Wave regimes of the vapour film condensation

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Abstract

From theoretical analysis and numerical experiment carried out for the vapour film condensation, the wave film flow regimes induced by dependence of the condensate viscosity on temperature were predicted. The criteria for existence of such regimes have been obtained. Both wave and waveless regimes of film condensation on the vertical wall were investigated on the pilot plant. A good match between results of theoretical study and experiments has been obtained.

1 Introduction

It is known that increases of the mass consumption in condensate film may have an influence on the stability of waveless flow regime. B. Spindler [1] studied the influence of phase transition on the stability of condensate film flowing over a plane provided that liquid properties are supposed to be constant. However, it has been established now that the correct description of how the dependences of liquid viscosity and surface tension on temperature affect the film flow regime can't be reached by means of any amendments to solutions which were obtained disregarding these effects [2-5].

The thing is, that change of substance properties may lead to the instability of film flow and transfer processes [2]. The papers [4],[5] are concerned with the problem of thermal-convective instability of condensate film flow when liquid viscosity is strongly dependent on temperature. It was shown that the solution of stationary Nusselt problem didn't exist under certain conditions. This result may be interpreted as prediction for existence of the wave regimes
of non-isothermal film flow induced by changing the liquid viscosity along the flow direction. Proper experimental results for the films of condensate with viscosity strongly dependent on temperature were obtained a long time ago [6]. But, according to our view, it would be incorrect to correlate directly the findings from [4, 5] with experimental data from [6]. Indeed, the boundary conditions for the problem considered in [4, 5] are not comparable with experiment setting in the work [6]. Moreover, such a condition as constant heat flux on the cooled wall under the film condensation [4] has never been realised in experimental devices, unless it is accessible by any unusual way [1].

Somewhat different aspect of the problem discussed is linked to investigations of arising, extension, and propagation of nonlinear waves in the condensate films. Much has been written about the waves in liquid films [1]. But waves on the condensate film surface as well as on the surface of films with changeable consumption and properties have been studied to a lesser extent [7]. Consequently, it is desirable whenever it is possible to identify the special features intrinsic in the evolution and behaviour of surface waves in such liquid films. The well-known studies of this problem [4, 7] are based usually on the methods developed by Taniuti and Wei in their old work [8], and by R.Grimshaw [9]. But, despite many works it is necessary that some aspects of the problem should be examined with the greater thoroughness. Specifically, it is important to find the correct way for using perturbation methods in reference to situations when solutions of the stationary problem are not constant. Such a situation we have in case of the film condensation under the non-zero heat flux.

Our object here is to establish the existence criteria for the wave regimes of condensate film flow which are induced under the low Reynolds number by the condensate viscosity dependence on temperature. The other goal of this paper is to present the certain correct rearrangements resulting in the nonlinear evolutionary equations for propagating the surface waves in condensate films. We'll also give the previous analysis of the likely behaviour for solutions of the equations obtained.

2 Theoretical details

Let us consider the condensation of a pure vapour on the vertical or inclined plane with allowance for the condensate viscosity dependence on temperature. At the beginning, with reference to [1, 4] we have good reason for using Nusselt approach as permissible on low Reynolds numbers. In this limit, the basic momentum and energy equations in liquid phase read
where \( p \) is the condensate density, \( \mu \) - its dynamic viscosity, \( \alpha \) - its thermal diffusivity, \( T \) is the temperature, \( u \) is the liquid velocity along the plane, \( t \) is the time, \( y \) is the transversal coordinate. Here \( g_{ef} = g \sin \gamma \), where \( g \) is the gravitational acceleration, \( \gamma \) - angle of inclination of the plane to the horizontal.

For \( \mu(T) \) we can use well-known approximation [4]

\[
\mu(T) = \mu_0 \exp\left(-\beta(T - T_0)\right),
\]

The heat balance equation reads

\[
q_w = \frac{\lambda}{h} \left. \frac{\partial T}{\partial y} \right|_{y=0} = L \rho \left( \frac{\partial h}{\partial t} + \frac{\partial}{\partial x} \int_0^h u dy \right) + \rho C_p \frac{\partial}{\partial x} \int_0^h u(T_s - T) dy,
\]

where \( \lambda \) is the liquid thermal conductivity, \( C_p \) - its heat capacity, \( L \) is the heat of condensation, \( x \) is the longitudinal coordinate, \( h \) is the thickness of liquid film, \( T_s \) is the film surface temperature.

Boundary conditions at the plane \( (y = 0) \) are

- no-slip condition \( u = 0 \); heat condition \( q = q_w \).

Appropriate equation for the heat transfer through the liquid film and plane reads

\[
q_w = K(T_{cw} - T_{cx}),
\]

where \( K \) is the heat transfer coefficient with allowance for the thermal resistance of the solid wall. Here \( T_{cx} \) is the temperature at the core of cooling liquid flowing co-current with condensate film.

On the other hand, heat balance relation would read

\[
\int_0^X q_w dx = Q_e C_{pc} (T_{cx} - T_{c0}),
\]
where $Q_c$ is the consumption of cooling liquid beyond the plane per unit of film width, $C_{pc}$ - its heat capacity per unit of volume, $T_{cX}$ - its current temperature, $T_{c0}$ - its initial temperature.

The interfacial boundary conditions under the condensation of motionless pure vapour are ($y = h$):

$$T = T_s = \text{const}; \quad \frac{\partial u}{\partial y} = 0. \quad (8)$$

Solution for the stationary problem was founded by eliminating the time derivatives from the governing equations. Provided the Kutateladze number satisfies the inequality $K_u = C_p(T_s - T_w)/L \ll 1$, we ignore the heat of supercooling and assume that heat flux essentially allows for the phase transition to take place [1].

In terms of dimensionless variables

$$\tilde{j} = \frac{L_j p}{Q_c C_{pc} (T_s - T_{c0})}, \quad \tilde{h} = \frac{Kh}{\lambda}, \quad \eta = \frac{xK}{Q_c C_{pc}},$$

after integrating over the liquid film thickness with use of (3) we receive

$$\frac{d\tilde{j}}{d\eta} = \frac{1 - \tilde{j}}{1 + \tilde{h}}, \quad (9) \quad \frac{d\tilde{h}}{d\eta} = \frac{\tilde{j} - 1}{1 + \tilde{h}} \frac{\partial \Phi}{\partial \tilde{j}} \frac{\partial \Phi}{\partial \tilde{h}}, \quad (10)$$

where

$$\Phi(\tilde{j}, \tilde{h}) = \tilde{j} + \frac{E}{\Omega^3} \frac{\tilde{h}(1 + \tilde{h})^2}{(1 - \tilde{j})^2} \exp \left( \frac{\Omega(1 + \tilde{j}\tilde{h})}{1 + \tilde{h}} \right) \Phi_1,$$

$$\Phi_1 = 2 + \frac{\Omega(1 - \tilde{j})}{1 + \tilde{h}} + 2 - \frac{1 + \tilde{h}}{\Omega\tilde{h}(1 - \tilde{j})} \left[ 1 - \exp \left( \frac{\Omega\tilde{h}(1 - \tilde{j})}{1 + \tilde{h}} \right) \right].$$

The governing parameters $E$ and $\Omega$ are determined as follows:

$$E = \frac{\beta L \rho^2 g \varepsilon \lambda^3}{Q_c C_{pc} \mu_0 K^3}, \quad \Omega = \beta(T_s - T_{c0}) \quad (11)$$
After analogous integrating the non-stationary system (1), (2) over the film thickness and rearranging, the following dimensionless equations are obtained

\[ \Lambda \frac{\partial \bar{\eta}}{\partial \tau} = \frac{\Omega}{E} \mathcal{R}(\bar{h}, \bar{j}; \Omega), \]  \hspace{1cm} (12)

\[ \frac{\partial \bar{h}}{\partial \tau} + \frac{\partial \bar{\eta}}{\partial \eta} = \frac{1 - \bar{j}}{1 + \bar{h}}, \]  \hspace{1cm} (13)

where \( \tau = t \frac{K^2(T_s - T_{c0})}{L \lambda \rho} \) - dimensionless time, and \( \Lambda = \frac{Q_c C_p(T_s - T_{c0})^2 K^3}{L^2 \lambda^2 \rho^2 g_{ef}} \).

Here the expansion of function \( \mathcal{R} \) is omitted because of its bulky appearance. However, when the acceptable for physical reasons limitations: \( \bar{h} \ll 1 \) and \( E \gg 1 \) are proposed the above system can be recasted into the following form:

\[ \Lambda E \frac{\partial \bar{\eta}}{\partial \tau} = - \frac{3 \Omega \bar{j} \exp(-\Omega \bar{j})}{\bar{h}^2} + E \bar{h}, \]  \hspace{1cm} (14)

\[ \frac{\partial \bar{h}}{\partial \tau} + \frac{\partial \bar{\eta}}{\partial \eta} = 1 - \bar{j}. \]  \hspace{1cm} (15)

This asymptotic system has plain stationary solution:

\[ \bar{j}_0 = 1 - \exp(-\eta) ; \quad \bar{h}_0 = \left(3 \Omega \bar{j}_0 \exp(-\Omega \bar{j}_0)/E\right)^{1/3}. \]  \hspace{1cm} (16)

It is left to the readers to check that

\[ \frac{\partial \bar{h}_0}{\partial \eta} \bigg|_{\eta = \eta^*} = 0, \text{ where } \eta^* = \ln\left(\frac{\Omega}{\Omega - 1}\right). \]  \hspace{1cm} (17)

By the way \( \bar{j}_0 \big|_{\eta = \eta^*} = 1/\Omega \), and critical value \( \eta^* \) is real only if \( \Omega > 1 \). Thus the dimensionless consumption and film thickness when \( \eta \to \infty \) will behave like

\[ \bar{j}_0 \to 1 \text{ and } \bar{h}_0 \to \left(3 \Omega \exp(-\Omega)/E\right)^{1/3}. \]

Such non-monotonous behaviour of function describing the condensate film thickness allows for supposing the instability of the stationary film flow when values of \( E \) and \( \Omega \) are located in a certain area of the governing parameters space. It would also mean that there is possible generation of surface waves.
in the ambit of critical point $\eta = \eta^\ast$. In order to clarify this problem we start using the small perturbances $\xi_1 \ll 1$ and $\xi_2 \ll 1$. Thus, we look for a solution of (14), (15) in the form

$$\bar{j} = \bar{j}_0 (1 + \xi_1); \quad \bar{h} = \bar{h}_0 (1 + \xi_2).$$

Discarding all the nonlinear terms in the equations for perturbed flow we obtain the following system:

$$\bar{j}_0 \frac{\partial \xi_1}{\partial \eta} + \bar{h}_0 \frac{\partial \xi_2}{\partial \tau} = -\xi_1, \tag{18}$$

$$\Lambda \frac{\bar{j}_0}{\bar{h}_0} \frac{\partial \xi_1}{\partial \tau} = (\Omega \bar{j}_0 - 1) \xi_1 + 3 \xi_2. \tag{19}$$

Expelling $\xi_2$ from this system yields

$$\frac{\Lambda \bar{j}_0}{3} \frac{\partial^2 \xi_1}{\partial \tau^2} - \frac{\bar{h}_0 (\Omega \bar{j}_0 - 1)}{3} \frac{\partial \xi_1}{\partial \tau} + \bar{j}_0 \frac{\partial \xi_1}{\partial \eta} + \xi_1 = 0. \tag{20}$$

According to the local approximation method [1] we will consider the characteristic values of perturbations which initiate only at the given critical point $\eta = \eta^\ast$. Thus, the perturbations can be searched for the form

$$\xi_i = \Xi_i \exp(i(\alpha \eta - \omega \tau)), \quad \text{where the amplitudes } \Xi_i \text{ are defined at the fixed coordinate } \eta = \eta^\ast; \alpha \text{ is a dimensionless wave number of perturbations with close to the film thickness wavelength } [1]; \omega = (\omega_\tau + i \omega_\eta) \text{ is a dimensionless complex frequency.}$$

Analysis of the appropriate eigenrelation reveals the instability ($\omega_i > 0$) of condensate film flow below the critical distance $\eta^\ast$. This conclusion is correct for any wave number provided that limitations $\Omega > 1$ and $E \gg 1$ are satisfied. There exists the rest point of an «unstable focus» type. Nearby the origin ($\alpha \to 0$) we can estimate the oscillation frequency $\omega_\tau$ as

$$\omega_\tau \sim \sqrt{3 \bar{h}_0 / (\Lambda \bar{j}_0)} \tag{21}$$

Study of equations modelling wave phenomena is based on the boundary-layer approximation [7]. It reads

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{\partial}{\partial y} \left( \frac{\mu}{\rho} \frac{\partial u}{\partial y} \right) + g_{ef} + \frac{\sigma}{\rho} \frac{\partial K_s}{\partial x}, \tag{22}$$

Here $v$ is the transversal velocity component, $K_s$ is the curvature of the film surface, $\sigma$ is the surface tension.
Let us introduce the coordinate \( \tilde{y} = y/h \) and function \( f(\tilde{y}, x; h) \) such as \( u = u_s f(\tilde{y}, x; h) \), where \( u_s \) is the surface velocity of condensate film. This assumption is based on the automodel hypothesis which use in our problem can be substantiated with reference to [10]. Thus, integrating (22) over the film thickness with use of a continuity equation gives

\[
\frac{\partial j}{\partial t} + \frac{\partial}{\partial x} \left( \frac{f_2}{f_1^2} \tilde{y} \right) + \frac{j}{f_1 h^2} \left( f_3 \nu_w - \frac{\lambda(T_s - T_w)}{L \rho} \right) = g_{ef} h + \frac{\sigma h}{\rho} \frac{\partial K_e}{\partial x},
\]

where \( f_1 = \int_0^1 f d\tilde{y}; f_1 = \int_0^1 f^2 d\tilde{y} ; f_3 = \frac{\partial f}{\partial \tilde{y}} \bigg|_{\tilde{y}=0} ; \nu_w = \frac{\mu}{\rho} \bigg|_{\tilde{y}=0} \).

The mass balance equation must be of the form

\[
\frac{\partial h}{\partial t} + \frac{\partial j}{\partial x} = \frac{\lambda}{L \rho} \frac{\partial T}{\partial y} \bigg|_{y=h}.
\]

For the qualitative analysis of (23), (24) by means of asymptotic expansions [9] it may be assumed that both condensate consumption and film thickness rather slowly vary along the plane. Such assumption will be justified far from the initial point under the high heat of condensation. Thus, by introducing the small parameter \( \varepsilon = \lambda(T_s - T_w)/(L \rho j_0) \) the functions \( j \) and \( h \) are expanded by power series in \( \varepsilon \)

\[
j = \sum_{i=0}^{\infty} \varepsilon^i j_i(\tau, z, \chi) ; \ h = \sum_{i=0}^{\infty} \varepsilon^i H_i(\tau, z, \chi).
\]

Here \( \tau = \varepsilon t \) is slow time, \( z = \varepsilon x \) is slow longitudinal coordinate, \( \chi = \theta(\tau, z)/\varepsilon \) is fast phase variable. In this terms, eqns (23), (24) can be uncoupled in all orders of \( \varepsilon \). Thus, the zeroth-order equation for film thickness reads

\[
\left( \frac{\partial^2}{\partial \theta \partial z} \right)^2 \left( \frac{f_2}{f_1^2} - 1 \right) \frac{\partial H_0}{\partial \chi} - \frac{\sigma}{\rho} \left( \frac{\partial \theta}{\partial z} \right)^3 H_0 \frac{\partial^2 H_0}{\partial \chi^2} = g_{ef} H_0 + \frac{\partial \theta/\partial \tau}{\partial \theta/\partial z} \nu_w f_3 \frac{1}{f_1 H_0}.
\]

Moreover, there exists quasi-linear correlation between the consumption and film thickness in the next-orders equations:

\[
J_i = -\frac{\partial \theta/\partial \tau}{\partial \theta/\partial z} H_i + \Gamma(H_{i-1}, J_{i-1}, \tau, z).
\]
It follows from (26), (27) that surface pressure gradient induced by the variation of surface curvature can be considered as a factor of dispersion for zeroth order waves only. The waves of higher orders evolve without dispersion. According to the previous analysis it can be also concluded that solutions describing the surface ripple waves would appear whenever the following inequality is fulfilled

$$\left(\frac{f_2}{f_1} - 1\right)\left(\frac{\partial \theta}{\partial z}\right) < 0.$$  \hspace{1cm} (28)

The above inequality links the wave number of sweeping wave to the integral characteristics of film velocity profile which depends on temperature field. It follows from theoretical estimations that non-stationary regimes of condensate film flows induced by the dependence of condensate viscosity on temperature may be realized under the low Reynolds number.

3 Experimental details

The pilot plant consists of the apparatus for glicerin evaporation with vapour superheater, of the condenser, and of the measuring facility. A heat power of the plant varied within the bounds of 200-1500 W. In order to preclude decomposition of the glicerin under the high temperature the experiments were carried out at vacuum. The pure vapour of glicerin condensed on the outside surface of the vertical pipe which was made of copper and was cooled by inside circulation of water. The dimensions of the pipe are 8 mm exterior diameter and 550 mm length. The main part of measuring system is the special device based on the optical knife edge test. This device is intended for the investigation of the outline of condensate film flowing along the pipe. The light source is a laser LG-208B of Soviet make. The experimental set-up allows also to obtain snapshots of the film surface. The control unit is intended to adjust the temperature and pressure of the vapour at the inlet of condenser, and the temperature and consumption of cooling fluid at the same place. The mobile thermo-couple is used to measure the temperature of cooling liquid inside the pipe. The following experimental situation was chosen: $T_{rup} = 140 - 150 ^\circ C$; $T_c = 10 - 50 ^\circ C$; $Q_c = 3 \cdot 10^{-5} - 6 \cdot 10^{-4} \text{ m}^3/\text{ms}$. All the experimental data obtained corresponded to the following Reynolds numbers for the condensate film: $Re = 0.1 - 1$. The vapour velocity in the condenser was less than 0.3 m/s. Thus, we were not observing just a little influence of this factor on the film flow.

This part of work was performed in alliance with Dr. Harry Almendinger.
4 Results and discussion

Solutions of eqn (26) were tested in the form of travelling solitary waves by means of the method [10] based on the rapid Fourier transformation. The main result of numerical experiments is that the governing parameters space \((E, \Omega)\) can be divided into two non-intersecting parts corresponding to different behaviour of function \(\overline{h}(\eta)\). Typical plot is shown in Fig. 1. When the appropriate point \((E, \Omega)\) is located below the certain boundary curve we observed a monotonous increase of stationary solution \(\overline{h}(\eta)\). Otherwise the function \(\overline{h}(\eta)\) has a maximum at certain value \(\eta^*\), and appropriate stationary solution proves to be unstable.

![Figure 1: Regimes of waveless (I) and wave (II)](image1)

Figure 2: Experimental results. Film condensation. Numerical study.

After comparing the wave amplitudes measured by means of the optical method under the different values of governing parameters \(E\) and \(\Omega\) we were able to draw the experimental curve \(E(\Omega)\) which separated in parameters space the area corresponding to sharp increase of the dimensionless amplitudes \((\overline{A} = AK/\lambda)\) of waves observed. In that case the waveless smooth flow took place only on the initial site about 25 mm long. The distinguished area is also remarkable for the high heat transfer coefficients \((\alpha_c = \lambda/h)\). In Fig. 2 the above curve is depicted (here symbol "o" corresponds to \(\overline{A} < 0.02, \alpha_c < 300 \text{ W/m}^2 \text{ K}\), and symbol "*" corresponds to \(\overline{A} > 0.29, \alpha_c > 900 \text{ W/m}^2 \text{ K}\)). Experimental points located below this curve correspond to rather low waves amplitudes, which are close to experimental errors. Comparison of the plots in Fig. 1 and Fig. 2 confirms a good match between the experimental and theoretical boundary curves. Thus, experimental data obtained fully agree with our theoretical predictions. It seems unlikely to search for other interpretation of the phenomenon observed than depend-
ence of the condensate viscosity on temperature. Apparently, in case when cooling liquid flows co-current with condensate film the instability observed may arise from the interaction between two opposite tendencies. On the one hand, increase of condensate consumption along the plane tends to intensify the dissipation of energy. However, the average temperature of condensate increases along the flow. Owing to that the average viscosity of condensate decreases. It conduces to the retardation of dissipation.

5 Conclusions

As detailed in the discussion, it can be concluded that dependence of condensate viscosity on temperature causes the instability of waveless film flow. As a result, the wave regimes of condensate film flow have been arisen under the low Reynolds number. Experimental data supports the correctness of theoretical estimations for the existence criteria of such film flow regime. The discovered wave regime of film condensation is remarkable for the higher heat transfer coefficients than the waveless regime under the same Reynolds number.

References


