Analysis of ductile crack growth by means of a damage yield strip model

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Abstract

Based on Gurson’s continuum damage model, a damage cohesive zone model of the Dugdale-Barenblatt-type is developed. The model is applied to describe quasi-static crack growth in a compact tension specimen and dynamic crack propagation under small-scale-yielding conditions. Crack tip and cohesive zone parameters are calculated as functions of crack advance and crack tip velocity. In both cases, the algorithm is numerically stable and rather efficient. A direct mesh dependence of the results is not observed.

1 Introduction

The application of material models including damage effects to the analysis of propagating cracks has the advantage that no external fracture criterion based on macroscopic parameters like J-Integral or $\delta_c$ is required. The description of the fracture process by the total loss of macroscopic stress carrying capacity is already incorporated in the constitutive model of the material. On the other hand, the direct application of damage models can cause severe numerical problems. Especially in investigations of localisation problems by finite element calculations the results often display a strong mesh dependence. To deal with this problem, a number of methods have been developed introducing non-local descriptions of the damage effects using additional length-scale parameters (e.g. [2]) or special regularised descriptions of the discontinuities in the displacement field (e.g. [11]).

In the present study, an alternative approach similar to the work of Tvergaard and Hutchinson [13] is employed. Gurson’s continuum damage model [6] in modified form as given by Tvergaard and Needleman [12, 14]
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is implemented into a cohesive zone model of the Dugdale-Barenblatt-type [1, 4]. This method was first used by Zhang and Gross [15] in a plane stress analysis considering rate-independent plasticity. In the present study, this model is extended to cover the plane strain state as well as viscoplastic material behaviour.

2 Damage Yield Strip Model

2.1 Constitutive Equations

The ductile damage process in metallic materials consists in nucleation, growth and coalescence of microvoids. One of the most successful models describing this process is the model proposed by Gurson [6] in its modified form [12, 14] which employes the void volume fraction \( f \) as a damage parameter and uses the following approximate yield function

\[
\Phi = \left( \frac{\sigma_e}{\sigma_M} \right)^2 + 2q_1 f^* \cosh \left( \frac{\sigma_{kk}}{2\sigma_M} \right) - \left( 1 + (q_1 f^*)^2 \right)
\]

where \( \sigma_e \) and \( \sigma_M \) denote macroscopic and microscopic equivalent stress respectively, \( q_1 \) is a material parameter and \( f^* \) the effective void volume fraction according to Tvergaard [12].

The well known evolution equation (see e.g. [12]) for the void volume fraction \( f \) is used in a form restricted to strain controlled void nucleation. Thus, the void volume rate is given by

\[
\frac{df}{dt} = (1 - f) \dot{\varepsilon}_{kk}^{pl} + D \dot{\varepsilon}_M^{pl}, \quad D = \frac{f_N}{s_N^2 \sqrt{2\pi}} \left( \frac{\varepsilon_M^{pl} - \varepsilon_N}{s_N} \right)^2
\]

where \( \dot{\varepsilon}_{ij}^{pl} \) is the macroscopic plastic strain rate and \( \dot{\varepsilon}_M^{pl} \) the microscopic equivalent plastic strain rate. The quantities \( f_N, s_N \) and \( \varepsilon_N \) are material parameters describing the void nucleation process.

A complete set of constitutive equations is obtained, if the total strain increment is splitted into an elastic part governed by Hooke’s law and a plastic part, which is determined by eqns. (1) and (2) in conjunction with the flow rule and consistency condition \( \dot{\Phi} = 0 \).

2.2 Cohesive Zone Formulation

In the present study, a cohesive zone model is used to describe the fracture process. For this purpose, the crack is extended virtually by a cohesive zone of length \( l_{coh} \) which is loaded by the stress \( \sigma_{coh} \) (see Fig. 1). A finite width \( w_{coh} \) is introduced to enable the definition of a strain \( \varepsilon_{coh} \) normal to the cohesive zone by

\[
\varepsilon_{coh}(x_1) = \frac{2u_2(x_1)}{w_{coh}}
\]
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Figure 1: Cohesive zone model

where \( u_2 \) is the displacement of the upper crack surface in \( x_2 \)-direction. Using this definition, arbitrary plastic material models can be implemented into the cohesive zone model by reformulation into one-dimensional form and the assumption of \( \sigma_{coh} = \sigma_{22} \) and \( \varepsilon_{coh} = \varepsilon_{22} \).

Application of this procedure to Gurson's model in conjunction with some minor simplifications (for details see [7] or [15]) yields the following system of equations defining the behaviour of the cohesive zone

\[
\sigma_{coh} = c_\sigma (1 - q_1 f^*) \sigma_M (c_\sigma \varepsilon_{coh})
\]

\[
\frac{\partial f}{\partial \varepsilon_{coh}} = c_\sigma \left( \frac{3}{2} q_1 f^* \frac{1 - f}{1 - q_1 f^*} \sinh \left( \frac{c_f}{2} (1 - q_1 f^*) \right) + D \right)
\]

where \( c_\sigma = \left\{ \begin{array}{ll} 1 & \text{for plain stress} \\ \frac{2}{3} \sqrt{3} & \text{for plain strain} \end{array} \right. , c_f = \left\{ \begin{array}{ll} 1 & \text{for plain stress} \\ \sqrt{3} & \text{for plain strain} \end{array} \right. .

In eqns. (4) and (5), the microscopic equivalent stress \( \sigma_M \) may be related to \( c_\sigma\varepsilon_{coh} \) by an arbitrary microscopic material model.

The cohesive zone parameters \( l_{coh} \) and \( u_{coh} \) can be determined from the conditions, that no strain singularity occurs at the virtual crack tip and that the calculated crack driving force at the time of crack initiation must reach its experimentally determined critical value.

3 Quasi-Static Crack Growth

The first problem to be examined is the problem of ductile crack growth in a compact tension specimen under quasi-static loading conditions. Thus all rate effects in the material constitutive description are neglected and the power-law

\[
\varepsilon = \varepsilon_0 \left( \frac{\sigma}{\sigma_0} \right)^N
\]

is used to describe the plastic material response, where \( \sigma_0 \) is the initial yield stress, \( \varepsilon_0 \) the corresponding strain and \( N \) the hardening exponent. Eqn. (6) is used for the description of the matrix material in the cohesive zone as well as for the non damaging plastic material behaviour in the surrounding
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The material parameters for ductile steel according to Table 1 are used, which have been presented by Klingbeil et al. [8].

Table 1: Material parameters (by Klingbeil et al. [8])

<table>
<thead>
<tr>
<th>matrix material</th>
<th>damage parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$ [MPa]</td>
<td>$\nu$</td>
</tr>
<tr>
<td>$2.1 \cdot 10^5$</td>
<td>0.3</td>
</tr>
</tbody>
</table>

The problem is solved by the finite element method using standard six-node small strain triangular elements for the discretisation of the specimen. The cohesive zone is modelled by special spar elements with a material behaviour according to eqns. (4) to (6). Details are given in [7].

First, the dependence of the results on the discretisation is examined. In Fig. 2, the value of the $J$-Integral at a certain load level is plotted versus the total number of elements in the problem. Clearly, no direct mesh dependence of the results is observed, if a sufficient number of elements is chosen.

The $J$-Integral based crack resistance curve is presented in the second plot of Fig. 2. Experimental results given by Klingbeil et al. [8] have been added for comparison. The typical dependence of the $J_R$-resistance curve on the crack propagation $\Delta a$ can be observed. The numerical results of the present study agree well with the experimental data given in [8].

4 Dynamic Crack Propagation

The second problem considered in the present study is the problem of a semi-infinite crack propagating at constant velocity $\dot{a}$ under dynamic conditions through an infinite plate. In this case, the strain-rate sensitivity of the material cannot be neglected. Therefore, the material behaviour is assumed...
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to be governed by a Perzyna-type [10] viscoplastic constitutive equation

$$\dot{\varepsilon}_{\text{coh}} = \frac{1}{\eta} \left( \frac{\sigma_M}{\sigma(\varepsilon_{\text{coh}}, \Delta T)} - 1 \right)^M$$  \hspace{1cm} (7)

where \( \eta \) and \( M \) are the viscosity parameter and the viscoplastic hardening exponent respectively. According to Lee [9], work hardening and thermal softening can be taken into consideration by assuming

$$\sigma(\varepsilon_{\text{coh}}, \Delta T) = \sigma_0 \left( \frac{C_0 \varepsilon_{\text{coh}}}{\varepsilon_0} + 1 \right)^{\frac{1}{N}} \left( \frac{\Delta T}{T_0} + 1 \right)^{N^\text{th}}.$$  \hspace{1cm} (8)

In (8), \( \sigma_0 \) and \( \varepsilon_0 \) denote initial yield stress and corresponding strain respectively, \( N \) is the work hardening exponent, \( \Delta T \) the local temperature rise, \( T_0 \) a reference temperature and \( N^\text{th} < 0 \) the thermal softening exponent.

A moving cartesian coordinate system with the origin at the physical crack tip is used. Therefore, the derivative with respect to time can be replaced by a derivative with respect to the coordinate \( x_i \) by the relation \( \partial(\cdot)/\partial t = -a \partial(\cdot)/\partial x_i \).

In the sense of a small scale yielding problem, the area surrounding the cohesive zone is regarded to be linear elastic. Thus, the mechanical problem can be solved by superposition of the elastodynamic crack tip field described by the dynamic stress intensity factor \( K_{Id} \) and the field induced by the cohesive zone stress \( \sigma_{\text{coh}} \), where the latter one is determined by integration of the fundamental solution describing a moving pair of concentrated forces applied to the crack surfaces at a constant distance to the crack tip [5]. In conjunction with the condition of regularity of the elastic strains at the virtual crack tip, the following system of equations is obtained

$$\varepsilon_{\text{coh}} = \frac{4V(\dot{a})}{\pi E w_{\text{coh}}} \int_0^{l_{\text{coh}}} \sigma(x_1^*) \left( 2\frac{\sqrt{l_{\text{coh}} - x_1^*}}{\sqrt{l_{\text{coh}} - x_1^*}} - \log \frac{\sqrt{l_{\text{coh}} - x_1^*} + \sqrt{l_{\text{coh}} - x_1^*}}{\sqrt{l_{\text{coh}} - x_1} - \sqrt{l_{\text{coh}} - x_1^*}} \right) dx_1^*$$  \hspace{1cm} (9)

$$0 = K_{Id} - \sqrt{\frac{2}{\pi}} \int_0^{l_{\text{coh}}} \frac{\sigma(x_1^*)}{\sqrt{l_{\text{coh}} - x_1^*}} dx_1^*$$  \hspace{1cm} (10)

where \( V(\dot{a}) = \frac{\dot{a}^2}{c_s^2} \frac{(1 + \nu)\alpha_d}{4\alpha_d \alpha_s - (1 + \alpha_s)^2} \), \( \alpha_s = \sqrt{1 - \frac{\dot{a}^2}{c_s^2}}, \) \( \alpha_d = \sqrt{1 - \frac{\dot{a}^2}{c_d^2}} \).

\( E \) and \( \nu \) denote the elastic constants of the surrounding material, \( c_d \) and \( c_s \) are the dilatational and shear wave velocity respectively.

The thermal problem can be solved by integrating the fundamental solution of Fourier’s equation given by Carslaw and Jaeger [3] assuming the area \( A_{\text{coh}} \) of the cohesive zone as a distributed travelling heat source

$$\Delta T(x_1, x_2) = \int_{A_{\text{coh}}} \frac{\sigma_{\text{coh}}\varepsilon_{\text{coh}}}{2\pi \lambda^\text{th}} e^{-\frac{a(x_1 - x_1^*)}{2\kappa^\text{th}}} \cdot K_0 \left( \frac{a\sqrt{(x_1 - x_1^*)^2 + (x_2 - x_2^*)^2}}{2\kappa^\text{th}} \right) dx_1^* dx_2^*$$  \hspace{1cm} (11)
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\[ K_0(\cdot) \] is the modified zero order Bessel function of the second kind. Thermal conductivity and diffusivity are denoted by \( \lambda^{th} \) and \( \kappa^{th} \) respectively.

The evaluation of the integrals in eqns. (9) to (11) is performed by a numerical integration scheme using a singular approximation of the integrand in the case of eqns. (9) and (10). The resulting algebraic system of equations is solved by the Newton-Raphson-method.

The calculation is carried out using the material according to Table 1 and assuming the viscoplastic, thermal and dynamic properties given in Table 2.

Table 2: Viscoplastic, thermal and dynamic material parameters

<table>
<thead>
<tr>
<th>( \eta ) [s]</th>
<th>( M )</th>
<th>( T_0 ) [K]</th>
<th>( N^{th} )</th>
<th>( \lambda^{th} ) [W/Km]</th>
<th>( \kappa^{th} ) [m²/s]</th>
<th>( c_d ) [m/s]</th>
<th>( c_d' ) [m/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>5</td>
<td>300</td>
<td>-0.5</td>
<td>35</td>
<td>( 8.75 \times 10^{-6} )</td>
<td>5944</td>
<td>3177</td>
</tr>
</tbody>
</table>

In Fig. 3, cohesive stress \( \sigma_{coh} \), strain \( \varepsilon_{coh} \), void volume fraction \( f \) and temperature rise \( \Delta T \) along the \( x_1 \)-axis in the cohesive zone are plotted at different velocity levels \( \dot{a} \). It can be observed that with increasing velocity the maximum value of the cohesive stress \( \sigma_{coh} \) rises due to viscous effects. Although the deformation rate increases with the velocity, the plastic strain at the crack tip remains the same for all velocities due to the fact that the void volume fraction \( f \) is a function of the plastic strain only and therefore
the final void volume fraction $f_v$ can be reached only at a certain amount of plastic strain. The increasing cohesive stress $\sigma_{coh}$ in conjunction with the increasing strain rate $\dot{\varepsilon}_{coh}$ causes a significant local temperature rise at the crack tip which is strongly dependent on the crack tip speed.

Note that the virtual crack extension remains perfectly closed in a part of the cohesive zone especially at larger velocities so that the material behaves linear elastic although the yield condition is satisfied. In Fig. 4, the "active" length $l_{coh}$ of the cohesive zone in which a relevant crack opening displacement can be observed, is plotted as a function of the crack tip velocity. The plastic zone dimension decreases with increasing $\dot{a}$ until it vanishes totally if the Rayleigh wave velocity is reached. In contrast, the crack driving force becomes infinite if the Rayleigh wave velocity is approached. Neither vanishing plastic zone size nor infinite crack driving force occur in a pure quasi-static analysis neglecting inertia effects.

5 Conclusions

A cohesive zone model based on the continuum damage model of Gurson was presented. Like in other approaches a characteristic length-scale parameter, the cohesive zone width $w_{coh}$, is involved. This parameter can be determined experimentally in a straightforward way from $J$-Integral initiation values or $G$-measurements.

The model was applied to ductile crack growth under quasi-static loading conditions and to dynamic crack growth under steady-state conditions. The results agree well with experimental data taken from [8], a direct mesh dependence of the results is not observed.

References

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