Comparison of two volume tracking algorithms for bubbly flow simulation

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Abstract

There are several volume tracking algorithms for numerical simulation of gas-liquid two-phase bubbly flow based on an interface tracking method. Among them, FLAIR (Flux Line-Segment Model for Advection and Interface Reconstruction) was reported as the best algorithm to predict convection of interface. However, FLAIR has not been tested for actual bubbly flow in which the effects of surface tension, drag force and buoyancy forces are dominant. In the present study, we applied two representative volume tracking algorithms, FLAIR and Donor-Acceptor methods to single bubbles rising through stagnant liquids to examine whether or not these two methods can yield good predictions. As a result, we could confirm that shapes and rising velocities of single bubbles predicted by both methods agreed with experimental data.

1 Introduction

Although a number of studies have been conducted to develop reliable models for the prediction of gas-liquid bubbly flow, we cannot still make good predictions due to the lack of knowledge on bubble dynamics, which depends on a bubble size, fluid properties, flow field around a bubble, gravity and so on. If we can obtain detailed information on the velocity and pressure fields around a bubble, we will be able to develop mathematical models for complicated bubble dynamics such as bubble intrinsic fluctuating motion, lateral migration in a shear field, virtual mass force and drag force. Recent rapid progress in computer performance and computational fluid dynamics gives us a large possibility of obtaining the necessary information by numerical simulation. As
can be found in Ref.[1], the volume of fluid (VOF) method is one of the promising methods for this purpose. In the VOF method, the motion of the gas-liquid interface is tracked by solving the liquid volume balance. In order to keep the sharpness of the interface, the Donor-Acceptor (DA) method, which is one of the volume tracking algorithms, was adopted in the original VOF method (Hirt & Nichols^) for the calculation of the liquid volume balance.

To examine the accuracy of volume tracking, Bugg and Naghashzadegan^ compared five volume-tracking algorithms, i.e., SLIC (Simple Line Interface Calculation), modified SLIC, DA, modified DA and FLAIR(FLux Line-segment model for Advection and Interface Reconstruction). They concluded that FLAIR is the best volume-tracking algorithm and DA is the worst for non-straining flows. However, FLAIR has not been tested for actual bubbly flow in which the effects of surface tension, drag force and buoyancy are dominant. Hence, in the present study, we implemented DA and FLAIR into the VOF method and examined the applicability of these two volume-tracking algorithms to the simulation of actual bubbles.

2 Numerical methods

2.1 Outline of the VOF method

An axisymmetric cylindrical coordinate system was used in the simulation. In the (r, z) system, local-instantaneous mass and momentum equations for incompressible two-phase flow are given by

\[
\frac{\partial u_k}{\partial t} + u_k \frac{\partial u_k}{\partial z} + v_k \frac{\partial u_k}{\partial r} = -\frac{1}{\rho_k} \frac{\partial p}{\partial r} + v_k \left( \frac{\partial^2 u_k}{\partial r^2} + \frac{\partial^2 u_k}{\partial z^2} + \frac{1}{r} \frac{\partial u_k}{\partial r} - \frac{u_k}{r^2} \right),
\]

\[
\frac{\partial v_k}{\partial t} + u_k \frac{\partial v_k}{\partial z} + v_k \frac{\partial v_k}{\partial r} = -\frac{1}{\rho_k} \frac{\partial p}{\partial z} - g + v_k \left( \frac{\partial^2 v_k}{\partial r^2} + \frac{\partial^2 v_k}{\partial z^2} + \frac{1}{r} \frac{\partial v_k}{\partial r} \right),
\]

where \( u \) denotes the velocity in the r direction, \( v \) the one in the z direction, \( t \) the time, \( p \) the pressure, \( \nu \) the kinematic viscosity, \( g \) the gravity constant, and the subscript \( k \) the phase index (k=G or L). The density \( \rho \) in Eqs.(2) and (3) is defined by

\[
\rho = (1-F)p_G + FP_L,
\]

where F is the volumetric fraction of the liquid phase in a computational cell. A cell is filled with the liquid when \( F=1 \) or with the gas when \( F=0 \). If \( 0<F<1 \), an interface exists in the cell (interface cell). The location and curvature \( \kappa \) of the interface are determined using the values of F in the cells surrounding the interface cell. Then, the pressure due to surface tension, \( p_s \), is calculated using \( p_s=\kappa \sigma \), where \( \sigma \) is the surface tension. The surface tension force per unit volume, \( f_s \), is normal to the interface and directs toward the center of curvature. It follows that
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\[ f_s = \frac{p_s S}{\Delta V}, \quad (5) \]

where \( S \) denotes the surface element in the interface cell directing toward the center of curvature, \( \Delta V \) the volume of the interface cell. In the interface cells, \( f_s \) is added to the right-hand side of momentum equations.

The volume conservation equation of the liquid phase,

\[ \frac{\partial F}{\partial t} + \frac{1}{r} \frac{\partial (F u_k)}{\partial r} + \frac{\partial F}{\partial z} = 0, \quad (6) \]

is used to advance the interface location. Solving Eq.(6) using a conventional difference scheme results in unexpected interface diffusion. In order to keep the sharpness of interface, special algorithms have to be employed to solve Eq.(6).

2.2 Volume tracking algorithm

DA and FLAIR algorithms were applied to solve the liquid volume conservation equation in the present study. An example of computational cells including interface is shown in Fig.1. In the figure, the gray region denotes the liquid volume, the white one the gas volume, and the black one the liquid volume \( \theta \) transferred from the donor cell to the acceptor cell in one time step \( \Delta t \). In DA, the transferred liquid volume, \( \theta_D \), is a rectangle as shown in Fig.1(a), while \( \theta_F \) in FLAIR is a trapezoid as shown in Fig.1(b). So that \( \theta_D \) is not equal to \( \theta_F \) in most cases. The difference of the transported volumes may cause a significant difference in the time-evolution of bubble shapes.

![Figure 1: Transferred liquid volume in DA method and FLAIR method](gray region: liquid volume, black region: transferred liquid volume)

3 Test problem

Bhaga & Weber\(^5\) proposed a reliable graphical correlation of the shapes and rising velocities of single bubbles in stagnant liquids shown in Fig. 2. The three dimensionless numbers, \( E_0, M \) and \( Re \), in Fig. 2 are the Eötvös, Morton and bubble Reynolds numbers defined by

\[ E_0 = \frac{g(\rho_L - \rho_G)d^2}{\sigma}, \quad M = \frac{g \mu_L^{\frac{4}{3}} (\rho_L - \rho_G)}{\rho_L^{\frac{2}{3}} \sigma^3}, \quad Re = \frac{\rho_L V_L d}{\mu_L}, \quad (7),(8),(9) \]
where $d$ denotes the equivalent bubble diameter, $\mu$ the viscosity and $V_T$ the terminal velocity. The Eötvös number represents the effects of buoyancy and surface tension forces. The Morton number accounts for the effects of fluid properties. The bubble Reynolds number is a dimensionless terminal velocity. The numbers, 1-7, in Fig. 2 represent the test conditions which cover spherical, ellipsoidal, and spherical cap bubbles. The values of Eötvös and Morton numbers and corresponding bubble shapes are summarized in Table 1.

![Figure 2: Bhaga & Weber's Graphical correlation and test conditions(1-7)](image)

### Table 1: Test conditions

<table>
<thead>
<tr>
<th>Case</th>
<th>$E_o$</th>
<th>$M$</th>
<th>Bubble shape</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8.67</td>
<td>711</td>
<td>Spherical</td>
</tr>
<tr>
<td>2</td>
<td>4.38</td>
<td>$2.98 \times 10^3$</td>
<td>Ellipsoidal</td>
</tr>
<tr>
<td>3</td>
<td>10.94</td>
<td>$2.98 \times 10^3$</td>
<td>Ellipsoidal</td>
</tr>
<tr>
<td>4</td>
<td>32.2</td>
<td>$8.20 \times 10^4$</td>
<td>Disc</td>
</tr>
<tr>
<td>5</td>
<td>243</td>
<td>266</td>
<td>Ellipsoidal cap</td>
</tr>
<tr>
<td>6</td>
<td>116</td>
<td>1.31</td>
<td>Ellipsoidal cap</td>
</tr>
<tr>
<td>7</td>
<td>115</td>
<td>$4.63 \times 10^3$</td>
<td>Spherical cap</td>
</tr>
</tbody>
</table>

![Figure 3: Computational domain](image)
Figure 3 shows the computational domain, initial condition and boundary conditions. A cylindrical tank was filled with a stagnant liquid. Initially, a single spherical bubble was located by 3.5d apart from the bottom of the tank. The radius of the tank was 5d, and its height was 13d. Simulations were conducted in a frame of reference fixed to the tank. A uniform square cells, $\Delta r=\Delta z$, were used. The number of cells assigned to the bubble diameter was 16 when bubble shape was spherical or ellipsoidal (cases 1-3), and 32 when it was disc, spherical cap or ellipsoidal cap (cases 4-7). The side and bottom walls of the tank were slip walls. Symmetric and continuous conditions were adopted for the left and top boundaries, respectively. The Courant number defined by $c=V_{\text{max}}\Delta t/\Delta z$ was ranging from $2\times10^4$ to $8\times10^2$.

4 Results and discussion

Figure 4 shows a comparison between measured and calculated bubble shapes. The shapes shown in the left column (Fig.4(a)) were obtained using DA and the ones in the right column (Fig.4(c)) were obtained using FLAIR. The photos shown in the center column (Fig.4(b)) were obtained by Bhaga & Weber\(^5\) (cases 1,4-7) and by Tomiyama et al.\(^6\) (cases 2,3).

When the bubble was spherical (case 1), FLAIR gave a slightly better result than DA. Ellipsoidal bubbles (cases 2 & 3) calculated using DA showed a little better agreement with the measured ones. For the cases when bubble became spherical or ellipsoidal (cases 1, 2 and 3), DA tended to make the nose and the equator of the bubbles slightly flattened, while the nose and the equator of bubbles obtained by FLAIR protruded outward, which made the radius of curvature at the nose of bubbles smaller.

Let us consider the causes for the above tendencies. Figure 5(a1) shows an example of calculated velocity distribution around a bubble (case 3, DA), which qualitatively agreed with the Hadamard-Rybczynski solution\(^7\) of the streamlines around a fluid sphere shown in Fig.5(a2). The velocity across the interface near the nose of bubble was almost perpendicular to the interface as schematically shown in Fig.5(b2), while the one in the vicinity of the equator was almost parallel to the interface as shown in Fig.5(d2). In the figures, the gray regions denote the gas volumes and the black regions the gas volumes advected from the donor cells to the acceptor cells in a time step. As shown in Fig.5(b1), DA may slightly underestimate the transferred gas volume $\theta_D$ near the nose of bubble, because the nose cannot move across the cell boundary until almost all the liquid in the donor cell has transferred. To the contrary, $\theta_P$ evaluated by FLAIR is apt to be slightly larger than the correct transferred volume $\theta_I$ as shown in Fig.5(b3). Figure 5(d) shows an example of interface cells where the interface is almost vertical and the velocity is almost parallel to the interface. In these cells, $\theta_D$ tends to be a little greater than $\theta_I$, whereas $\theta_P<\theta_I$. These errors in the transferred gas volume of the two methods will deform a bubble as shown in Fig.5(e), that is, DA tends to make bubble shape like a cylinder in the cylindrical coordinate as shown in Fig.5(e1), while the nose and the equator of a bubble obtained by FLAIR protrude outward as shown in Fig.5(e3). The reason why the
Figure 4: Comparison of bubble shapes ((a)DA, (b)measured, (c)FLAIR)
(a) Calculated velocity distribution near the bubble (case 3, DA)

(b1) DA

(b2) Horizontal interface

(b3) FLAIR

(c1) DA

(c2) Ideal bubble

(c3) FLAIR

(d1) DA

(d2) Vertical interface

(d3) FLAIR

(e1) DA

(e2) Ideal Shape

(e3) FLAIR

Figure 5: Transported volume of the gas phase calculated by DA and FLAIR methods
(gray regions : gas volume, black regions : transported gas volume)
spherical bubble (case 1) did not show the protrusion would be that strong effect of surface tension corrected the protruded part of the bubble.

In case 4, the thickness of disc bubble was predicted better by DA, while FLAIR gave a flat rear surface as shown in Fig.4(4). In the other test conditions (cases 5, 6 and 7), bubbles became ellipsoidal cap or spherical cap. The predicted bubble shapes obtained by the two methods agreed with each other. However, there were only a few cells between the nose and the tail of spherical cap bubble in case 7, which would induce certain errors in the calculation of the location and curvature of the interface, so that not only interface convection but also surface tension force acting on the interface would be predicted in the wrong way. In addition, the measured bubbles in cases 4 and 7 were much elongated in the horizontal direction so that we have to use much larger computational domain to remove the effects of the side walls of the tank on bubble shape prediction. Increasing the grid resolution and the size of computational domain will give us better predictions.

Through the above comparisons, we can conclude that all the calculated bubble shapes obtained by DA and FLAIR methods were good enough for a wide range of conditions.

Figure 6 shows a comparison of the bubble Reynolds numbers, Re, obtained using DA and FLAIR with the measured ones. FLAIR gave a slightly larger value of Re than DA when Reynolds number was low, which would be the result of the smaller radius of curvature at the nose of bubble when using FLAIR. However, calculated Re of DA and FLAIR agreed well with the measured Re for all the cases.

It was reported by Bugg and Naghahzadegan that FLAIR was the best volume-tracking algorithm and DA was the worst. The present result, however, confirmed that although both methods have their own advantages and disadvantages, they are applicable to the simulation of bubbles.
5 Conclusions

The applicability of the two volume tracking algorithms, the Donor-Acceptor (DA) method and the Flux Line-Segment Model for Advection and Interface Reconstruction (FLAIR) method, to single bubbles rising through stagnant liquids was examined in the present study. As a result, the following conclusions were obtained:

(1) shapes and terminal rising velocities of single bubbles predicted by DA and FLAIR agree with experimental data.

(2) although DA and FLAIR methods have their own advantages and disadvantages, they are applicable to the simulation of bubbles.

References


