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Turbulence modelling in casting processes - the challenges and an engineering approach

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Abstract

An engineering approach to modelling turbulent flows in the presence of solidification is presented based upon the widely used low-Reynolds k-ε turbulence model. Limitations in the approach are discussed as well as the challenges of incorporating the complexities of the moving solidification front into turbulence models. An industrial example related to near-net-shape continuous casting is analyzed to illustrate the usefulness of the method.

1 Introduction

In recent years, the modelling of solid/liquid transport phenomena has received considerable attention across a broad spectrum of materials processing disciplines. As for example numerous modelling efforts can be cited for unidirectional solidification systems where the influence of fluid flow on micro and macrosegregation processes has been of considerable interest [1,2]. In addition, and as another example, the sheet metal industry developed models to investigate in continuous casting processes the influence of fluid flow on important casting phenomena such as macrosegregation, cooling rates, mass transfer, and inclusion distribution, all of which play an important role in the quality of the final product [3,4]. Further effort is being directed towards the incorporation of solidification phenomena at the microscopic scale into the macroscopic equations typically used in engineering calculations [5]. In many of these applications fluid flow is primarily turbulent in nature which requires the implementation of an appropriate turbulence model in order to include the influence of turbulent mixing and associated phenomena into the solidification process.

At the present time very little experimental work on the interaction between solidification and turbulence is available, in particular quantitative turbulence data are lacking and they are extremely important for the development and validation of an engineering turbulence model. Furthermore, an extensive literature search has revealed little directed research toward the development of specific turbulence models to take into account the influence of solidification. Modifications to account for solidification effects therefore have been made to commonly used

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engineering turbulence models developed primarily for single-phase flows. These modifications are in many cases 'ad-hoc' in nature due to, as already mentioned, the lack of experimental data to provide adequate guidance [4].

Thomas and Najar [6] used the high-Reynolds number form of the k-ε turbulence model to model turbulent fluid flow and heat transfer in continuous casting. In their study a standard wall function was applied along the liquidus line at the boundaries with the surface roughness parameter in the wall function increased to simulate the increased roughness of a dendritic solidification front. This approach however does not extend the flow calculations into the mushy region and requires that the solidification front location be prespecified. Farouk *et al.* [7] modelled the twin-belt casting process using the low-Reynolds number k-ε model of Jones and Launder [8] without modification to any of the turbulence model functions or constants. In the work of Aboutalebi *et al.* [3] the low-Reynolds model version of Jones and Launder [8] was utilized to model turbulent flow in a stainless steel continuous caster, and Shyy *et al.* [4] used a similar approach with the low-Reynolds model of Launder and Sharma [9] to model continuous ingot casting. In both of the above studies the turbulent damping function f_μ is modified to include the effects of solid fraction on the turbulent length scales in the mushy zone.

The verification of these models against experimental data however is limited due to the complexity of the physical processes involved. Comparisons that have been carried out have been largely limited to the shape of solidification front profiles, as in the work of Shyy *et al.* [4] where the macroetched axial pool (solidification front) profile obtained experimentally was compared to their numerical results.

2 Turbulence/solidification modelling: Low-Reynolds number approach

The use of a modified low-Reynolds k-ε model at present, considering the constraints above-mentioned, seems to provide a reasonable compromise between an engineering approach and a complete phenomenological description of the processes. Even so, the model appears to be capable of taking into account important features along the solidification front, as for example:

i) Turbulent flow adjacent to a moving solid boundary, and within the mushy region, can be simulated with minor modifications to the standard single-phase form of the turbulent equations.
ii) The influence of solid fraction on the turbulent length scales in the mushy region can be incorporated through the model functions. This provides a means of maintaining the correct values for the turbulent diffusivities at the liquidus and solidus lines, as, the turbulent diffusivities approach zero at the solidus line and merge into the turbulent flow conditions at the liquidus line.
iii) The varying thickness of the wall region due to near-wall viscous effects and phase change can be handled since in low-Reynolds number k-ε turbulence models the computational grid can be extended into the mushy region up to the moving solid boundary.
iv) Source terms may be added to the turbulence equations to account for turbulent kinetic energy generation and dissipation resulting from vortex shedding as fluid flows through the dendrite network. A viable approach may be one similar to that developed by Tarada [10] for turbulent flows over rough surfaces.

Although the use of the low-Reynolds number k-ε models can be used with good results important limitations should be noted in what concerns the applicability of such models in the two-phase mushy region. Engineering turbulence models, including the low-Reynolds number forms, have been primarily developed on the basis of predicting single phase flow phenomena and therefore model source terms may have some limitations in the mushy region where the solid phase can interact with the mean liquid flow. This is particularly true at higher solid fraction levels where the solid velocity phase will significantly influence the liquid flow through the dendrites. Since the use of Darcy's law in the momentum equations yields mushy region velocities that are an average of the solid-liquid phase velocities, the turbulence model source terms will not be based on the state of the liquid phase only when the solid fraction levels are significant.

In the present macroscopic approach solidification processes at the microscale are not modelled although these processes may be influential in terms of turbulence. As for example, solute redistribution processes during solidification can generate solute driven buoyant flows [1] which in some processes may be turbulent. Solidification front morphology can also vary considerably between equiaxed and columnar forms, where in the case of equiaxed solidification, or by shearing away of dendrite arms in columnar solidification, solid particles are dispersed into the mean flow at low solid fraction levels which may enhance or attenuate turbulence. The influence of flow curvature due to the moving solidification front is also of importance.

Additional complexity in modelling turbulence/solidification interaction is the value of the turbulent Prandtl number, Pr_t, which can vary greatly near the wall and in flows with low-Reynolds number and low molecular Prandtl number [11]. Large changes in molecular viscosity during phase change will also influence the turbulent Prandtl number. Finally, fluctuations in the latent energy field with turbulent mixing, i.e. the $u'\Delta H'$ correlation, may also be important in the evaluation of the energy equation [12].

3 Governing equations

The primary interest of the present study is the solidification of binary alloys, and for this purpose the solution of transport equations is required in the liquid, solid/liquid and fully solid regions. In the case of alloy solidification it is computationally convenient to solve the governing equations over a fixed-grid rather than using a transformed coordinate system which is more amenable to the solidification of pure materials [13]. In the fixed grid approach the governing equations are formulated so as to apply to all three regions of the flow. Modifications to the standard single-phase equations for momentum, energy, turbulent kinetic energy and dissipation, to account for solidification, are as follows.

Continuity:

$$\frac{\partial(\rho u_j)}{\partial x_j}=0 \tag{1}$$

Momentum:

$$\frac{\partial(\rho u_j u_i)}{\partial x_j}=\frac{\partial}{\partial x_j}\left(\mu_{eff}\frac{\partial u_i}{\partial x_j}\right)-\frac{\partial P}{\partial x_j}+\frac{\mu_m}{K_o}(u_i-u_s) \tag{2}$$

Increasing resistance to fluid flow due to the dendrite network in the solid/liquid region is modelled using a porous medium approach in eqn (2), where Darcy's law is used with the permeability, $K_o=-bf_l^3/(1-f_l)^2$, related to the liquid fraction, f_l, via the Carmen-Kozeny equation for fluid flow through a packed network of solid spheres. The permeability is related to a specific material through the constant b. In the solid region all velocity gradients become zero as the solid fraction, f_s, approaches one and the permeability, K_o, zero. It should be noted that the relation $f_s+f_l=1$ applies throughout the computational domain, and that μ_{eff} is the effective viscosity equal to the sum of the molecular viscosity μ_m and the turbulent viscosity μ_t calculated from the turbulence model. The velocity of the solid phase is u_s and the density ($\rho=\rho_s=\rho_l$) is assumed the same between the phases.

Energy:

$$\frac{\partial(\rho u_j i)}{\partial x_j}=\frac{\partial}{\partial x_j}\left(\frac{k_{eff}}{c_p}\frac{\partial i}{\partial x_j}\right)-\frac{\partial(\rho u_j \Delta H)}{\partial x_j} \tag{3}$$

The governing energy equation is a conservation statement for the sensible energy, i, and the latent energy, ΔH, i.e. the total enthalpy $H=i+\Delta H$. The release of latent energy for binary substances is a function of solid fraction and solute concentration. In the modelling of industrial flows it is common, as a simplification of the analysis, to approximate the latent energy release as a linear function of temperature alone which is the approach used here. The release of latent energy is therefore governed by the following relations

$$\Delta H=\begin{cases} L, & f_s=0 \\ L(1-f_s), & 0<f_s<1 \\ 0, & f_s=1 \end{cases} \tag{4}$$

$$f_s=1-\frac{i-c_pT_s}{c_p(T_l-T_s)} \tag{5}$$

where the constants T_s and T_l are the solidus and liquidus temperatures and L the latent heat of fusion. The effective thermal conductivity, k_{eff}, in eqn 3 is equal to the sum of the molecular conductivity, k_m, and the turbulent thermal conductivity $c_p\mu_t/Pr_t$. The specific heat, c_p, is assumed the same between the phases.

Turbulent kinetic energy and dissipation:

$$\frac{\partial(\rho u_j k)}{\partial x_j}=\frac{\partial}{\partial x_j}\left(\frac{\mu_{eff}}{\sigma_k}\frac{\partial k}{\partial x_j}\right)+G_k+\rho\epsilon+\rho D \tag{6}$$

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$$\frac{\partial(\rho u_j \epsilon)}{\partial x_j} = \frac{\partial}{\partial x_j}\left(\frac{\mu_{eff}}{\sigma_\epsilon}\frac{\partial \epsilon}{\partial x_j}\right) + c_1 \frac{\epsilon}{k} G_k - c_2 f_2 \rho \frac{\epsilon^2}{k} + \rho E \tag{7}$$

The low-Reynolds number k-ε model of Launder and Sharma [9] has been found in previous studies for single-phase flows [14] to perform well in reproducing near-wall turbulent behaviour. In the present study the model of Launder and Sharma has been modified to include the influence of solid fraction on the turbulent length scales in the mushy zone. The Kolmogorov-Prandtl relation relates the effective viscosity, μ_t, to the turbulent kinetic energy and dissipation as follows:

$$\mu_t = \frac{C_\mu f_\mu l_\mu \rho k^2}{\epsilon} \tag{8}$$

where the influence of solid fraction on the turbulent length scale is through the function, l_μ, acting on the fully liquid length scale $l=c_D k^{3/2}/\varepsilon$ implicitly implied in eqn (8). The function l_μ should act to reduce the length scale to zero at $f_s=1$ and to retain the fully liquid value at $f_s=0$. This in effect controls the value of μ_t at the solidus and liquidus lines. The form of the function l_μ at present has little experimental basis, although some promising results have been obtained by Shyy *et al.* [4] for $l_\mu=(1-f_s)^{1/2}$. In the present study the form $l_\mu=(1-f_s)^2$, as proposed in [3] and tested in [15], will be used since it is expected that the solid fraction shall have a more significant effect on the turbulent length scale in the mushy region than that implied by the relation employed in [4].

The turbulence model constants for the simulations conducted were set to $C_\mu=0.09$, $C_1=1.45$, $C_2=1.9$, $\sigma_k=1.0$ and $\sigma_\varepsilon=1.30$, and the model functions f_μ and f_2 were unchanged from that in Launder and Sharma's model [9]. It is apparent from eqn (8) that the functions f_μ and l_μ act to modify the standard Kolmogorov-Prandtl relation for high Reynolds number flows (in the case of k-ε models) in the presence of near wall viscous effects and solid fraction respectively. A dimensionless near wall scaling parameter must also be determined for accurate predictions, such as R_t, R_y or y^+. Since the wall is moving choosing a turbulence model where the model functions are evaluated as a function of the dimensionless turbulent Reynolds number, $R_t=k^2/\nu\varepsilon$, is advantageous since the distance to the wall is not explicitly required nor the wall shear stress, both of which are difficult to calculate in the mushy zone where the dendrites form a highly irregular surface. In the models of Jones and Launder [8] and Launder and Sharma [9] the model functions are evaluated using R_t.

Solution of governing equations:

The foregoing equations are cast into a discrete form using the control-volume approach of Patankar [16] with the convective/diffusive link coefficients modelled using a power law formulation. The governing equations are then solved using a segregated solution procedure based on the *SIMPLEC* methodology [17].

4 Application to near-net-shape casting processes

In fig. 1 is shown a schematic of the entrance region typical of near-net-shape continuous casting devices such as the rotary-strip and twin-belt casting processes [15]. Turbulent molten metal flow enters the mould over a backward-facing step onto a highly cooled moving substrate over which the solidification front begins to develop. For the present simulation symmetry is assumed at the centreline for all variables along with one-way flow conditions at the exit. As an initial test the model equations are solved for the case of single-phase flow over a backward-facing step with the moving substrate set to a velocity of zero ($u_s=0$). The results are compared in fig. 2a with the experimental results of Vogel and Eaton [18].

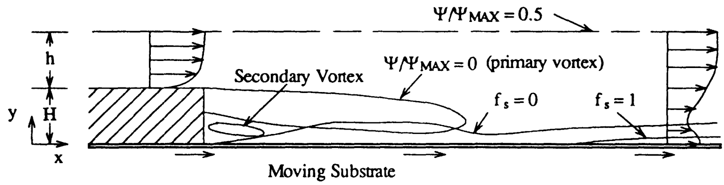

Figure 1 Schematic of entrance region for some near-net-shape casting process [15].

For the two-phase conditions, with the substrate velocity still zero, results are presented for two levels of cooling, $\alpha=3500\ W/m^2K$ and $5000\ W/m^2K$, where α is the heat transfer coefficient. In figures 2b and 2c comparison is made between the two levels of cooling at locations $x/H=5$ and 15. Reattachment occurs prior to $x/H=5$ for both levels of cooling since the recirculation region is reduced when solidification at the wall is present. From the turbulent viscosity profiles in figures 2b and 2c the turbulent viscosity values can be seen to be reduced to zero at increasing levels of solid fraction. The turbulent kinetic energy and dissipation rates are also reduced to zero as the solid fraction increases so that the necessary boundary conditions are maintained along the solidification front (i.e. $k=0$ and $\varepsilon=0$). By changing the exponent in the function l_μ the rate at which μ_t is decreased to zero can be controlled.

For the case when the substrate is moving along with higher inflow turbulence levels fig. 3 is provided. The moving substrate induces a large secondary recirculation region as is clearly visible in fig. 3a. In the recirculation region cooler temperatures prevail leading to higher solid fraction levels, as seen in fig. 3b, which drop off and then increase following reattachment. The influence of the solid fraction field on the turbulent viscosity field is visible along the substrate in fig. 3c. The influence of turbulent mixing is limited to solid fraction levels up to approximately $f_s=0.3$.

5 Conclusion

An engineering approach for modelling turbulent flows in the presence of solidification has been presented with examples from near-net-shape continuous casting. In this approach the influence of the two-phase mushy region is easily implemented, and the presence of a moving solid boundary accounted for, within the single-phase form of the k-ε model equations and has been shown to give physically realistic results. A full assessment of the approach must wait until quantitative experimental data are available.

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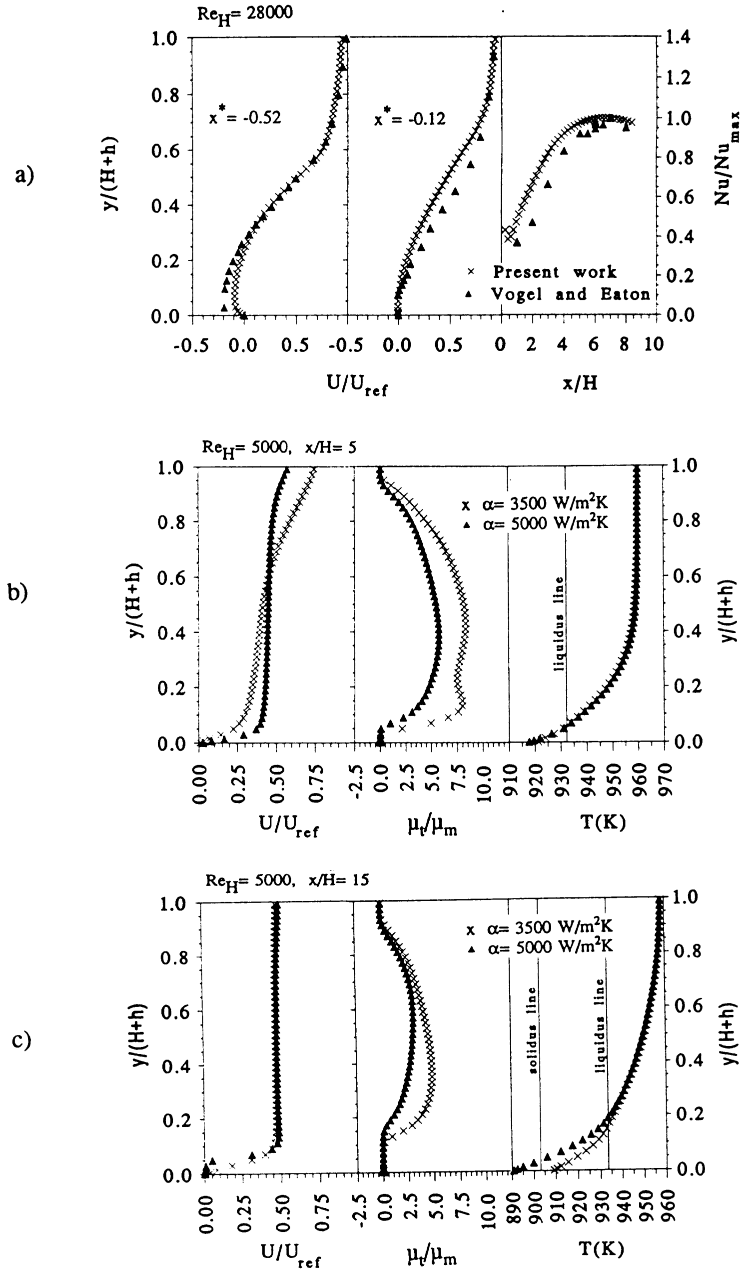

Figure 2 Backward facing step flow; a) single-phase flow, b) two-phase flow at $x/H=5$, c) two-phase flow at $x/H=15$. ($x^*=(x-x_r)/x_r$ where x_r is the streamwise reattachment length)

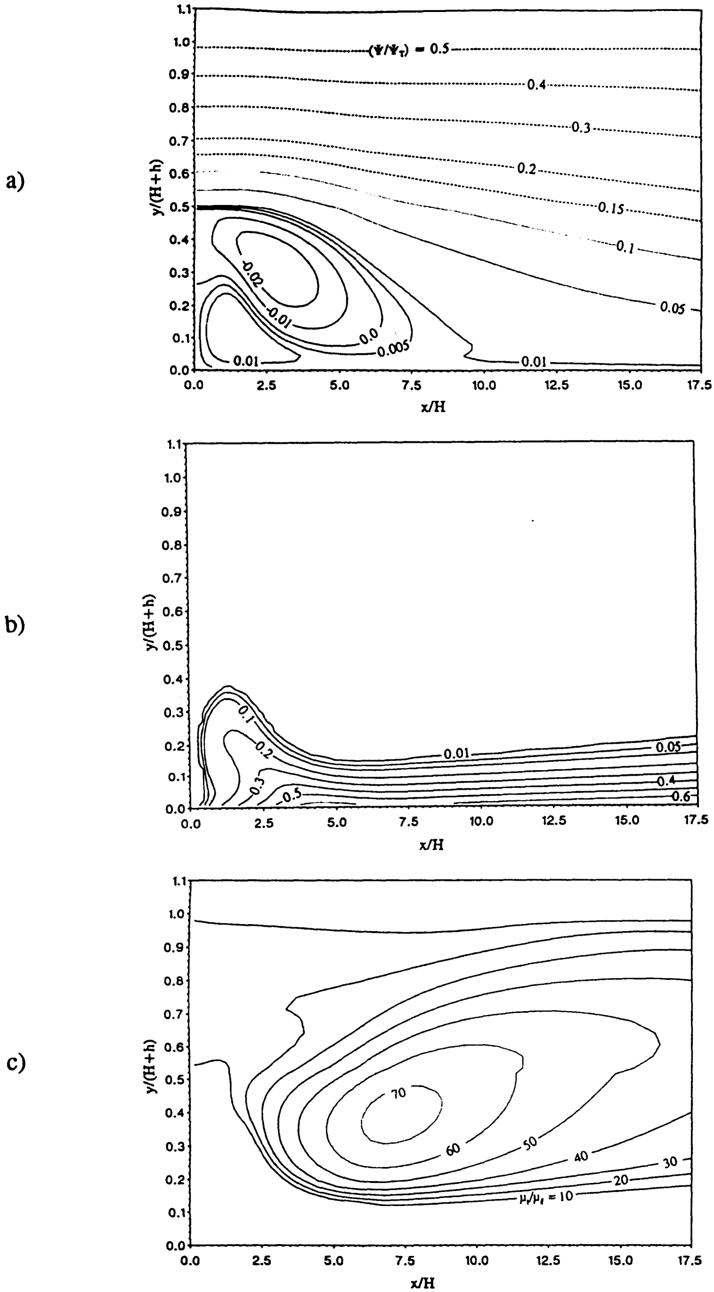

Figure 3 Flow over a backward facing step with a moving substrate at $Re_{2h}= 9{,}000$ and $\alpha=3000\ W/m^2K$; a) streamlines, b) solid fraction contours, c) turbulent viscosity field.

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