Form-finding bending-active structures with temporary ultra-elastic contraction elements

J. Lienhard1,2, R. La Magna1 & J. Knippers1
1Institute of Building Structures and Structural Design, University of Stuttgart, Germany
2str.ucture GmbH, Stuttgart, Germany

Abstract

Bending-active structures as basis of a structural form defined by elastic deformation, pose both numeric as well as strategic problems to a computational form-finding process. As the geometry of a bending-active structure becomes more complex in a coupled system, so do the equilibrium paths in the deformation process. Here, it is no longer possible to simply deform a structure into its elastically deformed shape by a number of linear support displacements. As a new approach for form-finding coupled bending-active systems in FEM, the authors developed a strategy using contracting cable elements to pull associated points from an initially planar system into an elastically deformed configuration. This paper will discuss the strategy of form-finding bending-active structures with ultra-elastic contraction elements by looking at basic samples of coupled bending-active structures, complemented with some considerations of the numerical background. In addition, samples of complex applications in built prototype structures are presented to highlight the effectiveness of this approach.

Keywords: bending-active, form-finding.

1 Introduction

Especially in the field of flexible structures, it is often impossible to determine beforehand the geometric description of an equilibrium system. This is well known in the field of tensile structures, as the output geometry is the result of a tight interaction between form, forces, material characteristics and boundary conditions. Thus we speak of form-finding rather than designing form. The indeterminacy of the form-finding process often poses a complicated problem
from a simulation point of view, as it is non-trivial to identify the kinematics and motion paths of key elements of the structural system. The proposed approach greatly simplifies the setup of the simulation model, freeing the analyst from the difficult and in some cases impossible task of precisely defining the paths of nodal displacement needed to perform the form-finding of bending-active structures. The main difference between a simple action type like support displacement and the proposed approach which employs contracting cables is exemplified in Figure 1: In the first case the motion path of the supports is completely determined beforehand. In the latter, where the bending-active elements is constrained by coupling it in space, the position of the displacements are unknown; here, a definition of the bending shape via support displacements is practically not possible. The employed cable elements work with a temporary reduction of elastic stiffness which enables large deformations under constant pre-stress. The cable element thereby becomes a free load defined by the magnitude of pre-tension and the rigidity of the elements in between which it is contracting.

Figure 1: Example of a basic bending-active component form-found via support displacements (a) and contracting cable elements (b).

2 Finite-element approach

The necessity for simulation of large elastic deformations in order to form-find bending-active structures poses no problem to modern nonlinear finite element analysis. However, since Finite-Element programs do not serve well as a design environment, the input data for the pre-processing of the simulation has to be generated with either physical form-finding or behaviour-based computational modelling techniques. The necessity and advantage of FEM lies in the possibility of a complete mechanical description of the system. Provided that form-finding solvers are included in the software, the possibility of freely combining shell, beam, cable, coupling and spring elements enables FEM to simulate the exact physical properties of the system in an uninterrupted mechanical description. These include: mechanical material properties, asymmetrical and varying
cross-sections, eccentricities, coupling and interaction of individual components, nonlinear stress-stiffening effects, nonlinear simulation of stresses and deflections under external loads (e.g. wind and snow), patterning and compensation.

It is often said that dynamic relaxation has a computation time advantage over FEM for systems of large deflection; however, this is rapidly diminished with an increasing number of elements per beam and large deflections. Comparative tests by [1] showed that computation time for the form-finding of an elastic gridshell was faster with FEM when the element length was set sufficiently small. For larger elements, computation time increased significantly or no convergence was reached at all in FEM, while dynamic relaxation converges quicker with increasing element size. The use of step size adaptive incremental deformation additionally drastically reduces the iteration steps and time needed in a FEM simulation. A disadvantage of FEM based form-finding of bending-active structures is the necessity to always start the simulation from a planar configuration in order to track the residual bending stresses in the system. This problem led to the development of the elastic cable approach, through which large deformations of complex bending-active structures can be solved using the Finite Element Method.

2.1 Elastic cable approach

Based on Finite Element technology, the contracting elastic cable approach has been proven to be a reliable technique for the form-finding of flexible structures. Following, the model setup and simulation details will be further examined. The model is mainly composed of two parts: the geometry to simulate, and the contraction elements which enable the form-finding process. In the first place the geometry is designed and pre-processed as in any CAE environment. The model is then equipped with a set of linear cables which link together all the pair of nodes that have to be joined during the form-finding. The contracting cables have to be defined as one single element which does not contain further internal nodes to avoid sagging and other collateral phenomena associated to the simulation of cable elements. Moreover, this reduces the computational complexity of the model as fewer nodes are present in the system, thus cutting down the total amount of degrees of freedom for which the system has to be solved. Having defined the whole geometrical and mechanical setup, a pre-stress value is then assigned to the cables, which is later translated into a pre-stressing load acting onto the elements. Due to the assigned pre-stress the cable elements will start contracting accordingly.

The cable elements work with a temporary reduction of elastic stiffness which enables large deformations under constant pre-stress. This method was originally developed for the form-finding of tensile membrane structures using, for example, the transient or modified stiffness method. For further information on this method (see [2] and [1]). For the form-finding of coupled bending-active systems, the great advantage is that the cables allow complete freedom of the equilibrium paths that are followed during the deformation process. The
pre-stress that is independent of the change in element length also allows the simultaneous use of several cable elements in the different positions of the system. Once the cables are contracted to nearly zero length they can be replaced by various types of coupling elements for the following structural analysis.

The value of nodal displacement can be deduced by multiplying the axial geometric stiffness of the element with the applied load. Referring to the basic equilibrium equation of a cable element in nonlinear FEM we can derive equation (1) for the pre-stressing cable used here:

\[
\left( \frac{EA}{l} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + \frac{P_x}{l} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \right) \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}
\] (1)

If Young’s modulus \( E \) is temporarily set to zero, the elastic stiffness would disappear and the tangential stiffness would be entirely based on the ratio pre-tension to length. Material stiffness therefore has no influence on the behaviour of the elastic cable which is thereby only constrained by the elements it is attached to. As such, the cable has become a directionless load defined by the magnitude of pre-tension and the rigidity of the elements in between which it is contracting.

Figure 2: Sample system with a spring attached to a contracting cable element; increasing the stiffness results in smaller nodal displacements per iteration (dashed line).

This artifice presents two advantages: in the first place the element becomes independent from its mechanical properties, thus creating a material-less device which serves only the purpose of pulling the system into shape; on the other
hand reducing the stiffness speeds up the contracting process, as the element exerts less resistance to the pre-stressing, thus shortening its length more rapidly.

It is worth noting that for each calculation step the amount of contraction is not constant or generally predictable during the form-finding process. Figure 2 displays a simple system where a spring of given stiffness is attached to a contracting cable. The graph illustrates the nodal displacement of the attached spring, which varies according to the stiffness of the spring itself. From the graph it becomes clear that the total displacement per step is reduced, as the overall resistance of the system has increased. It is therefore generally complicated to evaluate the number of contracting steps needed to fulfill the form-finding process, as a stiffer system will necessitate more iterations to reach the final configuration with the cable fully contracted.

The advantage of form-finding complex coupled equilibrium systems is shown by the example in Figure 3. This system is composed of two differently sized masts with fixed supports which are connected at the top and an additional inner point. Both cables of the system are simultaneously pre-stressed incrementally. The graph in Figure 3 shows the reciprocal dependency of cable pre-stress and length for the two cables in relation to the iterations of the linear incremental loop. Even though both cables are assigned with the same pre-stress increments, their actual pre-stress differs depending on the deformation of the system. Also, the moment of total contraction of the cable elements is reached at different iterations and still, the calculation continues after one cable has already contracted to a length close to zero. This astonishingly stable convergence behaviour enables the form-finding of far more complex systems introduced by the case studies below.

Figure 3: Left: Sample system composed of two differently sized masts with fixed supports. Right: Behaviour of the elastic cable approach for the system.
2.2 Incremental load steps

The large deformations that beam and cable elements undergo during the form-finding of bending-active structures cannot be simulated in a single load step. In order to still take advantage of a civil engineering FEM environment (in the presented cases Sofistik®), the authors developed custom programmed routines which enable incremental-iterative calculation.

The basic calculation procedure for incremental load steps is based on the option of primary load cases. Here, the deformations and strains of an already solved load case may be referred to by defining it as a primary load case for the calculation of a current load step. Therefore, deformations and stresses from a prior load step may be linked to another load case, thus reducing the calculation time.

An aspect to be considered is the exit criteria of the form-finding process. As the cable elements start contracting, they will continue doing so indefinitely, as the amount of displacement is proportional to the flexibility of the cable (i.e. the inverse of its stiffness) which is in turn proportional to its length. By successive contractions, the cable element shortens its length diminishing the total amount of length reduction that occurs at each pre-stressing load step. This entails that the element will never reach a zero-length configuration, but will rather converge asymptotically to zero as the graph in Figure 2 and 3 displays. Given this fact, it is necessary to define a threshold length under which the simulation can be deemed acceptable. This can be achieved by retrieving the length of each contracting element, checking whether all have reached the selected threshold, and finally halting the process if this is the case.

The asymptotical behaviour is reflected also in the global form-finding procedure, as the amount of total nodal displacement decreases as the cables tend to reach zero. This problem can be leveraged by introducing an adaptive framework that increases the pre-stressing load at each step depending on the number of iterations needed to find the equilibrium solution of the nonlinear problem.

3 Case studies

The following two case studies are briefly introduced to exhibit the strength of the elastic cable approach in form-finding complex bending-active structures.

3.1 Research pavilion ICD/ITKE

At the end of July 2010 the Institute of Computer based Design (ICD) and the Institute of building structures and structural design (ITKE) at the University Stuttgart realised a temporary research pavilion made of plywood. The design of the pavilion was the result of a student workshop which focused on the integration of physical experiments and computational design tools to develop bending-active structures. The project was discussed in detail in 0.
The Pavilion structure is based on a radial arrangement and interconnection of the self-equilibrating arch system made of 6.5 mm ply wood. Due to the reduced structural height, the connection points locally weaken the coupled arch system. In order to prevent these local points from reducing the structural capacity of the entire pavilion, the locations of the connection points between the strips need to change along the structure, resulting in 80 different strip patterns constructed from more than 500 geometrically unique parts.

Figure 4: Form-finding of the pavilion using the elastic cable (red lines) approach in three subsequent form-finding steps.

Given the unrolled geometry and connection points of the coupled arches, it was possible to form-find the pavilion structure with the elastic cable approach. The form-finding was separated into three subsequent steps (Figure 4). In a first step, 80 contracting cables simultaneously pull the flat plywood strips into the simply curved half torus shape. Thereafter, two sets of 205 contracting cables simultaneously form the coupled arch systems and then couple the arc pairs to a continuous torus shell. The nodal positions of the final equilibrium shape are entirely defined by the inner constraints. Such a system could not have been form found in FEM without the elastic cable approach. In fact, it was this project that ignited the research and development of this approach, and therefore laid the basis for later case study structures.

3.2 M1

The Textile Hybrid M1 at La Tour de l’Architecte showcases the research on hybrid form- and bending-active structure systems by the Institute for Computational Design (ICD) and Institute for Building Structures and Structural Design (ITKE) with students of the University of Stuttgart. The scientific goal of the project was the exploration of formal and functional possibilities in highly integrated equilibrium systems of bending-active elements and multi-dimensional form-active membranes (termed “Deep Surfaces”). The resulting multi-layered membrane surfaces allowed not only for structural integration, but also served as a functional integration by differentiating the geometry and orientation of the membrane surfaces. The project was discussed in detail in [5].
For the form-finding and analysis of the structure, FEM was utilised. Here, the parameters of the complex equilibrium system were explored to determine the exact geometry and evaluate the structural viability. Based on the elastic cable approach, the beams were initialised as straight elements and gradually deformed into interconnected curved geometries, finally being reshaped by the inclusion of pre-stressed membrane surfaces (Figure 5). The geometric data therein was determined initially by the physical form-finding models in defining the lengths and association points on the rods for the topology of FE beam elements. Given the unrolled geometry and connection points of the rods, it was possible to simulate the erection process and therefore the residual stress in a Finite -Element based form-finding process.

Figure 5: FEM form-finding of the Textile Hybrid based on the rod topology given by the physical model.

The form-finding of the beam elements in the M1 was separated into four subsequent steps. Starting with the straight beams, the loop modules were all form-found simultaneously by first forming an arc and then droplet shape. Based on the topological connection points given from the physical model, the loop elements were successively joined into a coupled system by contracting a final set of cables between the common nodes of the structure.

By means of automatic mesh generation, the membrane surfaces were added and a final form-finding of the fully coupled textile hybrid was undertaken. This form-found structural analysis model allowed verification of the geometrical shape, including its residual stress, as well as analysing the deformations and stress levels under external wind loads. Furthermore, the form-found membrane surfaces could be processed directly by the textile module of the software for patterning.
4 Conclusion

The use of ultra-elastic contraction elements in FEM has proven to be a powerful instrument for the form-finding of complex bending-active structures via large elastic deformations. An approach that can also be applied to surface elements, which may be used for the contraction of edge conditions rather than distinct points.

This approach enables the setup of an uninterrupted mechanical description of bending-active structures in a general purpose FEM environment. The three main process steps of form-finding, structural analysis and unrolling/patterning can thereby be represented in a single modelling environment. This ensures that structurally substantial residual stresses can be traced throughout all stages of design.

References


