Numerical simulation of induced currents on a slowly moving PEC plane under the illumination of EM pulses

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Abstract

In this paper, numerical simulation of induced currents on a PEC plane in slow motion under the illumination of EM pulses is reported. In the study numerical models are simplified as one-dimensional where PEC planes are assumed to be infinite in size and moving with a constant velocity no faster than 10% of the speed of light. The present work employs a characteristic-based algorithm to numerically predict the effects of moving PEC planes on the interaction between the incident and the PEC surface. The predicted induced currents are recorded as functions of time, whose corresponding frequency responses are also reported.

1 Introduction

In many cases, electromagnetic (EM) scattering problems involving objects in motion have been driven great deal of attentions of physicists and engineers. They include the scattering fields as well as the reflection of EM waves from moving bodies. The well-known Doppler shift is the consequence of a relative motion between the reflecting body and the direction and speed of the observed waves, which causes the reflected waves a shift in frequency. When targets experiences vibrating or rotating, there are many other frequencies contained in the reflected wave. The latter is not within the scope of this effort. Since Yee successfully applied finite-difference time-domain (FDTD) technique for solutions of the time-dependent Maxwell's equations in Cartesian coordinates in the mid-60's [1], FDTD method has been received tremendous attentions and applications to analyze electromagnetic problems in various areas of research. One is for analyzing electromagnetic wave scattering from moving surface.
Special arrangements are required to accomplish this attempt, the electric field at the surface is stored at electric field grid point closest to the surface depending on the position of the moving surface, the H-field adjacent to the surface is computed through the use of Faraday’s law and the E-field adjacent to the surface is obtained via Ampere’s law [2].

An alternative approach proposed in the present study is a characteristic-based algorithm providing numerical approximations to the time-dependent Maxwell curl equations. Thanks to its attribute of placement of the electric and magnetic field components in the model, it might turn out to be a better candidate for simulation problems involving time-dependent cell volume. For the present algorithm there is no need to modify boundary conditions as surface moves along neither to introduce extra storage for the field components for the varying cell volume that is updated every numerical time step.

In this paper, the basis of the characteristic-based algorithm is briefly summarized in second section. Description of characteristic variable boundary conditions and the definition of the problem are given in the following section. Also presented is how the change of grid cell volume associated with the treatment of field components as PEC surface advances. Results of simulation are illustrated and followed by conclusion.

2 Governing equations and boundary conditions

For two-dimensional formulation, the governing equations in conservation-law form and in curvilinear coordinate system are given

\[
\frac{\partial Q}{\partial t} + \frac{\partial F}{\partial \xi} + \frac{\partial G}{\partial \eta} = 0
\]

where \( Q = Jq \), \( F = \frac{\partial (\xi f + \eta g)}{\partial \xi} \), \( G = \frac{\partial (\xi f + \eta g)}{\partial \eta} \), \( J \) is the Jacobian of the reverse transformation, and the three variable vectors are \( q = [B_x, B_y, D_z]^T \), \( f = [0, -E_z, -H_z]^T \), and \( g = [E_z, 0, H_z]^T \), respectively.

Considering the case where the cell volume may be time-dependent, by the definition of variables in curvilinear coordinate system and that in Cartesian system, we rewrite the temporal increment as \( Q^{n+1} - Q^n = V^{n+1} (q^{n+1} - q^n) + (V^{n+1} - V^n) q^n \). In above expression, super-scripts \( n \) and \( n+1 \) stand for the present and previous numerical time step, and letter \( V \) is the cell volume. It’s quite obvious that by incorporating it into the formulation the change of cell volume is taken into consideration.

With the concern of geometric conservation law included in the numerical formulation, referring to Figure (1a) and focusing our attention on cell \( N \), we can simply apply the present method through out the numerical procedure without any additional modification. As the PEC surface moving along the x-direction and since in characteristic-based method field components are “placed” in the center of remaining cell, it is therefore straightforward to apply the present scheme to such cell as shown in Figure (1b).
There are two types of boundary condition used in the numerical formulation: one is applied to the object boundaries; the other is to the computational domain boundaries. On the surface of target, the boundary conditions are obtained through the concept of characteristic variables and known as the characteristic variable (CV) boundary conditions and defined as the product of the row eigenvector matrix and the instantaneous variable vector. From the definition, every CV designates the direction and speed of the information propagation. For the CV associated with the zero eigenvalue can be interpreted as the normal component of the total flux densities that must continue across the interface or vanish on a PEC surface. Also at the interface of media, the boundary conditions must be that the tangential components of both electric and magnetic field intensities are continuous and that the normal components of both electric and magnetic flux densities are continuous as well. Therefore, for the most outer cell surface, the boundary conditions should ensure that they are transparent to the out-going scattering fields.

Figure 1: (a) One-dimensional computational cell indexing; (b) Variable Q is averaged over the remaining cell and positioned at the center of the cell.

3 Cases with uniformly moving plane

The incident fields are assumed to be Gaussian electromagnetic pulse traveling initially in the positive x-axis. Pulses having unit peak value are truncated by a Gaussian window with a cut-off level of 100 dB below unity. Provided that the
incident Gaussian pulse has width of 5 ps, the highest frequency content and the corresponding wavelength are approximately 150 GHz and 2 nm, respectively.

The Doppler effect describes the frequency shift due to a relative motion between the source and the detector. If there exists a relative motion of uniform speed $v$, the frequency ($f_d$) detected by the detector can be evaluated by carrying out the following:

$$f_d = f_s \sqrt{\frac{1 - \frac{v}{c}}{1 + \frac{v}{c}}},$$

supposed that $c$ is the speed of light and $f_s$ is the frequency of electromagnetic signals emitted from the source. Furthermore, the relative speed $v$ is taken to be a negative value if they are approaching to each other. In this simulation, value of $v$ is taken in the range of $-0.1c$ and $+0.1c$.

(a) PEC approaching the incident.  
(b) PEC moving away the incident.

Figure 2: Evolution of electric field in time.

4 Results

To demonstrate how moving PEC surface interact with EM pulse, two sequences of electric field as time evolves are displayed in Figures 2(a) and 2(b) where boundaries on the far right travel in two different directions, respectively. A group of induced currents corresponding to various speeds is depicted in Figure 3. The induced currents are expected to be dependent upon the speed of PEC surface and the resultant pulse widths are summarized in Table 1. Due to the relative motion, the width becomes wider if PEC plate moves in the positive $x$-direction and slimmer when it moves in opposite way. The induced current spectra are given in Figure 4 along with the calculated values of the corresponding highest frequency. From the plot and Table 1, the predicted highest frequencies are in fairly good agreement with the expected values.
Figure 3: Induced currents as function of the speed of PEC plate.

Figure 4: Induced currents spectra as function of the speed of PEC plate.

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Table 1: Predicted and calculated (*) results.

<table>
<thead>
<tr>
<th>Speed of PEC Plane (c)</th>
<th>+ 0.10</th>
<th>+ 0.05</th>
<th>0</th>
<th>- 0.05</th>
<th>- 0.10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pulse width (ps)</td>
<td>5.568</td>
<td>5.263</td>
<td>5.000</td>
<td>4.763</td>
<td>4.492</td>
</tr>
<tr>
<td>Highest frequency (GHz)</td>
<td>137.3</td>
<td>145.0</td>
<td>152.7</td>
<td>160.4</td>
<td>168.1</td>
</tr>
<tr>
<td></td>
<td>138.2*</td>
<td>145.3*</td>
<td>152.7*</td>
<td>160.6*</td>
<td>168.9*</td>
</tr>
</tbody>
</table>

5 Conclusion

This paper has demonstrated that characteristic-based method can handle problems involving time-dependent cell volume. The predictive results are quite encouraging. It is our goal to extend the existing code to solve problems with vibrating boundaries either uniformly or sinusoidally and to introduce the second even the third dimension into the formulation for finite size of moving or vibrating boundaries.

References
