The structural modelling of deconstructable beams, fabricated using friction-grip shear connection

M. A. Bradford
Centre for Infrastructure Engineering and Safety, School of Civil and Environmental Engineering, The University of New South Wales, Australia

Abstract

Many new medium-sized office-building structures are “temporary” and in some cases are demolished within ten years of their construction. Composite steel-concrete structural systems are very popular for medium-sized office buildings, but deconstruction and re-use in deference to demolition is difficult because headed stud shear connectors in the composite slab and steel beam flooring system cannot be detached easily, and reuse is virtually impossible. As an alternative, it is proposed that precast concrete panels can be attached to steel beams using pre-tensioned bolts, instead of cast-in-situ floors with pre-welded headed stud connectors. The proposed floor system can be deconstructed by unbolting the precast panels, enabling recyclability of the system and providing significant advantages in a paradigm within the construction industry focussed on low emissions. This paper presents an analysis of this slab and beam system, with the composite beams comprising of two elastic elements connected at their interface by a stiff but finite frictional connection. Under loading, it is shown that three distinct lengthwise domains exist along the member, and a model for this is presented and discussed.

Keywords: deconstructability, composite beams, friction-grip bolting, partial interaction, precast.
1 Introduction

Most new medium-sized office building structures have a somewhat short lifespan. With composite steel-concrete structural systems being very popular in constructing medium-sized office buildings, deconstruction and re-use in deference to demolition is very difficult because headed stud shear connectors in the composite slab and steel beam flooring system cannot be detached easily, and reuse is virtually impossible (Figure 1). As an alternative, it is proposed that (1) precast concrete panels can be attached to steel beams using (2) friction-grip or pretensioned bolts, instead of cast-\textit{in-situ} floors with pre-welded headed stud connectors. The proposed demountable floor system can be deconstructed by unbolting the precast panels, enabling recyclability of the system and providing significant advantages in a paradigm in the construction industry focused on low emissions [1–3].

The paper presents an analysis of this slab and beam system in a generic fashion, with the composite beam comprising of two elastic elements connected at their interface by a stiff frictional connection provided by the pre-tensioned bolts whose response can be considered as being rigid-plastic. It is shown that relatively low pretension forces are needed to produce a very stiff beam with close to full shear interaction throughout the service load range of a typical composite beam.

Figure 1: Demolition of framed structure in Epping, NSW, Australia.

2 Theoretical model

2.1 General

A simply supported prismatic composite beam is considered to comprise of two elements: the top ($\Omega_c$) of area $A_c$ and bottom ($\Omega_s$) of area $A_s$ joined by pre-tensioned high-strength bolts tensioned and distributed uniformly along the member of length $2L$. The beam is further assumed to be subjected to a uniformly distributed load $q$, with the pre-tension providing a friction at the interface between $\Omega_c$ and $\Omega_s$ distributed uniformly of magnitude $f_i$. 


Pragmatically, the bolts are considered to be placed in clearance holes rather than being fitted bolts [4], with a potential to slip on average by a value $s_b$ determined based on the diameters of the bolts and the clearance holes.

Under monotonically increasing load $q$ starting from $q = 0$, a shear flow force develops at the interface, being given by

$$ f = \frac{qxQ}{I}, $$

in which $x$ is measured from the origin located at mid-span, $Q$ the first moment of area of region $\Omega$ about the elastic centroid of the cross-section denoted $\Omega = \Omega_e \cup \Omega_s$ and $I$ the second moment of $\Omega$ about this axis. The maximum value of the shear flow force $\pm qLQ/I$ is at the ends of the member, and so assuming the frictional force prevents any slip, the beam will have full interaction provided $q \leq q_1 = fQ/I$. When $q > q_1$, slip takes place in the regions that flank the beam until a value of $q = q_2$ is attained, and the slip $s$ at these flanking regions ($s = s_b$) is sufficient for the end bolts to bear against their clearance holes. Further loading then defines three lengthwise domains along the beam as $\Gamma_1: x \in [-a, a]$; $\Gamma_2: x \in [-b, -a] \cup [a, b]$ and $\Gamma_3: x \in [-L, -b] \cup [b, L]$. These domains are depicted in Figure 2.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{Regions of different shear connection.}
\end{figure}

### 2.2 Equilibrium and compatibility

The differential equation of bending is

$$ \frac{d^2 v}{dx^2} = v'' = \frac{q(x^2 - L^2)}{2\psi_i}, $$

in which $v$ is the transverse deformation of the beam and $\psi_i$ the flexural rigidity appropriate to the relevant domain $\Gamma_i$ ($i = 1, 2, 3$). As a sensible approximation and based on push test results [5], it is assumed that $\psi_1 = \psi_3 = EI$ and $\psi_2 = EI_0$. 
where $EI_0$ is the flexural rigidity of the counterpart non-composite beam. A more refined model would, of course, be based on elastic partial interaction shear stiffnesses of $k_i$ [6] within each $\Gamma_i$ ($i = 1, 2, 3$), and the current model uses the special case that $k_1 = k_3 = \infty$ and $k_2 = 0$.

Integrating eqn (2) within the domain $\psi_i$ produces

$$v = \frac{qL^4 \xi^2}{24\psi_i} \left(\xi^2 - 6\right) + LK_{i1} \xi + K_{2i}$$

and

$$v' = \frac{qL^3 \xi}{6\psi_i} \left(\xi^2 - 3\right) + K_n$$

where $K_{i1}$ and $K_{2i}$ are constants of integration relevant for each region $\Gamma_i$ and where $\xi = x/L$. Specifically,

$$v = \frac{qL^4 \xi^2}{24\psi_1} \left(\xi^2 - 6\right) + C_2$$

and

$$v' = \frac{qL^3 \xi}{6\psi_1} \left(\xi^2 - 3\right)$$

for the region $\Gamma_1$ when the symmetry condition $v'(0) = dv(0)/dx = 0$ is invoked;

$$v = \frac{qL^4 \xi^2}{24\psi_2} \left(\xi^2 - 6\right) + C_3L\xi + C_4$$

and

$$v' = \frac{qL^3 \xi}{6\psi_2} \left(\xi^2 - 3\right) + C_3$$

for the region $\Gamma_2$; and

$$v = \frac{qL^4 \xi^2}{24\psi_1} \left(\xi^2 - 6\right) + C_5L\xi + C_6$$

and

$$v' = \frac{qL^3 \xi}{6\psi_1} \left(\xi^2 - 3\right) + C_5$$

for the region $\Gamma_3$, in which $C_2$, $\ldots$, $C_6$ are constants of integration. These may be found by imposing the kinematic conditions $v(L) = 0$, as well as continuity of the displacement $v$ and slope $v'$ at $x = a$ and $x = b$ (the static conditions are imposed 
\textit{a priori} by a knowledge of the bending moment distribution for the statically determinate member). This leads to the constants of integration as being

$$C_2 = \frac{qL^4}{24\psi_1} \left\{5 + (1 - n)[(3\alpha^3 - 4\alpha^2 - 6\alpha + 12]\alpha - (3\beta^3 - 4\beta^2 - 6\beta + 12] \beta]\right\}$$

and

$$C_3 = \frac{qL^3}{6\psi_1} (1 - n)\alpha(\alpha^2 - 3)$$
\[ C_4 = \frac{qL^4}{24\psi_1} \left( 5 + (1-n)\beta^2(\beta^2 - 6) + 4(n-1) \left( \alpha\beta(\alpha^2 - 1) + (\beta - 1)[\beta(\beta^2 - 3) - \alpha(\alpha^2 - 3)] \right) \right), \]  
\[ C_5 = \frac{qL^3}{6\psi_1} (n-1)\left[\beta(\beta^2 - 3) - \alpha(\alpha^2 - 3)\right] \] (9)

and

\[ C_6 = \frac{qL^4}{24\psi_1} \left[ 5 + 4(1-n)\left[\beta(\beta^2 - 3) - \alpha(\alpha^2 - 3)\right] \right] \] (10)

For the portion of the beam having full interaction, the axial force in the steel is

\[ N_s = \frac{A_s y_s M}{I} = \frac{A_s y_s qL^2}{2I} (1 - \xi^2) = A_s E_s u_s' \] (12)

where \( E_s \) is the elastic modulus of the steel section, and \( y_s \) the (positive) distance to its centroid from that of the transformed cross-section. Solving eqn (12) subject to the symmetry condition \( u_s(0) = 0 \) gives

\[ u_s = \frac{qL^3 y_s \xi}{6E_s I} \left( 3 - \xi^2 \right) \] (13)

and specifically at the end of the region of full interaction defined by \( x = a \),

\[ u_s(\alpha) = \frac{qL^3 y_s \alpha}{6E_s I} \left( 3 - \alpha^2 \right) \] (14)

By a similar argument for the concrete slab, it can be concluded that

\[ u_c(\alpha) = \frac{qL^3 y_c \alpha}{6E_c I} \left( 3 - \alpha^2 \right) \] (15)

in which \( E_c \) is the elastic modulus of the concrete and \( y_c \) the (negative) distance from its centroid to that of the transformed cross-section.
The slip deformation \( s \) at the interface between the concrete slab and steel beam can be expressed by simple geometry as

\[
s = u_s - u_c + h'\alpha
\]  
(16)

where \( h = y_s - y_c \) is the distance between the element centroids, and for the case that \( \alpha = 1 \) and the beam deflections are governed by eqn (4), using eqns (4), (13) and (16) produces \( s = 0 \) as expected.

Figure 3 shows a free body diagram of the concrete and steel elements for \( q > q_1 \) (i.e. \( \alpha < 1 \)), for which the sliding friction is taken as \( f_i \) and for which the bolts have not commenced to bear in their clearance holes (\( \beta = 1 \)). For the steel,

\[
N_s = f_i(L - x) = A_s E_s u_s'
\]  
(17)

which can be integrated subject to the boundary condition given in eqn (14) to yield

\[
u_s = \frac{f_i L^2}{2 A_s E_s} \left[ \xi(2 - \xi) - \alpha(2 - \xi) \right] + \frac{q L^2 y_s \alpha}{6 E_s I} (3 - \alpha^2)
\]  
(18)

while for the concrete,

\[
N_c = f_i(L - x) = -A_c E_c u_c'
\]  
(19)

which produces

\[
u_c = \frac{f_i L^2}{2 A_c E_c} \left[ \alpha(2 - \alpha) - \xi(2 - \xi) \right] + \frac{q L^2 y_c \alpha}{6 E_c I} (3 - \alpha^2)
\]  
(20)
The slip can then be found from eqns (16), (5), (8), (19) and (20) as

\[ s = \frac{f_i L^2}{AE} \left[ \xi (2 - \xi) - \alpha (2 - \alpha) \right] + s q E h n \left[ \xi (\xi^2 - 3) - \alpha (\alpha^2 - 3) \right] \]  

(21)
in which

\[ \frac{1}{AE} = \frac{1}{AE_c} + \frac{1}{AE_s} \]  

(22)

Noting that \( q = f_i L / (\alpha Q L) \) allows the slip in eqn (21) to be written in the dimensionless form

\[ \frac{s}{L} = \left( \frac{f_i L}{AE} \right) \left[ \xi (2 - \xi) - \alpha (2 - \alpha) \right] + \left( \frac{hAE}{EQ} \right) \left( \xi (\xi^2 - 3) - \alpha (\alpha^2 - 3) \right) \]  

(23)

which identifies the significance of the dimensionless parameters \( f_i L / AE \) and \( hAE / EQ \).

The load at which bearing in the clearance holes commences is determined when \( s = s_b \) and is at \( \xi = 1 \). Substituting these into eqn (23) then produces

\[ (1 - p)\alpha^3 - 2\alpha^2 + (1 - m + 3p)\alpha - 2p = 0 \]  

(24)

which defines the location of the point at first slip \((\alpha L)\) as a function of the dimensionless variables

\[ m = \frac{AEs_b}{f_i L^2} \quad \text{and} \quad p = \frac{AEh}{EQ} \]  

(25)

When \( s_b = 0 \), the bolts commence to bear adjacent to the location of first slip, and so the solution of eqn (24) is \( \alpha = 1 \), while if the bolts are allowed to slip without restraint, \( m \to \infty \) and the solution of eqn (24) is \( \alpha = 0 \), as expected. Following this, the location of the first bearing of the bolts can be determined from eqn (23) by setting \( \xi = \beta \), producing the cubic equation

\[ p\beta^3 - \alpha \beta^2 + (2\alpha - 3p)\beta + (1 - p)\alpha^3 - 2\alpha^2 + (3p - m)\alpha = 0 \]  

(26)
3 Illustration

Figure 4 shows a family of curves of $\beta$ against $\alpha$ when $p = 1.0$, while Figure 5 gives the counterpart curves for $p = 5.0$. It can be seen that as the limiting value of the slip related to the size of the clearance hole relative to the bolt diameter $s_b$ increases (and so $-m$ increases), the difference between $\alpha$ and $\beta$ increases, indicating an increase in the region $\Gamma_2$ in which the bolts slip. An increase in the parameter $p$, viz. an increase in the distance between the centroids of the elements $h$, decreases the region $\Gamma_2$ slightly.

![Figure 4](image1.png)

**Figure 4:** Plot of $\beta$ against $\alpha$ for $p = 1.0$.

![Figure 5](image2.png)

**Figure 5:** Plot of $\beta$ against $\alpha$ for $p = 5.0$. 
The central deflection \( v(0) \) is obtained from eqns (5) and (7) as

\[
v_0 = \frac{qL^4}{24\psi_1} \left\{ 5 + (1 - n) \left[ (3\alpha^3 - 4\alpha^2 - 6\alpha + 12)\alpha - (3\beta^3 - 4\beta^2 - 6\beta + 12)\beta \right] \right\}
\]  
(27)

which can be recast in dimensionless form by normalising with respect to the central deflection of a beam with flexural stiffness throughout \((5qL^4/24\psi_1)\) as

\[
\omega = 1 + \frac{(1 - n)}{5} \left[ (3\alpha^3 - 4\alpha^2 - 6\alpha + 12)\alpha - (3\beta^3 - 4\beta^2 - 6\beta + 12)\beta \right]
\]  
(28)

When the member has full interaction throughout (being imposed by \( f_i \to \infty \)), \( \alpha = \beta = 1 \), and \( \omega = 1 \) in eqn (28), while when it has no interaction (being imposed by \( f_i = 0 \)), \( \alpha = 0 \) and \( \beta = 1 \), and \( \omega = n \) in eqn (28) as expected.

Figure 6: Dimensionless deformation versus load for \( p = 1.0 \) and \( n = 2.0 \).

The evolution of the deformation of a composite beam is depicted in Figure 5, in which the dimensionless load \( q/q_1 = 1/\alpha \). For this beam, it is twice as stiff when composite compared with being non-composite (viz. \( n = 2.0 \)), and the parameter \( p \) in eqn (25) is taken as 1.0. The parameter \( m \) in eqn (25) captures both the frictional resistance \( f_i \) and the limiting slip \( s_b \). When \( s_b = 0 \) and so \( p = 0 \), there is no region of slip and so the beam has full interaction throughout. However, as \( s_b \) increases, the beam has increasing regions of slip with no interaction, and hence the stiffness of the beam is reduced, as can be seen from Figure 5. The maxima of the curves correspond to the location of \( q_2 \) when the bolts commence to bear in their clearance holes.
Figure 7 shows a typical cross-section of a composite beam, for which it is assumed that the steel section is an Australian 460UB82-1, that the elastic moduli of the steel and concrete are 200 GPa and 25 GPa respectively and that the simply supported member of length \(2L = 9\) m is subjected to a uniformly distributed load. Based on modular ratio theory, \(\psi_1 = 246.6 \times 10^6\) kNmm, \(\psi_2 = 88.5 \times 10^6\) kNmm and so \(n = 2.79\). If the beam has 16 M20 bolts tensioned to 150 kN (which is typical), then \(f_i = 150 \times 16 \times 0.45 / 9000 = 120\) N/mm, which assumes that the coefficient of friction is 0.45, as has been found in push testing [5]. Also for this beam, \(I = 1233 \times 10^6\) mm\(^4\) and \(Q = 2.5 \times 10^6\) mm\(^3\), which leads to \(q_1 = 13.2\) kN/m and which represents a load of 66 kPa. The deflection magnification for this beam is shown in Figure 8, in which it can be seen that the deflections increase above those for the counterpart beam will full interaction once the load \(q_1\) is exceeded. However, when the load to cause first bearing is reached, the flanking regions of the beam enter a regime of full interaction again and the beam stiffens with an increase of load.

Figure 8: Deflection magnification for composite beam in Figure 7.
4 Conclusions

This paper has proposed the use of bolted shear connectors in composite beams as a sustainable replacement of welded headed shear connectors, whose deconstructability and re-use is not possible. High-strength friction grip bolts are more expensive to manufacture and to install than are high-strength bolts, but when the cost of deconstructability and re-use, as well as the need for fewer high-strength bolts than headed connectors are taken into account, the technology shows considerable promise. The pre-tensioning delays the onset of interface slip, and has ramifications on the structural response at service load levels, as does the slip of the bolts in oversized clearance holes. These variables can be captured by a prescriptive equation that was developed in the paper.

Acknowledgement

The work reported in this paper was supported by an Australian Laureate Fellowship awarded to the author by the Australian Research Council.

References


