Thermal numerical simulation of the laminar construction of RCC Dams

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Abstract

The use of the Roller Compacted Concrete (RCC) in the core of the body of gravity dams can considerably speed up the procedure of the construction. Since there is no post-cooling treatment for RCC dams, it is necessary to analyze the temperature profile of the concrete body for desired plan of the construction. A diffusive equation is coupled to the concrete heat generation equation in order to solve the temperature field. The discrete form of the equations is derived using Galerkin method by application of piece wise linear approximate function. The solution domain is divided into hybrid structured/unstructured triangular finite elements, considering gradual movement of the top boundary of the concreting. Application of the software in simulation of temperature profile in a typical RCC dam section is also demonstrated.

1 Introduction

Heat generation due to cement hydration in mass concrete dams and the heat exchange with the surrounding ambient result in time varying temperature profiles in the structure. Tensile stresses develop in the structure due to restrained thermal movements, which can result in thermal cracking of the concrete. The use of RCC method and fast construction of thin layers makes major difficulties for the application of post cooling techniques. Hence thermal considerations are critical tasks in RCC dams. This fact makes temperature profile simulation an important part of the design and construction process.
In this paper after introducing the governing equations of the phenomenon, the numerical techniques employed for the computer solution is described. In the solution algorithm, the layered construction technique is taken into account and numerical movement of the top surface boundary of the concrete structure is introduced.

2 Governing equations

Since heat exchange in dams principally takes place in the transverse direction of the dam axis, two-dimensional models are generally adopted for temperature studies [1]. Assuming isotropic thermal properties for the solid materials, the familiar equation defining heat generation and transfer is of the form,

\[ \alpha \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \frac{\alpha}{\kappa} \frac{\partial T}{\partial t} = - \frac{\partial Q}{\partial x} \]

where, thermal diffusion is defined as \[ \alpha = \frac{\kappa}{\rho c} \].

The parameters of the governing equation are \( \rho (Kg/m^3) \) density, \( c (KJ/Kg^o c) \) specific heat, \( T (^o c) \) temperature, \( \kappa (W/m^o c) \) heat conduction coefficient and \( Q (KJ/m^3 h) \) rate of heat generation per unit volume. The rate of heat evaluation \( \bar{Q} \) for the rock foundation of the dam is taken as zero and for the concrete dam body, it is determined as below.

3 Heat generation in concrete

Due to the considerable influence of the temperature on hydration rate and thereby heat generation of cementitious materials, it is necessary to take the temperature history of various points in concrete body into account [2,3,4]. Various functions have been proposed for considering this effect and are referred to as maturity function. Using a well-known example of these functions which is proposed by Rastrup [3], the relative rate of reaction will be taken as:

\[ H(T) = 2^{0.4(T - T_r)} \]

By this function the relative rate of reaction at \( T \) to the reaction rate of a reference temperature \( T_r \) can be determined. Having defined the relative speed of reaction, the following procedure is utilized for determining \( \bar{Q} \) at various points in the concrete section. The volumetric rate of heat evolution of the concrete mix per unit volume is determined under a known temperature regime. This data is then converted into heat evolution as a function of equivalent time, \( t_e \), under a reference temperature, which can then be used as the base data for
heat generation of concrete. $t_e$ is determined using the relative rate of reaction concept given above in equation (1):

$$t_e = \int H(T) dt$$

(3)

Therefore, heat evolution of concrete as a function of equivalent time $Q(t_e)$ is obtained. Various functions have been used by researchers for defining the heat evolution as function of $t_e$. An example of which takes the form [5],

$$Q(t_e) = A + E \exp\left\{-b\left[t_e\right]^n\right\}$$

(4)

where, $A$, $E$, $b$ and $n$ are constants obtained by regression of experimental heat evolution data. Now the rate of heat evolution at $T_e$ can be determined as,

$$\dot{Q}(t_e) = \left(\frac{dQ(t_e)}{dt}\right) = nbE[t_e]^{n-1} \exp\left\{-b\left[t_e\right]^n\right\} \left(t_e^{1/2}\right)$$

(5)

Therefore, for every point in the concrete body, the rate of heat generation can be derived as a function of $t_e$, taking the temperature history into account.

4 Numerical Solution

The governing equation for heat generation and transfer can be written in the following form,

$$\frac{\partial T_i}{\partial t} + \frac{\partial}{\partial x_i}\left(\alpha \frac{\partial T}{\partial x_i}\right) = S_i \quad (i=1,2)$$

(6)

where $T$ is the unknown parameter and $S_i$ is the heat source. If temperature flux in $i$ direction, is defined as $F^r_i = a \frac{\partial T}{\partial x_i}$ the equation takes the form,

$$\frac{\partial T_i}{\partial t} + \left( \frac{\partial}{\partial x_i} F^r_i \right) = S_i \quad (i=1,2)$$

(7)

Application of the Galerkin method on triangular finite elements with three nodes (using piece wise linear approximate function) ends up with an efficient model for solving parabolic equation of above type. Based on this method, after multiplying the residual of the above equation by the weight function $\phi$ and integrating over a sub-domain $\Omega$ (associated with its central node n) we have,
Note that the linear interpolation function takes the value of unity at node \( n \) (the center of \( \Omega \)) and zero at other neighboring nodes. Integrating above equation by part over \( \Omega \) and omitting zero terms the equation takes the form,

\[
\int_\Omega \left( \frac{\partial T_n}{\partial t} \right) \phi \, d\Omega + \int_\Omega \left( \frac{\partial F_i^d}{\partial x_i} \right)_n \phi \, d\Omega = \int_\Omega S_n \phi \, d\Omega \quad (i=1,2)
\]  

For a triangular element the transient term can be written as,

\[
\frac{\partial}{\partial t} \int_\Lambda \phi \, T \, d\Lambda = \frac{\Lambda}{3} \frac{dT}{dt}
\]

where \( \Lambda \) is the area of the triangular element. Similarly, the source term of the equation (11) takes the form,

\[
\int_\Lambda \phi \, S \, d\Lambda = \frac{\Lambda}{3} S
\]

The interpolation function is chosen piece wise linear, hence, the temperature flux is constant over an element (since it is formed by first derivative of the unknown primitive variable). Since the interpolation function is zero at the nodes other than \( k \), the temperature flux can be written for a triangular element as,

\[
\int_\Lambda F_i^d \frac{\partial \phi}{\partial x_i} \, d\Lambda = -\frac{1}{2} F_i^d \sum_{k=1}^{3} (\Delta l)_k
\]

Where \( \Delta l_i \) is the component of the normal vector of the edge of the triangle (opposite to the node \( k \)) in \( i \) direction. Summing the terms for all the triangles sharing the node \( n \) (the center of \( \Omega \)), the contribution of the inside sub-domain edges of triangles will be canceled due to considering counter-clock wise direction for calculation. After mathematical manipulations and simplifications equation (9) can be written for \( \Omega \) with center node \( n \) as:

\[
\Omega_n \frac{dT_n}{dt} = \Omega_n S_n \sum_{k=1}^{3} (\sum_{i=1}^{n} F_i^d \Delta l_i) \quad (i=1,2)
\]
where $N$ is the number of perimeter sides of $\Omega$ and $\Delta l_i$ is the component of its normal vector in $i$ direction. The resulting numerical model can explicitly be solved for every node $n$ (the center of $\Omega$ which is formed by gathering triangles sharing node $n$). The explicit solution of temperature at every node of the domain of interest can be modeled as,

\[
T_n^{i+\Delta t} = T_n^i + \Delta t \left[ S_n - 3 \frac{\Delta}{2 \Omega_n} \left( \sum_{k=1}^{N} F_{i}^{d} \Delta l_i \right) \right] \quad (i=1,2)
\]  

The heat source for each node $n$ in concrete body is defined by $S_n = \alpha_n \mathcal{Q}(e)_n / \kappa_n$.

If $\alpha$ and $\kappa$ at node $n$ are considered independent of time and temperature and maturity of the concrete, then for determination of the heat source in every time step, only the value of heat generation rate $\mathcal{Q}(e)_n$ must be updated for the nodes.

Considering thermal diffusivity as $\alpha = \kappa / \rho C$ with the unit (m$^2$/s), the criterion for measuring the ability of a material for temperature change. On the other hand, using unstructured meshes with different sizes of sub-domains and application of different materials implies that the minimum magnitude of the above relation be considered. Hence, to maintain the stability and accuracy of the explicit time stepping the minimum time step of the global domain should be considered as,

\[
\Delta t = k \left( \frac{\Omega_n}{\alpha_n} \right)_{\min}
\]  

where, $k$ can be considered as a proportionality constant coefficient, which its magnitude is less than unity. Hence, $F_{i}^{d}$ at each triangular element can be calculated using divergence theorem,

\[
\int_{\mathcal{L}} F_{i}^{d} d\Lambda = \int \left( \frac{\partial T}{\partial x_i} \right) d\Lambda = \sum_{m=1}^{3} \left( \bar{T} \Delta l_i \right)_m
\]  

where, $\Delta l_i$ is the $i$ direction component of normal vector of $m$th edge of a triangle and $\bar{T}$ is the average temperature of that edge. Therefore, we can consider following algebraic equation for calculation of temperature flux.

\[
F_{i}^{d} = \frac{1}{\Lambda} \sum_{m=1}^{3} \left( \bar{T} \Delta l_i \right)_m
\]
5 Initial and boundary conditions

As initial condition, the concrete placing temperature and initial rock foundation temperatures are considered which should be defined. The boundary condition on concrete external surface is taken as,

\[ \kappa \frac{dT}{dn} + q = 0 \]  

(18)

where \( q \) is the rate of heat exchange of concrete surface with the surrounding and is taken into consideration through three mechanisms; \( q_c \) (convection), \( q_r \) (long wave radiation) and \( q_s \) (solar radiation).

\[ q = \pm q_c + q_r - q_s \]  

(19)

\( q_c \) is through air movement and depends on the difference between temperature of the concrete surface with surrounding air,

\[ q_c = h_c (T_s - T_{air}) \]  

(20)

A similar relationship to above is used for long wave radiation [5],

\[ q_r = h_r (T_s - T_{air}) \]  

(21)

Short wave exchange rate \( q_s \) is given by,

\[ q_s = \alpha I_n \]  

(22)

Here, \( h_c \) is coefficient of thermal convection, \( h_r \) is coefficient of thermal radiation and \( \alpha \) is surface absorption coefficient and \( I_n \) is incident normal solar radiation.

In numerical modeling, the natural boundary conditions are used at boundaries in vicinity of the air [8] and nonreflecting boundary conditions is used at far foundation boundary nodes, by computing the temperature of the boundary, using internal nodes of the domain [9].

6 Discretization of solution domain

The solution domain comprises of the dam body and the foundation. Considering the regular geometry of dam cross section and in order to facilitate the movement of the upper boundary of concrete as layered sequences of
construction progresses, the use of structured mesh was considered for dam body. For construction of structured mesh, the nodal points are obtained by intersection of series of columns and rows. Then by the use of inclined lines each area within the four nodes is divided into two triangles. In this type of mesh the required shape of upstream and downstream boundaries of a dam can easily be taken into account. Since the age of concrete varies from layer to layer in the body of RCC dam, the use of regular mesh spacing allows moving the top boundary by gradual increasing the number of layers with constant thickness. The structured mesh is therefore quite appropriate because of offering control on the layer thickness and sequence of construction.

The foundation of the dam, where moving boundary is not required is discretized using unstructured mesh. This will facilitate consideration of foundations with irregular layers of materials (with different properties). The irregular triangular mesh was produced using Deluaney Triangulation technique. Such a mesh generation method allows local refinement of triangular elements by using source points and lines. In this type of mesh the address of each node (coordinates) plus the three nodes making up each triangle are explicitly defined. The use of unstructured mesh for the foundation of the dam has thus the advantage of allowing finer mesh size near the dam body, where heat flux is high. In addition it provides the ability to use coarser mesh spacing at points of the far foundation points, where the temperature gradients have small magnitudes. This increases the speed and accuracy of calculations [10].

7 Verification of the model

In order to verify the accuracy of the model a set of experimental measurements on a concrete cube with 60 (cm) dimensions with 450 (kg/m³) cement is used. The concrete block was insulated on all faces. The ambient temperature was varying between 10 and 20 °C. The applied heat generation function is that of reference [5]. A two dimensional rectangular structured triangular mesh with 6 (cm) spacing was generated for numerical simulations. The placing concrete temperature was considered approximately 15 °C. The properties of concrete was considered as \( \alpha_c = 0.0038 \ (m^2/h) \) and \( k = 9(KJ/m^0ch) \).

![Figure 1: Comparison of the computed results with the experimental data](image-url)
The result of computer model is compared with the average experimental measurements at three points of concrete block (one point at the center and two points near the faces). The comparison of the results of the model with the experimental data for the first ten days of the concrete age presents reasonable agreement (Figure 1).

8 Application to a typical case

For presenting the ability of the model to deal with real cases, it has been applied to a typical gravity dam section. Both the base and the height of the section are 10 (m), while the dam crest width is 2.5 (m). Upstream and downstream slopes are 0.1:1.0 and 0.8:1.0, respectively. The regular mesh of the RCC dam part is generated with 0.2 (m) vertical spacing, while horizontal spacing at the base of the dam is considered 0.3 (m) and reduced in upper layers proportional to the dam width. The horizontal mesh spacing at the dam base was taken as the finest part of the unstructured triangular mesh of the foundation part, where the size of triangles increase far from the dam base. The radius of the half circle shape far field boundary was considered 1.5 times of the dam base.

Figure 2: Four stages of layered concrete placing (RCC Dam).
Following assumptions were made for concrete placing program. The layers are constructed every 24 hours at the thickness of 0.4 (m). The cement content of the concrete is 150 (kg/m$^3$). The concrete placing temperature is considered 25°C. The ambient temperature variation is assumed to be between 20 and 30°C. In order to have high gradients at dam wall boundaries, no insulations were considered. The existence of high gradients makes severe conditions for the numerical computations. The properties of concrete is considered as $\alpha = 0.0038$ (m$^2$/h) and $\kappa = 9$ (kJ/m$^o$C h). Rock foundation is considered to be quartzite with $\alpha = 0.009$ (m$^2$/h) and $\kappa = 6$ (kJ/m$^o$C h).

Figure 3: Temperature after completion Figure 4: Temperature history of points

The results of the computational model for four different stages are shown at Figure 2. Figure 3 shows the temperature distribution after 10 days of dam completion. The computed temperature history of four points at the levels 2, 4, 6 and 8 (m) from the base (at center of the dam width) are compared with the maximum temperature of the computational domain at Figure 4. As can be seen the maximum temperature of the central points located at 2, 4 and 6 (m) above the dam base coincide with the maximum temperature of the domain. While the maximum temperature at central point 8 (m) above dam base, which is near the dam crest (where the width of the section is narrow), is less than maximum temperature of the field. This shows that, for the design and construction program considered above, during the final stages of the dam construction, the maximum temperature will occur somewhere else rather than the newly placed concrete layers (i.e. at the core of the section). This is in general agreement with the observed temperature fields of real RCC structures.

9 Conclusions

The equation of heat generation and transfer is solved on hybrid structured-unstructured triangular finite element mesh by using Galerkin method. The numerical model was verified by comparison of the predicted results with the available experimental measurements. Efficient modeling of the layered concrete
structure (RCC Dam) is achieved by application of gradual movement of the top boundary of the structure during the construction period. The developed model was applied to a typical RCC dam section and the results of the temperature fields obtained showed the general pattern expected for such structures.

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11 References