Ablation problems using a finite control volume technique

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ABSTRACT

An element based finite control volume procedure is applied to the solution of ablation problems for 2-D axisymmetric geometries. A mesh consisting of four node quadrilateral elements was used. The nodes are allowed to move in response to the surface recession rate. The computational domain is divided into a region with a structured mesh with moving nodes and a region with an unstructured mesh with stationary nodes. The mesh is constrained to move along spines associated with the original mesh. Example problems are presented for the ablation of a realistic nose tip geometry exposed to aerodynamic heating from a uniform free stream environment.

INTRODUCTION

Early attempts at solving ablation problems utilized 1-D finite difference techniques on a grid that allowed the surface node to move while keeping the interior nodes fixed in space. When the surface node became sufficiently close to its nearest neighbor node, it was removed from the computational domain. The work of Brogan\(^1\) was typical of this approach. When nodes were removed from the computational domain, perturbations in recession rate (and possibly surface temperature) commonly occurred. Moyer and Rindal\(^2\) circumvented this problem by attaching a coordinate system to the receding surface; all nodes moved with the velocity of the ablating surface, with the exception of the last ablating node which was stationary. The Charring Materials Ablation (CMA) code that
resulted from this work is still used extensively throughout the aerospace industry. Enhancements to this translating grid technique were reported by Blackwell\textsuperscript{3}. While the translating grid technique works quite well for 1-D geometries, it is not readily extendable to multi-dimensions.

Landau\textsuperscript{4} introduced the concept of a 1-D contracting finite difference grid in which the computational domain shrinks while keeping the number of nodes in the ablator fixed. This was accomplished by introducing a spatial coordinate transformation that mapped the remaining ablator thickness into the domain $0 \leq \eta \leq 1$ where $\eta = 1$ is always the ablating surface and $\eta = 0$ is the fixed surface. The resulting mesh moves with a local velocity of $\eta \dot{s}$; the ablating surface velocity is $s$ while the last ablator nodal velocity is zero. Additional refinements of this one-dimensional method have been presented in Blackwell and Hogan\textsuperscript{5}.

Several authors have utilized Landau-like transformations to solve two- and three-dimensional ablation problems; see References 6-11 for a representative sampling. The resulting transformed energy equation is more complicated than the original partial differential equation because of the addition of convection like terms due to the moving grid and other geometry change related terms. Finite difference techniques applied to a regular mesh were used to solve the resulting energy equation. Although the above works can handle reasonably complex exterior geometries, they all suffer from the inability to simultaneously model the complicated interior geometries of realistic nose tips.

Element-based methods have been more successful in modeling conduction phenomena in geometrically complex objects than have finite difference, structured grid based methods. Hogge and Gerrekens\textsuperscript{12} applied the finite element method to 1-D ablation problems with pyrolysis; a deforming mesh based on penetration depth concepts was utilized. Chin\textsuperscript{13} applied the finite element method to 1- and 2-D ablation problems with pyrolysis using a fixed mesh. In comparing his results with the CMA\textsuperscript{2} code, oscillations occurred in the surface energy balance terms. It is likely that these oscillations were due to surface elements being removed from the mesh just as were those oscillations reported in Brogran\textsuperscript{1}. Hogge and Gerrekens\textsuperscript{14} applied the finite element method to 2-D nose tip geometries. A regular mesh was applied to the material region experiencing ablation with one set of mesh lines approximately parallel to the ablating surface. The other mesh lines were formed by straight line rays or spines. For a given ray, the surface node moves a distance $s \Delta t$ in the direction of the local surface (inward) normal. All other nodes along this ray are moved proportionally with the last ablating node (for this ray) being fixed in space. This causes the
rays to rotate with time and contract their length; however, the rays still remain straight lines. In the nose tip examples they presented, the initial mesh of the tip region was constructed based on a cylindrical coordinate system. This precludes ablation depths greater than one nose radius, which is unrealistic for small nose radius ballistic reentry vehicles.

In the work presented, an element-based numerical method was adopted because of the desire to solve problems with complicated internal geometries. However, it will not be the method of weighted residuals approach used in the traditional finite element method. Instead, energy will be conserved on control volumes of finite size. Additional details on this Finite Control Volume (FCV) method are given in Blackwell and Hogan.

**DEVELOPMENT OF BASIC EQUATIONS**

The starting point for our development is the integral form of conservation of energy on a moving and deforming control volume

\[
\frac{d}{dt} \iiint_{\Omega} \rho e \, d\Omega + \iint_{A} dA \cdot \dot{q} - \iint_{A} \rho \dot{V}_b \cdot dA = 0. \tag{1}
\]

where \( V_b \) is the local element velocity. The three terms above represent energy storage, heat conduction, and “apparent” convection of energy due to the mesh motion, respectively. For a solid, the internal energy \( e \) is approximately equal the enthalpy \( i \). In order to move the time derivative inside the integral sign in Equation (1), Leibnitz rule will be applied

\[
\frac{d}{dt} \iiint_{\Omega} \rho e \, d\Omega = \iiint_{\Omega} \frac{\partial}{\partial t} (\rho e) \, d\Omega + \iint_{A} \rho \dot{V}_b \cdot dA \tag{2}
\]

Applying Equation (2), the energy equation becomes

\[
\iiint_{\Omega} \frac{\partial}{\partial t} (\rho e) \, d\Omega + \iint_{A} dA \cdot \dot{q} = 0 \tag{3}
\]

Blackwell and Hogan solved Equation (1) using the FCV procedure on a four node quadrilateral mesh for non-ablating problems (stationary mesh) with a bilinear temperature profile; this technique was extended to moving mesh problems. Since the discretization of the conduction term for this work is identical for that of Reference 15, the details will not be repeated here.

In the discretization of the energy storage term in Equation (3), we assumed the density was constant over each element and that \( e = C_v T \). Expressing the elemental temperature distribution using the element shape function and nodal temperatures, the storage term becomes
For stationary mesh problems, the shape functions are independent of time; however, they are time dependent for a moving mesh. Following Lynch and Gray\(^1\) and Lynch and O’Neill\(^2\), the shape function derivatives can be computed from

$$\frac{dN_i}{dt} = \frac{\partial N_i}{\partial t} + \mathbf{V}_b \cdot \nabla N_i = 0$$  \hspace{1cm} (5)

Within a moving element, the shape function total derivative is zero while the partial derivative with respect to time is non-zero. Expanding Equation (4), the energy storage term becomes

$$\iint \int \frac{\partial T}{\partial t} dV = \iint \int \left[ \frac{\partial N_1}{\partial t}, \frac{\partial N_2}{\partial t}, \frac{\partial N_3}{\partial t}, \frac{\partial N_4}{\partial t} \right] (e) \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix} dV$$

$$+ \iint \int [N_1 N_2 N_3 N_4] (e) \frac{dT}{dt} dV$$ \hspace{1cm} (6)

The first term is the contribution due to the moving mesh while the second term is the stationary mesh energy storage term. The details of the numerical integration of the second term are given in Blackwell and Hogan\(^3\); the first term was evaluated using a similar technique.

**MESH MOTION ALGORITHM**

The computational domain for a nose tip will be divided into two regions. The first region (ablating) contains moving elements and must be slightly larger than the maximum anticipated ablation; within this region, a structured mesh was utilized. The second region will be the remainder of the nose tip; its mesh is
stationary but may be unstructured. Figure 1 shows a typical nose tip mesh with the structured grid for the moving region and the unstructured grid for the stationary region. The location of the interface between the ablating and non-ablat-

Figure 1. Schematic of initial mesh showing moving and stationary regions.

ing regions is somewhat arbitrary provided the thickness of the ablating region does not shrink to zero. The mesh lines that intersect the ablating surface will be called rays or spines and the mesh will be constrained to move along them. Figure 2 is a schematic of the mesh motion algorithm that will be utilized. The unit vector $\hat{a}$ lies along the spines while the unit vector $\hat{n}_j$ is the surface normal for

Figure 2. Schematic of mesh motion algorithm.
the j'th spine. The variable $\eta$ is a Landau type variable that gives the distance from the non-ablating boundary as a fraction of the total spine length and is computed from

$$\eta_{i,j} = 1 - \frac{1}{L_j} \sum_{k=1}^{i-1} \Delta L_{k,j}; \quad L_j = \sum_{k=1}^{I} \Delta L_{k,j}$$

(7)

The $\eta$ values are calculated for the initial mesh and do not change with subsequent mesh motion. For a rectangular mesh with recession parallel to one of the coordinate directions or 1-D geometries, $\eta$ is identical to the spatial coordinate transformation introduced by Landau. The motion in the direction of the spine will always be greater than or equal to the motion in the normal direction; see the inset in Figure 2.

COMPUTATIONAL RESULTS

To demonstrate the validity of this mesh motion algorithm, the nose tip given in Figure 1 was exposed to a uniform free stream environment. The Reynolds number was sufficiently low that the boundary layer remained laminar. The nose tip material was graphite and temperature dependent thermal properties were used. The boundary conditions used were a specified temperature and recession rate as a function of time. Figure 3 presents the nose tip shape, deformed mesh, and isotherms for four different times. The isotherm interval is 500 R with a maximum temperature of 7500 R. The mesh motion algorithm performed quite well. In practice, the aerodynamic heating distribution varies as the nose tip changes shape; this effect was ignored in this calculation. Future work will include simulations involving turbulent heat transfer on the spherical portion of the nose tip and inclusion of more realistic surface energy balance relationships.

REFERENCES

Figure 3. Computational results for 2.5, 5.0, 7.5, and 10.0 s.