A contribution to the problem of the suspension dewatering

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Abstract

During the sedimentation process of a suspension, a sharp interface develops separating the upper zone of free sedimentation from the lower zone of compression. The position of this interface results from the balance of mass exchange between both the zones. The weight of the solid phase in the compression zone increases the liquid phase pressure. Consequently, the liquid phase is pushed out which causes the required dewatering of the suspension.

The paper presents a numerical solution of the process. Based on the equations of Darcian mechanics of two-phase systems and the constitution relations, the governing equations of the problem are formulated.

Making use of the mass-balance equation as an additional boundary condition, the problem becomes a moving boundary problem for the compression zone. The form of the governing equations and the imposed boundary conditions make it possible to use the method of characteristics when solving the problem numerically. The achieved results are presented in the form of height of the compression zone and the development of the solid-phase concentration within the compression zone.

1 Introduction

Various technologies require to study the process of suspension sedimentation and dewatering. The reduction of the suspension volume minimizes, for example, the space of digestion tanks and improves the efficiency of filtration or centrifugation. The gravity thickening is, because of its low energetic demands, one of the most convenient processes of lowering volume of suspensions and significantly increasing the concentration of their solid phase. The investigated process can be divided into two parts: simple sedimentation of the solid-phase particles and subsequent
compression due to the height of the solid phase. Mls et al. [1] suggested to solve the problem by means the theory of water seepage through porous media, particularly applying the Darcy law. Published results of experimental research (e.g. Handová and Sladká [2]) made it possible to determine the required hydromechanical characteristics and to solve the process numerically. A numerical solution to the suspension sedimentation problem was presented by Mls [3].

2 Equations of Darcian mechanics

Under certain assumptions on the flow velocities and the hydraulic conductivity, Mls [4] formulated the following system of partial differential equations governing the Darcian mechanics of two-phase media.

\[
\frac{\partial}{\partial t} (\rho_w(x,t)n(x,t)) + \frac{\partial}{\partial x_j} (\rho_w(x,t)v_j(x,t)) = 0,
\]

\[
\frac{\partial}{\partial t} (\rho_s(x,t)(1-n(x,t))) + \frac{\partial}{\partial x_j} (\rho_s(x,t)v_j(x,t)) = 0,
\]

\[
\frac{\partial}{\partial t} (\rho_w(x,t)w_i(x,t)) + g \frac{\partial x_3}{\partial x_i} \rho_w(x,t)n(x,t)
\]

\[
+ n(x,t) \frac{\partial p}{\partial x_i}(x,t) + g\rho_w(x,t)n(x,t)k^{-1}_{ij}(x,t)v_j(x,t) = 0,
\]

\[
\frac{\partial}{\partial t} (\rho_s(x,t)v_i(x,t)) + g\rho_s(x,t)(1-n(x,t)) \frac{\partial x_3}{\partial x_i} - \frac{\partial \tau_{ij}}{\partial x_i}(x,t)
\]

\[
+ (1-n(x,t)) \frac{\partial p}{\partial x_i}(x,t) - g\rho_w(x,t)n(x,t)k^{-1}_{ij}(x,t)v_j(x,t) = 0,
\]

\[i = 1, 2, 3, \text{ where } t \text{ is time, } x = (x_1, x_2, x_3) \text{ are space coordinates with } x_3 \text{ axis oriented vertically upwards, } \rho_w \text{ and } \rho_s \text{ are densities of the liquid phase and the solid phase respectively, } n \text{ is porosity of the medium, } w \text{ and } v \text{ are the volumetric flux-density vectors of the liquid phase and the solid phase respectively, } g \text{ is the gravitational acceleration, } p \text{ is the liquid-phase pressure, } k \text{ is the hydraulic conductivity, } u \text{ is the relative flux-density vector of the liquid phase satisfying}
\]

\[u = w - \frac{n}{1-n}v,
\]

and \(\tau\) is the effective solid-phase stress.

3 The investigated process

Consider following process. An initially homogeneous suspension column of kaolin and water is placed into a vertical cylinder. Denote \(c\) the concentration of the suspension and \(L\) the initial height of the column, where

\[c = (1-n)\rho_s.
\]
Let us investigate the one-dimensional sedimentation process starting from these initial conditions. Under the assumption of impervious bottom of the cylinder, following Equation can be derived from the one-dimensional form of Equations (1) and (3)

\[ w(x, t) = -v(x, t) \]

for \( \forall x \in (0, L) \) and \( t > 0 \). Making further use of Equations (3) and (5), functions \( v, p \) and \( u \) can be excluded leaving Equations (1) and (4) as governing equations of the process with three unknown functions, \( w, s \) and \( \tau \), where

\[ s(x, t) = 1 - u(x, t). \]

During the sedimentation process, the condition

\[ \frac{\partial s}{\partial t}(m_1, t_1) \frac{\partial s}{\partial t}(m_2, t_2) \geq 0 \]

is satisfied for every two values \( m_1, m_2 \) of a material coordinate and every two values \( t_1, t_2 \) of time. This condition defines the monotonous process, for details see [MLs [3]]. Consequently, following constitutive relations derived for suspensions of kaolin and water and for similar monotonous processes can be utilized:

\[ k(c) = A_1 c^{A_2} \quad \text{(6)} \]

and

\[ \tau(c) = \begin{cases} 0 & \text{for } c \leq B_1 + B_2, \\ g \frac{\rho_w - \rho_s}{\rho_s} \left( c + B_1 \ln \left( \frac{c - B_1}{B_2} \right) - B_1 - B_2 \right) & \text{for } c > B_1 + B_2, \end{cases} \quad \text{(5)} \]

where \( A_1 = 18.6, A_2 = -2.468, B_1 = -67.206 \) and \( B_2 = 151.55 \) being \([k] = \text{m/s}, [\tau] = \text{Pa} \) and \([c] = \text{kg/m}^3\). Equation (7) defines the constitutive relation between the solid-phase stress and the suspension concentration. This relation enables us to exclude function \( \tau \) from Equation (4) and to reduce the number of unknown functions in the set of governing equations. We receive the final set

\[ \frac{\partial s}{\partial t}(x, t) - \frac{\partial w}{\partial x}(x, t) = 0 \quad \text{(8)} \]

and

\[ \frac{\partial w}{\partial t} - a^2(s) \frac{\partial s}{\partial x} = b(s, w), \quad \text{(9)} \]

of governing equations, where

\[ a^2(s) = \begin{cases} 0 & \text{for } c < B_1 + B_2, \\ \frac{gs(1 - s)}{(C_1 - s)(s - C_2)} & \text{for } c \geq B_1 + B_2 \end{cases} \quad \text{(6)} \]

and

\[ b(s, w) = \frac{g(1 - s)}{(C_1 - s)(s - C_3 w^C_4)}, \quad \text{(11)} \]

where \( C_1, C_2, C_3 \) and \( C_4 \) are constants resulting from the above relations.
4 Moving boundaries

Two visible interfaces develop in the suspension column during the sedimentation process. The upper one separates the zone of sedimenting suspension from the overlying layer of water. The lower interface separates the sedimentation zone from the zone of compression which develops at the bottom of the column as a result of the imperviousness of the bottom.

Denoting \( Z(t) \) the height of the upper interface above the bottom of the column at time \( t \), it holds

\[
Z(0) = L
\]

and

\[
Z(t) = \sup\{x; c(x, t) > 0\} \quad \text{for } t > 0. \tag{12}
\]

The mutual interactions of solid-phase particles cause non-zero values of the effective solid-phase stress within the compression zone. Hence, the following equations define the height \( Y(t) \) of the lower interface above the bottom at time \( t \):

\[
Y(0) = 0
\]

and

\[
Y(t) = \sup\{x; \tau(x, t) < 0\} \quad \text{for } t > 0. \tag{13}
\]

During the sedimentation process, both the interfaces move towards each other and they join at certain time \( t_m \). There exists only one interface in the column after this time. It separates the compression zone from the upper layer of water and it satisfies both Equations (12) and (13). The interface continues moving downwards until the steady state position is reached. The steady state represents the ultimate stage of the gravity thickening.

5 The zone of sedimentation

From the definition (13) it follows that in the space between the interfaces (12) and (13) the suspension is characterized by the absence of effective stress. Let us denote

\[
\Omega_s = \{(x, t); t \in (0, t_m), \ x \in (Y(t), Z(t))\}.
\]

The initial homogeneity of the suspension and the absence of effective stress make it possible to find the following solution of Equations (8) and (9) in the domain \( \Omega_s \):

\[
s(x, t) = s_0
\]

and

\[
w(x, t) = k_0 \frac{\rho_s - \rho_w}{\rho_w} s_0^3 \left( 1 - \exp \left( \frac{-g \rho_w (1 - s_0)t}{s_0 k_0 ((1 - s_0) \rho_s + s_0 \rho_w)} \right) \right),
\]

where \( c_0 \) is the initial suspension concentration and \( k_0 = k(c_0) \).
Taking into account the absence of solid-phase particles above the upper interface $Z(t)$ and making use of the above expression of the flux density $w(x,t)$, the following expression of the height $Z(t)$ can be derived for $t \in (0, t_m)$:

$$Z(t) = L - \frac{\rho_s - \rho_w}{\rho_w} k_0 s_0 \left[ t + \frac{((\rho_s - \rho_w) s_0 + \rho_w) (1 - s_0) k_0}{g s_0 \rho_w} \right] \times \left( 1 - \exp \left( \frac{-g \rho_w (1 - s_0) t}{s_0 k_0 ((1 - s_0) \rho_s + s_0 \rho_w)} \right) \right).$$

(7)

6 Numerical solution

The studied dewatering of the suspension reaches its maximum in the zone of compression. Let $\Omega_c$ denote the zone of compression. It holds

$$\Omega_c = \{(x,t); t > 0, x \in (0, Y(t))\}.$$  

In order to know the rate of compression and the time necessary to get a target thickness of the suspension, it is necessary to solve the given problem in the domain $\Omega_c$.

The lower boundary condition of this domain, i.e. at the bottom of the column, is

$$w(0, t) = 0, \quad \text{for} \ t > 0.$$

That means that the position of the boundary and the value of one of the unknown functions are given.

A boundary condition at the top of the compression zone results from the definition of the interface $Y(t)$. From Equations (13) and (7) it follows

$$c_-(t) = B_1 + B_2,$$

when denoting

$$c_- (t) = \lim_{x \to Y(t)^-} c(x,t), \quad c_+ (t) = \lim_{x \to Y(t)^+} c(x,t)$$

and similarly for function $s_-(t), s_+(t), w_-(t)$ and $w_+(t)$. Hence, a boundary condition at the top of the compression zone reads

$$s(Y(t), t) = s_-(t).$$

(14)

defining the value of one of the unknown functions but retaining the position of the boundary unknown. To solve the problem, it is possible to add the mass balance equation at the interface $Y(t)$ in the form

$$\frac{dY}{dt} (s_+(t) - s_-(t)) + w_+(t) - w_-(t) = 0.$$

(15)

The functions $s_+(t)$ and $w_+(t)$ are given by the solution of the sedimentation zone for $t < t_m$. For $t > t_m$, both functions $s_+$ and $w_+$ become zero since the
sedimentation process stopped. Equation (15) does not introduce any additional unknown function into the problem, and therefore Equations (14) and (15) supply all boundary information needed for the problem to be solved.

Since the set of Equations (8) and (9) is hyperbolic, it is possible to use the method of characteristics in order to get the numerical solution. Moreover, this method proved to be very efficient when incorporating the imposed boundary conditions into the numerical scheme.

Using the described method, the investigated problem of the suspension sedimentation and subsequent compression was solved numerically. The obtained results of two different experiments are presented in Figures 1 and 2.

The presented method was used to model two laboratory sedimentation tests numerically. In both the tests the suspension of water and kaolin was considered. The first experiment started with the initial suspension concentration $c_0 = 39.08$ kg/m$^3$ and the initial height of the suspension column $L = 2.008$ m. The second experiment started with the values $c_0 = 20.70$ kg/m$^3$ and $L = 1.906$ m.

The calculated developments of the heights $Y(t)$ of the compression zone are shown in Figure 1. The lines show the development of the height of the compres-
Figure 2: Concentration profiles of the compression zone.

sion zone for experiment 1 and 2. The values of times $t_m$ (marked by ↓) for both experiments are clearly visible.

The distributions of the suspension concentrations in the columns are shown in Figure 2. In each case, the concentration profile at the time $t_m$ and the concentration profile during the steady state ($t_\infty$) are presented.

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References
