Effects of damage-softening and heat generation on spall damage process

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Abstract

A systematic method to predict spall damage of plate impact based on continuum damage mechanics (CDM) was proposed. By representing the damage state by use of a scalar damage variable, the viscoplastic constitutive equation of Perzyna together with the damage evolution equation of Lemaitre & Chaboche were incorporated into the commercial finite-difference program MANJUSRI-3D for non-linear dynamic analysis. The effect of the surface energy due to the nucleation and growth of microvoids was included into the energy conservation law. The numerical results, such as particle velocity-time profiles at the center of the target rear surface and damage distribution, were compared with the corresponding results of the experiments. The differences between one-dimensional and two-dimensional analysis were discussed with respect to the behavior of stress wave propagation and the temperature distribution. The effect of the surface energy and that of the damage-softening were also elucidated.

1 Introduction

A phenomenon leading to local separation and fracture of materials caused by the reflection and interference of impulsive compressive waves is called spall damage. The spall damage in ductile materials is induced by the local development of distributed microscopic cavities brought about by the instantaneous passage of intensive stress waves. Since spall damage in metals may be induced by relatively low level of stress, it forms an important problem in the design and analysis of structural components under impulsive loading. In this work, we will discuss the applicability of continuum damage mechanics (CDM) to spall damage analysis in metals as a prospective systematic approach.
2 Damage Variable and Its Evolution Equation

In CDM, the mechanical effects of cavities due to damage are represented in terms of a proper macroscopic damage variable. When the development of cavities does not show salient anisotropy, the damage can be described by a scalar damage variable \( D \) (\( 0 \leq D \leq 1 \)):

\[
\begin{align*}
D &= 0 \quad (t = 0; \text{initial undamaged state}) \\
D &= 1 \quad (t = t_r; \text{final rupture state})
\end{align*}
\] (1)

The damage variable \( D \) is often interpreted as the reduction of load-carrying area (effective area), and thus, the effect of Cauchy stress is magnified by the damage to the following effective stress:

\[
\bar{\sigma}_{ij} = \frac{\sigma_{ij}}{1 - D}.
\] (2)

By postulating that the development of microcavities and the degradation of the mechanical properties are approximately isotropic, we employ the evolution equation of isotropic damage developed by Lemaitre & Chaboche:

\[
dD = \left\{ \frac{P - P_D}{(1 - D)S_D} \right\}^{q_D} \left\{ \frac{dP}{S_D} \right\},
\] (3)

where \( < > \) is Macauley bracket, \( P = (1/3)\sigma_{kk} \), \( P_D \) are the hydrostatic stress and its threshold value of damage development, and \( S_D, q_D \) are material parameters.

3 Constitutive Equations

The phenomenon discussed in this paper is the plate impact problem and then the hydrostatic pressure becomes dominative. Accordingly, we separate the constitutive equation into the spherical and deviatoric parts.

The stiffness of materials is usually decreased by the development of microcavities which form spall plane. We will postulate that the decrease of material stiffness is isotropic. Elastic constitutive equation of deviatoric stress is, then, obtained by applying the principle of strain equivalence to the equation concerned with deviatoric stress as follows:

\[
e_{ij}^e = \frac{1 + \nu}{E(1 - D)} s_{ij},
\] (4)

where \( e_{ij}^e, s_{ij} \) are the deviatoric components of elastic strain and stress, and \( E, \nu \) are the elastic modulus and Poisson’s ratio.

As for the collision of two bodies with relatively low speed, hydrostatic pressure generated in the bodies is not so high and the constitutive equation for volumetric strain is given by Hooke’s law. Therefore, the constitutive relation of damaged material with heat generation is given by

\[
\varepsilon_v = -\frac{p}{K(1 - D)} + 3\alpha(T - T_0),
\] (5)

where \( \varepsilon_v \), \( p \), \( K \), \( \alpha \), \( T \), \( T_0 \) are the volumetric strain, the hydrostatic pressure (\( p = -P \)), the elastic bulk modulus, the thermal expansion coefficient, temperature and its initial value.

As collision speed goes beyond a limit, on the contrary, the hydrostatic pressure becomes extremely high and the constitutive equation for volumetric
strain can not be given by Hooke's law. In such high speed collision case, the Mie-Grüneisen equation of state (Mcqueen, Marsh, Taylor, Fritz & Carter\(^3\)) is often used. Then, as the elastic bulk modulus is subject to the effect of damage development, the principle of strain equivalence is applied to the relation between the hydrostatic pressure and the volumetric strain:

\[
\begin{align*}
 p &= (1-D)(p_H - \Gamma \rho_0 e_H) + \Gamma \rho_0 e, \\
 p_H &= \frac{\rho_0 c_0^2 \eta}{(1-s\eta)^2}, \\
 e_H &= \frac{p_H \eta}{2 \rho_0}, \\
 \eta &= 1 - \frac{\rho_0}{\rho},
\end{align*}
\]

where \(e\), \(\rho\), \(\rho_0\) are the specific internal energy, the material density and its initial value and \(\Gamma\), \(p_H\) and \(e_H\) are the Grüneisen parameter, the Hugoniot pressure and energy. \(c_0\) is the bulk sound speed \(\sqrt{K/\rho_0}\) under room temperature which represents the lower limit of plastic stress wave propagation speed under plate impact. The hydrostatic pressure is also the implicit function of temperature because it is the function of the specific internal energy besides the volumetric strain and the damage.

Eqns (5), (6) provide the different constitutive equations for the material volumetric strain. However, we applied Taylor expansion to eqn (6) including eqn (7) about \(e_v\), \(e\) and \(D\) and then obtained its solution to the second order as follows:

\[
p(e_v, e, D) = -(1-D)Ke_v + \frac{\Gamma \rho_0 e}{2} K(\Gamma - 4s)e_v^2 + \ldots
\]

If the terms of \(e_v^2\) and the third and higher order can be neglected, this equation is reduced to the following form:

\[
e_v = -\frac{p}{K(1-D)} + \frac{\Gamma \rho_0 e}{K(1-D)}.\]

This is analogous to eqn (5). If the second terms concerned with the thermal strain in eqns (5), (6) are small, these equations coincides with each other.

In the following analyses, we use eqns (4) and (6) with (7) as elastic constitutive equations.

For inelastic deformation, viscoplastic constitutive equation of Perzyna\(^1\) extended to damaged materials is used:

\[
\dot{\varepsilon}_{ij}^p = \frac{\gamma}{2} \left( \frac{\sqrt{J_2}}{(1-D)\{(q + (\kappa_0 \beta q)e)^{-\beta}q\} - 1} \right)^m \frac{s_{ij}}{\sqrt{J_2(1-D)}},
\]

\[
\varepsilon_{eq}^p = \int_0^t \frac{2}{3 \sqrt{3}} (\dot{\varepsilon}_{pq}^p \dot{\varepsilon}_{pq}^p)^{1/2} dt',
\]

where \(\dot{\varepsilon}_{ij}^p\), \(s_{ij}\), \(\varepsilon_{eq}^p\) are the viscoplastic strain rate tensor, the deviatoric stress tensor and the equivalent viscoplastic strain. The symbol \(\kappa_0\) corresponds to the initial yield stress in shear, \(q\), \(\beta\) are the material hardening parameters (Eftis\(^4\)), and \(\gamma\) and \(m\) are the viscosity parameter and a material constant, respectively.

### 4 Field equations

The conservation laws of mass, momentum and energy in dynamic process are described with respect to the rectangular Cartesian coordinate system by the following forms:
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\[ \dot{\rho} + (\rho \dot{u}_i)_i = 0, \quad (12) \]
\[ \rho \dot{u}_i - \sigma_{ij,j} = 0, \quad (13) \]
\[ \rho \dot{\epsilon} = \sigma_{ij} \dot{\epsilon}_{ij}, \quad (14) \]

where \( u_i \) and \( \dot{\epsilon}_{ij} \) are the displacement and the total strain rate of damaged material, respectively. The effect of body force in the momentum conservation (13) is neglected, and it is further assumed that the duration of the spallation is so short that the heat conduction can be ignored in the energy conservation (14).

In spall damage process, local temperature rise is expected as a result of significant inelastic deformation around the fracture surface. In order to analyse this process, the temperature and the internal energy is related by the following relation:

\[ \dot{\epsilon} = C_v \dot{\theta}, \quad (15) \]

where \( C_v \) is the isovolumetric specific heat. In addition to the analysis by use of eqn (14) the analysis that allows for the surface energy due to the generation and growth of distributed microvoids is conducted as described later, and the effects of that is clarified.

5 Dissipation due to Surface Energy

The energy conservation law (14) in the preceding section is based on the assumption that all the external work changes into the internal energy. However, if the internal damage develops in the material, the external work dissipates as the surface energy due to the generation and growth of microvoids and microcracks besides thermal dissipation, too. Because nucleated surface develops in the form of distributed microvoids, the ratio of surface energy product per unit volume \( \dot{A} \) is as follows:

\[ \dot{A} = \gamma_s \dot{A}_{\text{nucl}}, \quad (16) \]

where \( \dot{A}_{\text{nucl}} \) is the rate of nucleated surface product. The surface energy \( \gamma_s \) is related to the critical rate of strain energy release \( G_c \), that is, \( G_c = 2\gamma_s \).

By postulating that a microvoid is a sphere with the average radius \( r \) and distributes with the average interval \( \lambda \), it is represented as a sphere in a unit cell. In the case that the load acts on the cell in the direction perpendicular to a face of the cell, the relation between \( r \) and \( D \) is given as follows:

\[ D = \pi r^2 / \lambda^2. \quad (17) \]

Thus, the area of nucleated surface per unit volume \( A_{\text{nucl}} \) is

\[ A_{\text{nucl}} = \frac{4\pi r^2}{\lambda^2} = \frac{4D}{\lambda}. \quad (18) \]

From eqns (16), (18), the energy conservation law that allows for the dissipation due to the surface energy is following:

\[ \rho \dot{\epsilon} + \frac{4\gamma_s}{\lambda} D = \sigma_{ij} \dot{\epsilon}_{ij}. \quad (19) \]

6 Analysis of Elastic-Viscoplastic Spall Damage

We first analyze the plate impact experiment of Curran, Seaman, & Shockey\(^5\), in which a target disc of 1.6 mm thick is impacted by a flyer disc of 0.6 mm thick at 160 m/s. In this work the radius of the plate is taken to be 3.2 mm, twice as
large as the thickness of the target plate for both plates. By use of the commercial finite-difference program MANJUSRI-3D (Itoh, Katayama, Obata, Moriya & Murakami\textsuperscript{6}) for non-linear dynamic analysis, the two-dimensional axisymmetric analysis for this experiment was conducted. The material constants for OFHC copper are employed with reference to literatures (Perzyna\textsuperscript{1}/Lemaitre & Chaboche\textsuperscript{2}/Eftis, Nemes & Randles\textsuperscript{4}/Johnson\textsuperscript{7}/Eds. Gomer & Smith\textsuperscript{8})

\[
\begin{align*}
C_v &= 3.924 \times 10^5 \text{ J/kg}\cdot\text{K} , \quad \gamma = 2.307 \times 10^6 \text{ s}^{-1} , \quad m = 5 \\
\kappa_0 &= 93.1 \text{ MPa} , \quad q = 125 \text{ MPa} , \quad \beta = 6.14 \\
P_d &= 1400 \text{ MPa} , \quad q_d = 0.7 , \quad S_d = 445 \text{ MPa} \\
c_0 &= 3901 \text{ m/s} , \quad \Gamma = 1.51 , \quad s = 1.99 , \quad \gamma_s = 1.65 \text{ J/m}^2
\end{align*}
\]

(20)

In the following, \( t = 0 \) is the moment of impact, \( z = 0 \) is the impact surface and \( r = 0 \) is the central axis of target plate. The distribution of the hydrostatic pressure \( p \) in the target plate are shown in Figs. 1 and 2. As shown in Fig. 1, the elastic wave of approximately 300 MPa precedes the plastic wave of 2900 MPa in the middle part. The front of the elastic wave already has arrived at and reflected from the rear surface and the reflected tensile stress wave has canceled the compressive stress in this part. The compressive waves that arrive at the rear surfaces of target and flyer are reflected to the tensile waves and cancel the compressive region to give the eventual hydrostatic pressure of vanishing magnitude. Then the region of the compression becomes narrow with the lapse of time. The stress wave distribution at \( t = 0.62 \mu\text{s} \) are shown in Fig. 2. Though the value of the hydrostatic pressure is nearly zero in the middle part at \( t = 0.55 \mu\text{s} \), immediately after that, a large localized tensile stress is induced around \( z/L = 0.63 \text{ mm} \) and the region of tensile stress spreads thereafter. However, the magnitude of tensile stress is constrained about 1500 MPa by damage development.

![Figure 1: Stress distribution in the cross section of target plate (\( t = 0.2 \mu\text{s} \))](image-url)
Around $z/L = 0.63$, the tensile stress is relaxed by damage softening and the elements there are eventually raptured. The hydrostatic pressure in this part reduced to zero. The damage distribution in the cross section of target plate is shown in Fig.3. The damage process proceeds rapidly after the emergence of tensile stress, and tends to the final rapture state $D = 1.0$ around $z/L = 0.63$.

![Figure 2: Stress distribution in the cross section of target plate ($t = 0.62 \mu s$)](image)

![Figure 3: Distribution of damage in the cross section of target plate ($t = 1.20 \mu s$)](image)
This location almost coincides with the spall surface of the experiment. Fig. 4 shows the temperature distributions calculated by eqn (14) in the central axis. The temperature is elevated with the damage development. In Fig. 5 the results of analysis by eqn (19) of the temperature change is shown. Compared with Fig. 4, the temperature elevation due to eqn (19) is estimated about 5 °C lower than that by eqn (14) in maximum, which can be ignored by taking account of the error due to the numerical method.

![Graph](Image)

Figure 4: Distribution of temperature in central axis (with eqn (14))

![Graph](Image)

Figure 5: Distribution of temperature in central axis (with eqn (19))

Furthermore, an experiment to measure the history of the back surface velocity of target plate was conducted by Ranjendran, Bless & Grove\(^9\), where a flyer plate of 2.0 mm thick impacts a target plate of 9.0 mm thick at 185 m/s. The plates are OFHC copper. The back surface velocity was measured by a velocity interferometer (VISAR). Fig. 6 shows the comparison among the results of the experiment and analyses. The largest values of the particle velocity in those cases show good agreement. The spall pullback velocity of the 2-dimensional analysis agrees with that of the corresponding experimental result.
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![Graph](image)

Figure 6: Particle velocity at the center of target back surface

7 Conclusions

The proposed method can predict the process of stress wave propagation, development of damage and the resulting temperature elevation. Moreover, it is thought that almost all the external work changes into the deformation, and the energy dissipation due to surface nucleation can be disregarded in this case. It is necessary to note that the process of this phenomenon is remarkably different from the process of crack growth in fracture mechanics.

REFERENCES