocations can be sought by this modeling and the only restriction needed is to know that no internal dislocations are probable. It means that every discontinuity induces a statically determined or undetermined problem. A typical example from practice verifies the theory based on the back of the analysis.
1 Introduction

In this paper we discuss a possible solution of the stability of a dislocation in the rock continuum under the assumption that the behavior of the continuum is approximately known from the physical model and the stability is studied by means of the BEM as a contact problem. In virtue of this technology the limit state analysis can be carried out. The interfacial conditions are partly derived from the physical model and partly show the critical material characteristics, which can lead to the bumps. The mathematical model originates from the Uzawa's algorithm solving the classical problem of the generalized Coulomb's friction with exclusion of tensile strength along the interfaces. The distribution of material properties is simulated according to parametrical study in the mathematical model at the face of stope.

This procedure was successfully applied to the estimation of the behavior of an opening in the vicinity of town of Ostrava, Czech Republic. Some results from this region are described using the study proposed in this paper.

2 Physical modeling

Principles of a new projecting method of underground construction in soft rocks can be formulated on the basis of results from extensive tests on physical models which were carried out in experimental department of Pardubice University.

Properties of these rocks differ from site to site. Rocks are frequently separated by discontinuity surfaces. These surfaces and the weakening zones cause disintegration or susceptibility to disintegration of the rock mass into structural units of various forms, size and properties. Their properties change also with the stress mode and depend on the stressing force, to which the rock was exposed in the past.

It results from the above-mentioned facts that the properties of the rock environment cannot be measured either on small rock samples or by isolated sporadic tests in situ. In the first case, we cannot evaluate the effect of weakening planes, and in the other one an unpleasant dispersion variance of material properties is caused by inhomogeneities of the rock mass. In both cases, the necessary conditions of the physical similitude are usually not observed. These conditions would require measuring the rock properties under stress conditions equal to stressing force, to which they are exposed during construction and after completion of construction works. These requirements can very easily be observed on physical models from equivalent materials.

Results from laboratory tests can be used directly as impute data for mathematical solution provided the constitutional relation between the stress tensor and strain (deformation) tensor is linear and provided the relative homogeneity and isotropy (e.g. of solid rocks) are supposed. This is not our case and large simulations in laboratory have to be carried out.

On the other hand the stresses are difficult to obtain from the physical modeling. This is the moment when the mathematical model can help and
coupling of both models can approximate the real state of the rock continuum and the structure.

A very important conclusion results from some selected rock mechanics problems using the method of physical and mathematical modeling, and from their mutual comparison. It is possible, by means of a test on a physical model, to establish conditions, which are of primary interest to us (e.g. when the resistance of internal forces against failure is optimal). For conditions thus established, the deformation and stress of the rock environment in the neighborhood of the underground opening are determined by the finite element method, boundary element method, or with the aid of the combination of both. A part of problem can therefore be resolved by tests on a physical model and the results obtained can be used as input values for final solution by some numerical methods. In this way, both procedures can be adequately combined and completed, advantages and drawbacks of both of them acting complementarily.

With respect to the extension of the modeling equipment, material properties of physical material, measurement tools, and time factor it has been selected geometric ratio 1:200 and time scale 1:120. The mutual relation between real parameters and model properties of the material is given by adequate rules of similarity. The modeling stand (basin with glassed front side reinforced by steel frame was 6 m high, 2 m long and 1 m wide). The rear side has been constructed from plastic plates with longitudinal openings in the shape of distinguished seams. Modeling equipment enables researchers to load the terrain by pneumatic pillows. Front view of the model in depicted in Fig. 1.

Figure 1: Front view of the experimental stands with the physical model.
The results of movements of the equivalent material in the front plane of
the stand are determined for vertical and horizontal displacements and have
been measured by photogrammetric method. This method enables one to get
stereo couples of photographic pictures; one of them is in the starting (virgin)
position. From the stereo couples of the model states the movement vectors at
selected points for given stages of physical model are derived.

3 Solution of underground continuum by the BEM

In this part we briefly describe an implementation of boundary element method
to the solution of specific problems of underground continuum, for which the
numerical method appears to be extraordinary advantageous.

The method, among others, reduces the problem by one. Further good
application of the boundary element method is the optimization and/or contact
problems which concern the boundary only. Then, in spite of the finite element
method it suffices to study a change of location of boundary elements only.

The problem is solved as two-dimensional, i.e. a possible dislocation is long
enough, and narrow seam is considered. Moreover, the nonlinear behavior is
considered in the region, which is sufficiently close to the dislocation, according
to Mises theory. Suspicious dislocation is given from the experimental model
from physically equivalent materials.

In our following consideration we will concentrate to the physically nonlinear
problems (nonlinear evolution is also included in boundary conditions). Let us
solve the problem on domain \( \Omega \). We originate from the Cauchy equations:

\[
(\alpha + \mu) \frac{\partial}{\partial x_i} \text{div} u + \mu \Delta u_i + X_i + \sum_j \frac{\partial \sigma_{ij}}{\partial x_j} = 0, \quad i = 1, \ldots, 3
\]

where

\[
\text{div} u = \frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2}, \quad \Delta = \frac{\partial}{\partial x_1} + \frac{\partial}{\partial x_2}
\]

and \( u = (u_1, u_2) \) is the displacement field, \((X_1, X_2)\) are components of the
volume weight and \( \sigma_{ij}^0 \) are components of the tensor of initial stress.

These equations will be solved in coordinate system \( 0x_1, x_2 \). In the sense of
BEM, (1) may be reformulated in an equivalent form:

\[
c_{k1}(\xi)u_i(\xi) = (\varepsilon_{ijk}, \sigma_{ij}) + [p_{ik}, u_i] - [u_{ik}^*, p_i] - (u_{ik}^*, x_i)
\]

where \([.\) are boundary integrals, \( .\) are plane integrals, \( c_{k1} \) is the matrix of
coefficients depending on a position of \( \xi \), \( p \) is the vector of external forces, \( \varepsilon \) is
the tensor of deformations and quantities with asterisk denote the relevant quantities of fundamental solution. Such the function was derived by Melan and can be found in publication [1], for instance.

Now the trick starting with the polarization tensor used in [2] is applied for describing the nonlinear behavior of the whole massif. In the polarization tensor also initial stress can be involved. From the Cauchy equations (respecting shearing stresses to be zero) we have well known relations for virgin state to get initial stresses:

\[
u_1^o = -\frac{1-2\nu}{4\mu(1-\nu)} X_1x_1^2 + \text{const.}, \quad u_2^o = 0
\]

\[
\sigma_{1,1}^o = -X_1x_1, \quad \sigma_{1,2}^o = 0, \quad \sigma_{2,2}^o = -\frac{\nu}{1-\nu} X_1x_1 \quad (3)
\]

4 Contact problem

Before we start the analysis preliminary considerations will be introduce. In order to explain the process of computation two-dimensional problem will be treated. The three-dimensional problems are solved similarly.

Let the problem be described from experimental study. The field of horizontal displacements is depicted in Fig. 2. In numerical version the problem is illustrated in Fig. 3 by domain \( \Omega \), \( \Gamma_c \) is a part of boundary splitting the plane into left \( \Omega_L \) and right half plane \( \Omega_R \), along \( \Gamma_p \) the distribution of given surface forces is done and \( \Gamma_c \) is fictitious slip surface (dislocation) either prescribed (this is our case, the location is estimated from the physical model), or the location is to be searched by an enlarged numerical process. \( \Omega_{\text{seam}} \) is the domain of the seam, for which the dislocation and the bearing capacity is to be assessed.

After discretization of (2) in the sense of boundary element method the problem leads to the system of algebraic equations:

\[
\begin{bmatrix}
H_{11} & H_{12} & H_{13} \\
H_{21} & H_{22} & A_{23} \\
H_{31} & H_{32} & A_{33}
\end{bmatrix}
\begin{bmatrix}
u \\
u^- \\
u^+
\end{bmatrix}
= \begin{bmatrix}
G_{11} & G_{12} & G_{13} \\
G_{21} & G_{22} & G_{23} \\
G_{31} & G_{32} & G_{33}
\end{bmatrix}
\begin{bmatrix}
g \\
g^- \\
g^+
\end{bmatrix}
+ \begin{bmatrix}
g^c \\
g^-^c \\
g^+^c
\end{bmatrix}
= \begin{bmatrix}
F \\
F^- \\
F^+
\end{bmatrix}
\quad (4)
\]

where the upper index - denotes "from the left" and + denotes "from the right", \( g \) is the vector of prescribed surface forces along the boundaries \( \Gamma \) and \( \Gamma_p \), \( p_c \) is the vector of surface forces on fictitious contact \( \Gamma_c \) and \( F \) includes the effect of volume weight.

As the vector \( g \) contains known quantities we can rearrange the previous equations to obtain:
Figure 2: Cross-section of the massif with horizontal displacements and seams.

\[
\begin{bmatrix}
H_{11} & H_{12} & H_{13} \\
H_{21} & H_{22} & A_{23} \\
H_{31} & H_{32} & A_{33}
\end{bmatrix}
\begin{bmatrix}
u \\ u_- \\ u_+
\end{bmatrix}
= 
\begin{bmatrix}
G_{12}p_+^c + G_{13}p_-^c \\
G_{21}p_+^c + G_{23}p_-^c \\
G_{31}p_+^c + G_{33}p_-^c
\end{bmatrix}
\begin{bmatrix}
F \\ F_-^c + G_{11}^g \\
F_-^c + G_{21}^g \\
F_-^c + G_{31}^g
\end{bmatrix}
\]

(5)

Suppose now that for example \(u_-^c\) and \(u_+^c\) is known. Then the problem is uniquely solvable, so that the matrix \(H_{11}\) is regular. For the similar reason the
matrices $H_{kk}$ are regular, too. Also, the same assertion holds for the matrices $G_{1i}$, $i = 1,2$. This is the general result of solvability of linear problems of elasticity by boundary element method. We can conclude that the matrix $H$ is singular, but the last submatrices are regular matrices. This is why it is possible to rearrange the system in the sense of matrix canonical transformations (in algorithm we use Gaussian elimination) to obtain:

\[
\begin{bmatrix}
H_{11} & H_{12} & H_{13} \\
0 & A_{22} & A_{23} \\
0 & A_{32} & A_{33}
\end{bmatrix}
\begin{bmatrix}
u \\
u^+_c \\
u^+_c
\end{bmatrix}
- \begin{bmatrix}
B_{11} \\
B_{21} \\
B_{31}
\end{bmatrix}
\begin{bmatrix}
p_c \\
p_c \\
p_c
\end{bmatrix}
= \begin{bmatrix}
C_{11} \\
C_{21} \\
C_{31}
\end{bmatrix}
\tag{6}
\]

where the balance condition

\[
p_c = p_c^- = -p_c^+
\tag{7}
\]

was employ. The matrices are known while the vectors $u$ and $p$ remain unknown. From the last form the reducibility follows and we can employ the following system of equations:

\[
A_{32}u^+_c + A_{33}u^+_c - B_{31}p_c = c_{21}
\]
\[
A_{22}u^+_c + A_{23}u^+_c - B_{21}p_c = c_{22}
\tag{8}
\]

Figure 3: Domain and denotation of the example under study.

Generally, along the contact line only balance condition holds and the compatibility is prescribed with the aid of more complicated relations. For example, suppose that at each nodal point along the contact line holds:
\[
[u]_n = u_n^1 - u_n^2 \geq 0
\]
\[
|p_t| \leq Tp_n + c
\]
\[
|p_t| \leq Tp_n + c \Rightarrow E\lambda > 0, [u]_n = -\lambda p_t
\]  \hspace{1cm} (9)

where \(T\) and \(c\) are prescribed coefficients (they may vary along the contact), \(p_n\) and \(p_t\) are projections of tractions to the normal and tangential direction with respect to the contact line, respectively. Then Uzawa’s algorithm can be applied – see, e.g., [3].

5 Example

In praxis we are interested in a measure of stability of the massif connected by the fictitious contact line of the dislocation. After iteration process we can get a length of a part of the contact on which the tangential bond \([u]_t\) is different from zero. Hence, we can talk about percentual exhausting of bearing capacity (e.g. on 50 percent of the length of the contact the conditions (9) are violated, so that the bearing capacity is also 50 percent). This is the way on how to introduce coefficient \(s\), which gives the measure of stability:

\[
s = \frac{\text{length of the part where } [u]_t \neq 0}{\text{total length of contact}} \times 100 \text{ percent}
\]  \hspace{1cm} (10)

The study on stability concerning the influence of backfill of the underground opening in Fig. 3 has been carried out. Along the boundaries \(\Gamma^p_u\) and \(\Gamma^p_d\) surface forces were implied to simulate admissible relative displacements between top and bottom openings. These forces are expressed by virtue of spring coefficient \(k\) [MPa/m]. Material coefficients of the massif \(\Omega\) have the following values: \(E = 52\ 500\ \text{MPa}, \nu = 0.29\), the peak values \(E_p = 38\ 000\ \text{MPa}\), and \(v_p = 0.38\), the residual values \(E_r = 5\ 000\ \text{MPa}\), the angle of internal friction is 42 degree, its residual value is 32 degrees, the shear strength \(c = 0.9\ \text{MPa}\) and its residual value is considered as 0.4 MPa. The capacity of the fictitious contact line is expressed by the relation (10) and given in Fig. 4.

6 Conclusions

In this paper a complex study based on coupled numerical end experimental studies has been proposed and applied to a real structure of underground work, in particular mines. It is aimed to assess the bearing capacity of dislocation, its position has been detected by experimental studies on scale models, and the results involving positioning of the dislocation have then been taken as input data of a great importance to observing stability (particularly to bumps) in the coal seams.

Many others studies have been carried out and in the future we intend to test with the aid of this procedure another approaches to improve the information.
about the three-dimensional behavior of the system of dislocations in the rock mass. This will lead to much more realistic information and helps to designers of underground works to prepare better plans.

The approach is very close to back analysis, which is widely used in other parts of civil and underground structures.

Figure 4: Values of tangential spring stiffnesses vs. safety margin.

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References

