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Calculation of the plane and axisymmetric Riabouchinsky cavitational flows by the direct boundary element method

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ABSTRACT

A numerical algorithm is proposed for the solution of the steady cavitating flows around the arbitrary plane and axisymmetrical blunt-ended bodies with the fixed separation point. The fluid is assumed as incompressible and weightless, and the flow is assumed as potential. For a description of the cavitational flow the Riabouchinsky cavitating scheme ("with the mirror") was used and the length of the free streamline was chosen as a parameter. Numerical results for the drag coefficients, the shape of cavities and cavitation numbers are presented for cavitating flows behind the cones and wedges with half-angle in range $[10^o, 120^o]$. These results are compared with the data of other authors for the disk (90^o) and the cones.

INTRODUCTION

The tasks of designing the high speed submarine apparatus and predicting their performance have preoccupied engineers for many years. Although there are only a small number of numerical axisymmetric fully cavitating flows calculation methods. The numerical calculation of axisymmetrical cavitating flows is based on two techniques:
- finite difference method proposed by Brennen [1]. The solutions are obtained for cavities behind a disk and a sphere in different size of solid wall tunnel. The same problem was also treated by Garabedian [2] who approached the axisymmetric case by successive corrections to the corresponding planar flow, each correction involving the solution of a linear mixed boundary-value problem.
- boundary integral equation methods in works of Amromin and Ivanov [3], Gyzevsky [4] and Kojouro [5]. In these works the vortex surface distribution was used and resulting system of nonlinear algebraic equations solved by the different numerical methods. In present work the new numerical iterative algorithm based on direct boundary element method is proposed

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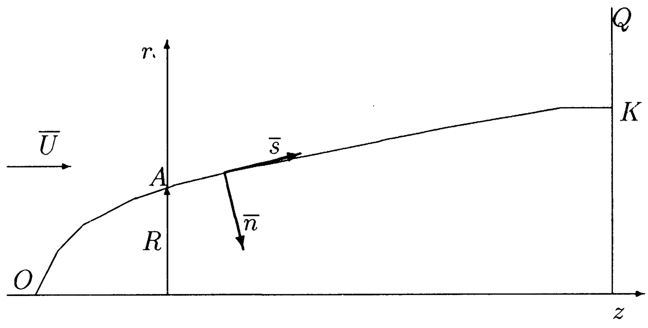

Figure 1. The sketch of the cavitational flow.

for the calculation of the cavitational flows.

PROBLEM DEFINITION

Cavitating flow around an axisymmetric body is generally a three-dimensional problem. If the case is limited to zero angle of attack, this problem may be described in the same manner as a two-dimensional problem.

The uniform stream with the velocity U_o flows round the blunt-ended body shown in figure 1. Axis Oz is a line (plane) of a symmetry. The line OA is a wetted surface of a body and AK is a free streamline. The line KQ is a plane of symmetry of Riabouchinsky flow. The positions of the boundaries AK and KQ as well as the shape of the latter are initially unknown.

The governing equation for the plane and axisymmetric potential flows in dimensionless variables is

$$\nabla^2\phi = 0, \tag{1}$$

where ϕ is the disturbed velocity potential; the velocity in coordinate system (r, z) is $V_z = 1 + \phi_{,z}, V_r = \phi_{,r}$. The boundary conditions can be described in following way. The kinematic condition on wetted and free surfaces is

$$\phi_{,n} = -r'(s), \tag{2}$$

where $\overline{n}$ is the unit outward normal vector. Equation (2) indicates that the flow rate through the boundary is zero. Assuming uniform cavity pressure, on free streamline is valid the dynamic condition

$$\phi_{,s} = U_c - z'(s), \tag{3}$$

where U_c is a constant velocity on the free surface; s represents the curvilinear abscissae of the point on free streamline. On the line of symmetry KQ

$$\phi = 0. \tag{4}$$

The equation (3) can be integrated over the free streamline with the condition (4) and it becomes

$$\phi(s) = z_K - z(s) + U_c(s - s_*), \tag{5}$$

where s_* is length of the free streamline.

For Riabouchinsky model, one parameter defines a unique solution: cavitation number, $\sigma = \frac{p_o - p_c}{\frac{1}{2}U_o^2} = \frac{U_c^2}{U_o^2} - 1$, where p_o, p_c are the remote upstream and cavity pressures and ρ is the density of the fluid; or half-length of the cavity $z_L = z_K - z_A$; or length of the free streamline s_*.The lengths of this curve are chosen arbitrary as input data.

NUMERICAL METHOD

Let the initial location of the free surface be known. The boundary integral representation of solution of the equation (1) is [8]

$$\frac{1}{2}\phi_M = \int_S [\phi_{P,n}(F_{MP} - F_{LP}) - \phi_P(F_{MP,n} - F_{LP,n})]\rho^\epsilon dS_P \tag{6}$$

where $\epsilon = 0$ for plane case and $\epsilon = 1$ for axisymmetric case; $M(r,z)$ and $P(\rho,\zeta)$ are points on the boundary, $L(r, 2z_K - z)$ is a point symmetric relatively plane KQ; $F_{MP} = -\frac{1}{\pi}\ln r_{MP}$ is a fundamental solution for isotropic $2D$ medium; $F_{MP} = \frac{1}{\pi}\frac{1}{\sqrt{(\rho+r)^2+(\zeta-z)^2}}K(\gamma)$ is fundamental solution for axisymmetric case; $\gamma^2 = \frac{4\rho r}{(\rho+r)^2+(\zeta-z)^2}$; r_{MP} is the distance between points $M(r,z)$ and $P(\rho,\zeta)$. In axisymmetric case, the complete elliptic integrals of the first $K(\gamma)$ and second kinds may be approximated by polynomial approximations [7]. In this equation, the integrals are considered singularity when P tends towards M.

The discretization of (1), which leads to the classic 'boundary element method' technique (see, Brebbia and others [8]) described below. In the boundary element method, the above integral equation is solved numerically by dividing the boundary S into $N + L$ elements (N intervals on the free boundary and L intervals on the wetted surface of the body in this case), in each of which ϕ and $\phi_{,n}$ are approximated by constants. We denote these values by ϕ_i and $\phi_{i,n}$, $i = 1, \ldots N + L$; and apply equation (6) at one nodal point M_i in each boundary element to obtain

$$\frac{1}{2}\phi_i = \sum_{j=1}^{N+L}(\phi_{j,n}\int_{S_j} G_{iP} dS_P - \phi_j \int_{S_j} G_{iP,n} dS_P), \tag{7}$$

where S_j denotes integration over the jth boundary element.In plane case the coefficients of the linear systemof equations (7) are integrated analytically over intervals. In an axisymmetric case numerical integration is used over the boundary element, parameterising this interval in an appropriate manner and taking into account the singularity in the integrals when $i = j$.

Eliminating the ϕ_i from each element on the free surface and $\phi_{i,n}$ from each element on the wetted boundary by applying the corresponding boundary condition in each nodal point, we thus obtain a system of $N + L$ simultaneous linear algebraic equations with $N + L + 1$ unknowns (N unknowns $\phi_{i,n}$ on free surface, L unknowns ϕ_i on body surface and value of the velocity on free boundary U_c). Additional equation for U_c we obtain by equating value of the potential in the separation point, calculated in (5) and value obtained by linear extrapolation by two points on the wetted surface

$$z_K - z_A - U_c s_* = \phi_{N+1} + (\phi_{N+1} - \phi_{N+2}) \frac{\Delta_{N+1}}{\Delta_{N+1} + \Delta_{N+2}}, \tag{8}$$

where Δ_j is the length of the jth boundary element. The system of $N+L+1$ linear algebraic equations (7,8) was solved by the direct Gaussian elimination method.

In order to solve the problem of cavitating flow, the shape of the cavity must be calculated by successive iterations. The new location of the free surface has been calculated by integration

$$r'_i = -\frac{\phi_{i,n}}{\sqrt{\phi_{i,n}^2 + z_i'^2}} \tag{9}$$

$$z'_i = sign(r'_{i+1} - r'_i)\sqrt{1 - r_i'^2} \tag{10}$$

with initial condition $r(0) = R$, $z(0) = z_A$. If the iteration process is coincided, then formula (9) translates in kinematic condition (2). The iterative procedure is continued until a converge criterion is satisfied. Usually, $6 \div 12$ iterations demanded for the coincidence. On the cavity, the velocity component normal to the cavity surface will be zero only at the convergence of the iterative process. After finishing the iterative process, the cavitation number σ must be retained as a solution parameter.

NUMERICAL RESULTS

To demonstrate the numerical scheme developed above, we consider a plane cavitational flow behind the wedges and plate. This problem has analytical solution in complex variables (see, Riabouchinsky [7]). Comparison of numerical results for plane flow (65 boundary elements) and analytical data

Table 1. Comparison of numerical and analytical results for wedges

α^o	σ	R_K/R calc.	R_K/R anal.	c_d calc.	c_d anal.	z_L/R calc.	z_L/R anal.
90	0.45	3.491	3.4847	1.2808	1.2788	15.58	15.568
90	0.30	4.727	4.7305	1.1470	1.1451	31.74	31.798
90	0.10	12.07	12.201	0.9644	0.9680	243.3	245.6
45	0.30	3.441	3.454	0.8360	0.8361	22.88	23.94

Table 2. Comparison of numerical C_d^* with data of Gyzevsky [4] ($\sigma = 0.25$)

α^o	90	60	30
Gyzevsky [4]	0.223	0.223	0.224
present work	0.2186	0.2198	0.2207

made in table 1. Here α is the half-angle of the wedge ($\alpha = 90^o$ corresponds by the plate).The differences between calculated and analytical values are very small. For anaxisymmetric case numerical values $c_d^* = F_d/\pi R_K^2$ for cones compare with the numerical data of Gyzevsky [4] in table 2. Coincidence between numerical data of Gyzevsky [4] and present work is close also.

The computed and experimentally observed (Brennen [1]) position free streamline behind the disk are presented in figure 2 for cavity number $\sigma = 0.2$. The agreement of these data is very close. The pressure distribution on the wetted surface of the disk for $\sigma = 0.24$ has been presented on the scale of the figure 3. There is close agreement with the experimental data of Rouse and McNown from [1]. The dependencies of the drag coefficient $c_d$and cavity radius R_K of the half-angle of the cone are presented in figure 4 for cavity number $\sigma = 0.1$. Here the value $\alpha = 90^o$ corresponds by the disk.

CONCLUSIONS

The boundary element method has been presented in this paper for the simulation of plane and axisymmetric cavitational flows in nonlinear formulations. Comparisons with analytical, numerical and experimental data suggests that the solutions obtained by the present numerical method are quite accurate. The flow is partially well modelled in the case of the blunt ended bodies with a fixed separation point, namely the disk.

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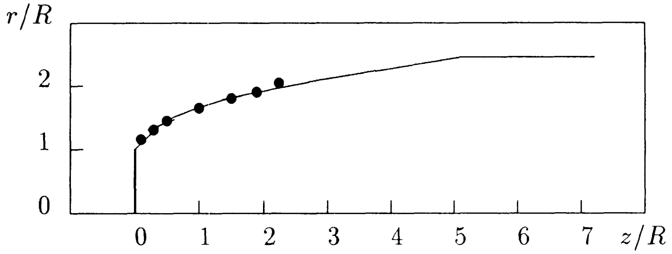

Figure 2. Comparison of theoretical and experimental cavity profiles for disk ($\sigma = 0.2$), • Brennen [1]

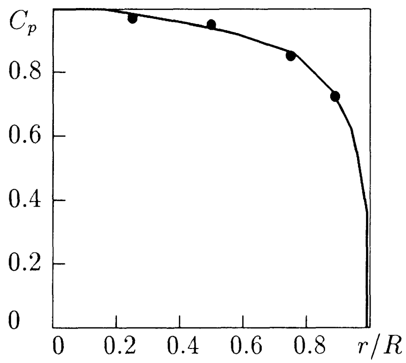

Figure 3. Pressure distribution on the surface of the disk ($\sigma = 0.24$), experimental data • for $\sigma = 0.24$ from [1].

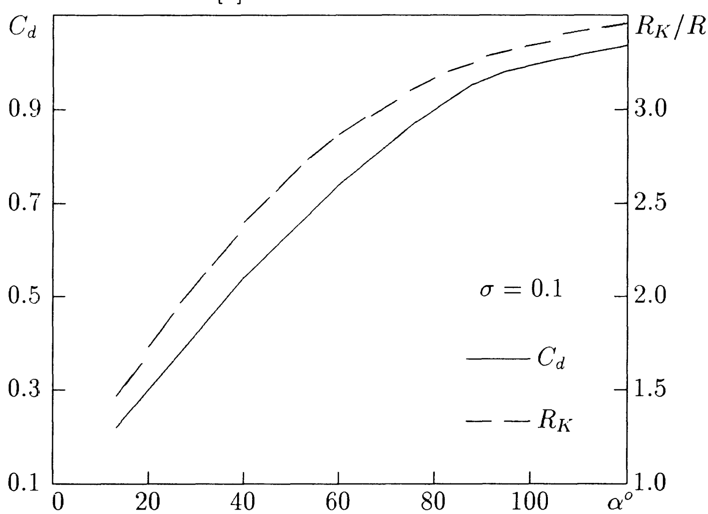

Figure 4. Variation of the drag coefficient and cavity radius with half-angle of the cone for $\sigma = 0.1$.

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