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Formula of stress concentration factors for round and flat bars with notches

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Abstract

Stress concentration of V-shaped notches in round and flat bars under various loading is important especially for the test specimen used to investigate the mechanical properties of materials. For notched bars, Neuber proposed simple approximate formula K_{tN} useful for wide range of notch shape: $1/(K_{tN}-1)^m = 1/(K_{ts}-1)^m + 1/(K_{td}-1)^m$ and m=2. Here, K_{ts} and K_{td} are exact solutions for shallow and deep notches, respectively. In this study, first, most suitable exponent m is determined so as to minimize the difference between K_{tN} and accurate K_t, that is, the results of body force method. Next, by applying the least squares method to the ratio K_t/K_{tN} more accurate formulae are proposed. The formulae proposed in this paper are found to give the stress concentration factors with better than 1% accuracy.

1 Introduction

To evaluate stress concentration factors (SCFs) of notched bars as shown in Fig. 1, Neuber proposed simple ingenious approximate formula K_{tN} useful for wide range of notch shape[1]:

$$\frac{1}{(K_{tN}-1)^m} = \frac{1}{(K_{ts}-1)^m} + \frac{1}{(K_{td}-1)^m} \text{ and m=2.} \tag{1}$$

Here, K_{ts} and K_{td} are exact solutions for shallow and deep notches, respectively. In our previous paper[2], on the basis of the Neuber formula correction factors were obtained by applying the least squares method to the ratio K_t/K_{tN} where K_t is the results of body force method[3-5]. In addition, the stress concentration factors are provided in a graphical way on the basis of the formulae so they can be used easily in design or research. In Neuber formula (1), however, the exponent m was assumed to be 2 without any good reasons. In this paper, first, the exponent of Neuber formula is considered so as to minimize the difference between K_{tN} and accurate K_t, that is, the results of body force method. Next, by applying the least squares method to the ratio K_t/K_{tN} more accurate formula are proposed. The stress concentration factors can be given by using these formula with less than 1% accuracy.

2 Definition of Stress Concentration Factors

In this paper, the stress concentration factors(SCFs) are based on the nominal stress at the minimum diameter or width and defined in eqn(2).

$$K_t = \sigma_{max}/\sigma_n \tag{2}$$

where σ_{max} is the maximum stress at the root of V-shaped notches. The problems treated in this paper are shown in Fig.1 with the definition of nominal net stress σ_n for each problem. In Fig.1 d is a diameter or width of minimum section, h is a plate thickness, P is the magnitude of external load, M is the magnitude of external bending moment, and T is the magnitude of external torsional moment. In the problems of (a) and (b) Poisson's ratio ν is assumed to be 0.3. In this study the following notations will be used.

$$\xi = \sqrt{t/\rho}, \quad \eta = \sqrt{\rho/t}, \quad \lambda = 2t/D, \quad \varepsilon = 2\rho/D \tag{3}$$

where the parameters ρ, t, D, d are indicated in Fig.1.

Problem(a): Round bar under tension [$\sigma_n = 4P/(\pi d^2)$]
Problem(b): Round bar under bending [$\sigma_n = 32M/(\pi d^3)$]
Problem(c): Round bar under torsion [$\tau_n = 16T/(\pi d^3)$]
Problem(d): Flat bar under tension [$\sigma_n = P/dh$]
Problem(e): Flat bar under bending [$\sigma_n = 6M/d^2h$]

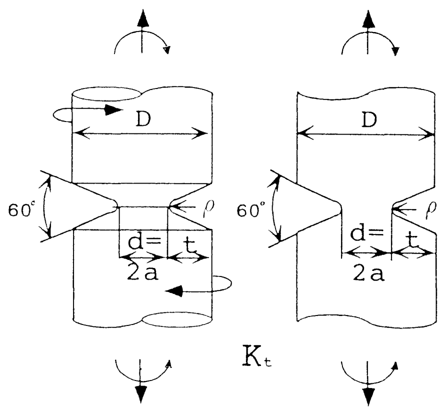

Figure 1: Round and flat bars with notchs, Round bar: (a)tention;(b)bending;(c)torsion, Flat bar: (d)tention;(e)bending

3 Most Suitable Exponent m in Neuber Formulae

To estimate SCFs of notched bars, the Neuber method makes use of the two exact solutions[1], that is, the solution of elliptical hole in an infinite plate K_{tH} as a shallow notch K_{ts} and the solution of hyperbolic notch as a deep notch K_{td}. From these values, the Neuber value K_{tN} is given by the following ingenious simple equation:

$$K_{tN} = \frac{(K_{ts}-1)(K_{td}-1)}{\left\{(K_{ts}-1)^m + (K_{td}-1)^m\right\}^{\frac{1}{m}}} + 1 \qquad (4)$$

The stress concentration factors can be estimated from the solution of a V-shaped notch in a semi-infinite plate K_{tv}. The very accurate formulae for K_{tv} can be obtained through applying the least squares method to the results of the body force method[5]. They are shown in eqn (5) with less than 0.2 % estimated errors. Here $K_{tH}=1+2\sqrt{t/\rho}$.

(1) $0 \leq \xi < 1.0$

$$K_{tV} = \left(1.000 - 0.120\xi + 0.2683\xi^2 - 0.1273\xi^3\right) K_{tH} \qquad (5a)$$

(2) $0 < \eta \leq 1.0 \quad (1.0 \leq \xi < \infty)$

$$K_{tV} = \left(1.035 + 0.0261\eta - 0.1451\eta^2 + 0.0842\eta^3\right) K_{tE}$$

$$K_{tE} = \left(1.121 - 0.2846\eta + 0.3397\eta^2 - 0.1544\eta^3\right) K_{tH} \qquad (5b)$$

For notched bars, Neuber proposed equation (4) with exponent m=2. However, in this study, most suitable exponent m is considered so as to minimize the difference between K_{tN} and accurate K_t, which is the results of body force method. As an example, in the case of round bar under tension, K_t/K_{tN} values are plotted in Figure 2 with varying m in the range 2.2~3.2. In Fig. 2, m=2.8 is found to be most suitable within $\pm 6\%$ error. In a similar way, the most

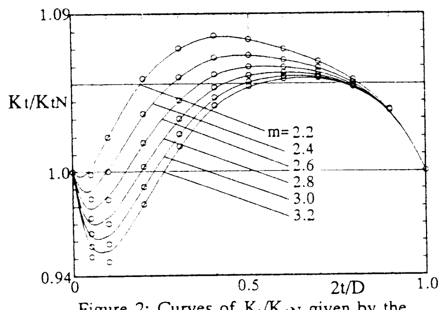

Figure 2: Curves of K_t/K_{tN} given by the approximate formulas of notch in round bar under tension with varying m (m=2.2~3.2).

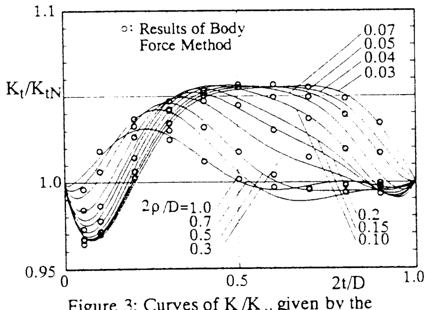

Figure 3: Curves of K_t/K_{tN} given by the approximate formulas of notch in round bar under tension (m=2.8) [$\sigma_n=4P/(\pi d^2)$]

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suitable exponent m is obtained in each problem. As a result, m=2.8 is also found to be most suitable in other problems. Figures 3-7 shows K_t/K_{tN} for wide range of notch shape when most suitable m=2.8 is used. It is found that maximum error is less than 5~7%.

4 Correction Factor K_t/K_{tN}

Modified Neuber formula (4) with m=2.8 has accuracy within 5~7% error. However, to improve the accuracy more, the correction factor for K_t/K_{tN} for each problems can be obtained by applying the least squares method. These obtained formulae are shown in equations (6)-(10). The curves given by the obtained formula is shown in Figures 3-7. Figures 3-7 indicate that the proposed formulae give SCFs with better than 1% accuracy. Table 1 show K_{tN} and correction factor for each problem. Stress concentration factors for problems (a)-(e) are provided directly in a graphical way as shown in Figures 8-12 so they can be used easily in design or research.

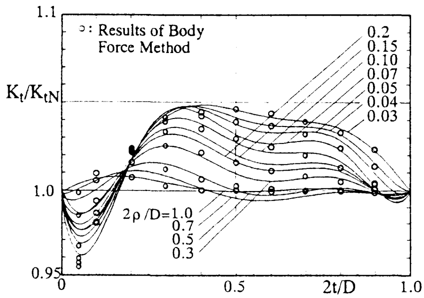

Figure 4: Curves of K_t/K_{tN} given by the approximate formulas of notch in round bar under bending (m=2.8) [$\sigma_n=32M/(\pi d^3)$]

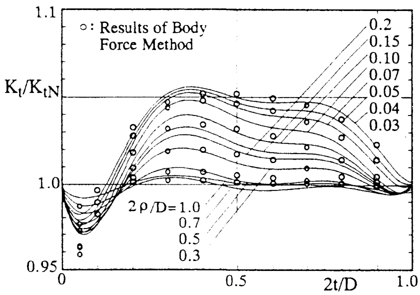

Figure 5: Curves of K_t/K_{tN} given by the approximate formulas of notch in round bar under torsion (m=2.8) [$\tau_n=16T/(\pi d^3)$]

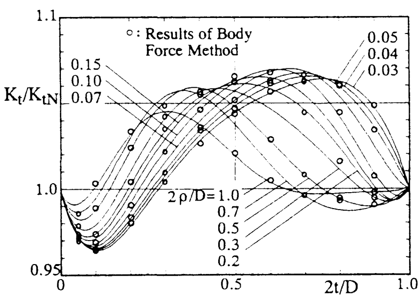

Figure 6: Curves of K_t/K_{tN} given by the approximate formulas of notch in flat bar under tension (m=2.8) [$\sigma_n=P/dh$]

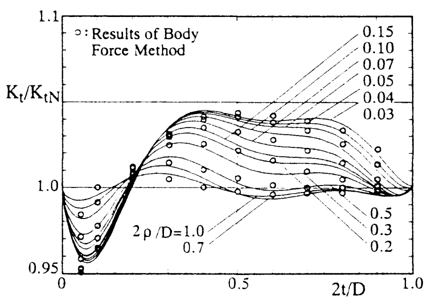

Figure 7: Curves of K_t/K_{tN} given by the approximate formulas of notch in flat bar under bending (m=2.8) [$\sigma_n=6M/(d^2h)$]

Problem (a)

$$K_t/K_{tN} = \left(0.9964+0.0802\varepsilon-0.6029\varepsilon^2\right) + \left(-0.8480-5.4969\varepsilon+44.774\varepsilon^2\right)\lambda + \left(7.1608+88.196\varepsilon-570.64\varepsilon^2\right)\lambda^2 + \left(-19.356-424.51\varepsilon+2591.4\varepsilon^2\right)\lambda^3 + \left(22.775+917.30\varepsilon-5508.4\varepsilon^2\right)\lambda^4 + \left(-10.648-917.30\varepsilon+5478.1\varepsilon^2\right)\lambda^5 + \left(0.9216+343.36\varepsilon-2034.3\varepsilon^2\right)\lambda^6 \quad (0.03 \le 2\rho/D \le 0.1)$$

$$K_t/K_{tN} = \left(0.9989-0.0057\varepsilon+0.0043\varepsilon^2\right) + \left(-1.1142+1.8361\varepsilon-0.6894\varepsilon^2\right)\lambda + \left(11.128-9.8563\varepsilon+1.3437\varepsilon^2\right)\lambda^2 + \left(-37.165+19.945\varepsilon+2.3816\varepsilon^2\right)\lambda^3 + \left(60.165-28.727-1.3686\varepsilon\varepsilon^2\right)\lambda^4 + \left(-48.307+28.945\varepsilon-7.0406\varepsilon^2\right)\lambda^5 + \left(15.295-12.135\varepsilon+5.3659\varepsilon^2\right)\lambda^6 \quad (0.1 \le 2\rho/D \le 1.0) \tag{6}$$

Problem (b)

$$K_t/K_{tN} = \left(1.0186-0.6912\varepsilon+4.5957\varepsilon^2\right) + \left(0.6062-50.602\varepsilon+352.67\varepsilon^2\right)\lambda + \left(-7.4522+598.27\varepsilon-4113.6\varepsilon^2\right)\lambda^2 + \left(34.013-2400.7\varepsilon+16321.0\varepsilon^2\right)\lambda^3 + \left(-68.143+4377.1\varepsilon-29536.0\varepsilon^2\right)\lambda^4 + \left(62.146-3728.8\varepsilon+25024.0\varepsilon^2\right)\lambda^5 + \left(-21.188\varepsilon+1205.4\varepsilon-8052.6^2\right)\lambda^6 \quad (0.03 \le 2\rho/D \le 0.1)$$

$$K_t/K_{tN} = \left(0.9950+0.0027\varepsilon+0.0008\varepsilon^2\right) + \left(-1.1718+2.7125\varepsilon-1.4235\varepsilon^2\right)\lambda + \left(13.813-27.858\varepsilon+13.70\varepsilon^2\right)\lambda^2 + \left(-51.966+96.962\varepsilon-45.583\varepsilon^2\right)\lambda^3 + \left(89.257-159.41\varepsilon+73.355\varepsilon^2\right)\lambda^4 + \left(-72.365+126.48\varepsilon-58.031\varepsilon^2\right)\lambda^5 + \left(22.438-38.891\varepsilon+17.983\varepsilon^2\right)\lambda^6 \quad (0.1 \le 2\rho/D \le 1.0) \tag{7}$$

Problem (c)

$$K_t/K_{tN} = \left(0.9976-0.0574\varepsilon+0.380\varepsilon^2\right) + \left(-0.2251-16.873\varepsilon+95.074\varepsilon^2\right)\lambda + \left(5.0686+151.60\varepsilon-916.29\varepsilon^2\right)\lambda^2 + \left(-20.884-518.45\varepsilon+3169.4\varepsilon^2\right)\lambda^3 + \left(36.943+850.30\varepsilon-5210.0\varepsilon^2\right)\lambda^4 + \left(-29.877-684.95\varepsilon+4200.6\varepsilon^2\right)\lambda^5 + \left(8.9809+218.32\varepsilon-1338.4\varepsilon^2\right)\lambda^6 \quad (0.03 \le 2\rho/D \le 0.1)$$

$$K_t/K_{tN} = \left(0.9951+0.0054\varepsilon-0.0023\varepsilon^2\right) + \left(-1.1092+1.5223\varepsilon-0.6373\varepsilon^2\right)\lambda + \left(12.722-18.340\varepsilon+8.2051\varepsilon^2\right)\lambda^2 + \left(-46.846+65.859\varepsilon-28.905\varepsilon^2\right)\lambda^3 + \left(79.242-108.28\varepsilon+46.221\varepsilon^2\right)\lambda^4 + \left(-63.563+84.842\varepsilon-35.260\varepsilon^2\right)\lambda^5 + \left(19.561-25.614\varepsilon+10.378\varepsilon^2\right)\lambda^6 \quad (0.1 \le 2\rho/D \le 1.0) \tag{8}$$

Problem (d)

$$K_t/K_{tN} = \left(0.9960+0.1255\varepsilon-0.850\varepsilon^2\right) + \left(-0.6607-4.8067\varepsilon+24.253\varepsilon^2\right)\lambda + \left(3.7414+70.527\varepsilon-286.53\varepsilon^2\right)\lambda^2 + \left(-5.2254-336.79\varepsilon+1376.7\varepsilon^2\right)\lambda^3 + \left(-2.2323+746.90\varepsilon-3151.9\varepsilon^2\right)\lambda^4 + \left(9.8254-764.89\varepsilon+3269.3\varepsilon^2\right)\lambda^5 + \left(-5.4466+289.0\varepsilon-1231.5\varepsilon^2\right)\lambda^6 \quad (0.03 \le 2\rho/D \le 0.1)$$

$$K_t/K_{tN} = \left(1.0008-0.0073\varepsilon+0.0045\varepsilon^2\right) + \left(-0.9811+0.8331\varepsilon-0.2538\varepsilon^2\right)\lambda + \left(8.2135-2.3324\varepsilon+0.5838\varepsilon^2\right)\lambda^2 + \left(-26.539+13.041\varepsilon-11.808\varepsilon^2\right)\lambda^3 + \left(46.069-50.304\varepsilon+46.229\varepsilon^2\right)\lambda^4 + \left(-40.927+67.240\varepsilon-58.324\varepsilon^2\right)\lambda^5 + \left(14.164-28.469\varepsilon+23.568\varepsilon^2\right)\lambda^6 \quad (0.1 \le 2\rho/D \le 1.0) \tag{9}$$

Problem (e)

$$K_t/K_{tN} = \left(0.9954+0.0441\varepsilon-0.3514\varepsilon^2\right)$$
$$+\left(-1.4237+4.7736\varepsilon-26.357\varepsilon^2\right)\lambda$$
$$+\left(14.229-40.619\varepsilon+265.86\varepsilon^2\right)\lambda^2$$
$$+\left(-48.353+111.76\varepsilon-868.29\varepsilon^2\right)\lambda^3$$
$$+\left(76.567-114.86\varepsilon+1118.3\varepsilon^2\right)\lambda^4$$
$$+\left(-57.422+17.607\varepsilon-480.71\varepsilon^2\right)\lambda^5$$
$$+\left(16.412+21.133\varepsilon-7.2857\varepsilon^2\right)\lambda^6$$
$$(0.03\leq 2\rho/D\leq 0.1)$$

$$K_t/K_{tN} = \left(0.9963-0.0004\varepsilon+0.0015\varepsilon^2\right)$$
$$+\left(-1.3941+2.0845\varepsilon-0.9283\varepsilon^2\right)\lambda$$
$$+\left(14.513-18.574\varepsilon+7.4069\varepsilon^2\right)\lambda^2$$
$$+\left(-51.118+57.209\varepsilon-20.271\varepsilon^2\right)\lambda^3$$
$$+\left(84.096-85.414\varepsilon+27.527\varepsilon^2\right)\lambda^4$$
$$+\left(-66.207+63.192\varepsilon-19.221\varepsilon^2\right)\lambda^5$$
$$+\left(20.114-18.493\varepsilon+5.4805\varepsilon^2\right)\lambda^6$$
$$(0.1\leq 2\rho/D\leq 1.0) \qquad (10)$$

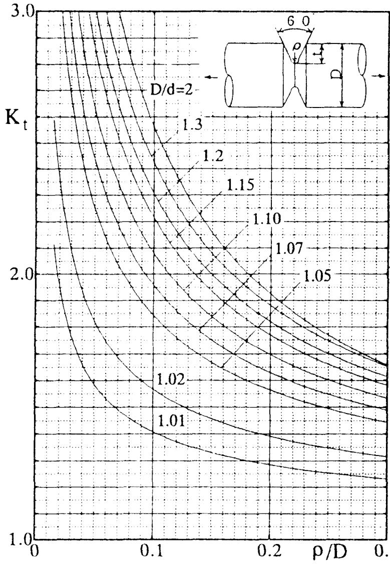

Figure 8: K_t of notch round bar under tension [$\sigma_n=4P/(\pi d^2)$, P=magnitude of external load, d=diameter of minimum section].

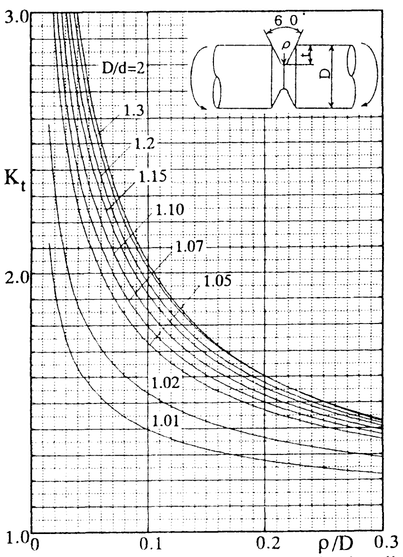

Figure 9: K_t of notch round bar under bending [$\sigma_n=32M/(\pi d^3)$, P=magnitude of external load, d=diameter of minimum section].

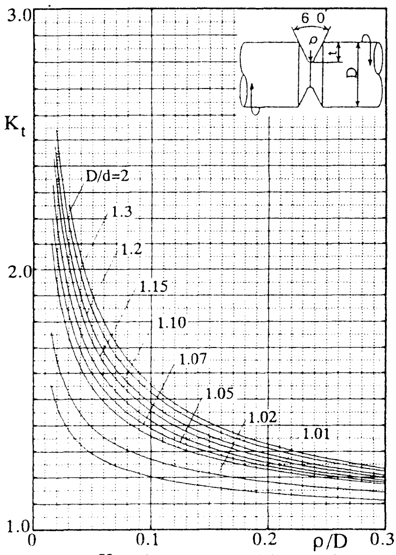

Figure 10: K_t of notch round bar under torsion [$\tau_n=16T/(\pi d^3)$, P=magnitude of external load, d=diameter of minimum section].

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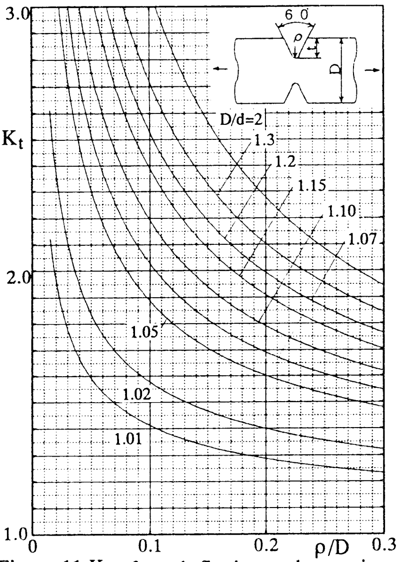

Figure 11: K_t of notch flat bar under tension [$\sigma_n = P/dh$, P=magnitude of external load, d=diameter of minimum section].

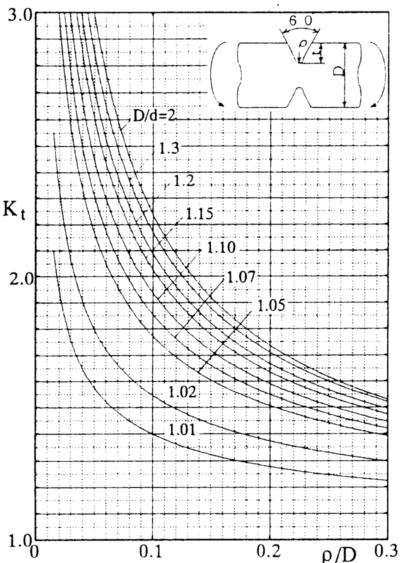

Figure 12: K_t of notch flat bar under bending [$\sigma_n = 6M/(d^2h)$, P=magnitude of external load, d=diameter of minimum section].

Conclusions

The stress concentration of V-shaped notches in round and flat bars under various loading is often encountered in mechanical design of shafts. It is also important for test specimens used to investigate the fatigue strength of materials. Accurate stress concentration factors (SCFs) have been given in a recent analysis of the body force method. In this paper, approximate formulae that are suitable for engineering applications are proposed. First, most suitable exponent m of Neuber formula eqn (1) K_{tN} is determined so as to minimize the difference between K_{tN} and accurate K_t, that is, the results of body force method. Next, by applying the least squares method to the ratio K_t/K_{tN} more accurate formulae are proposed. SCFs are also provided in a graphical way on the basis of the formulae so they can be used easily in design or research.

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Table 1: K_{tN} and Correction Factor.

	Kts	Ktd	KtN (maximam error)	Correction Factor Kt/KtN (maximam error)
Problem (a)	$K_{ts}=K_{tV}$ (Eqn (5))	$K_{td}=\frac{1}{N}\left\{\frac{a}{\rho}\sqrt{\frac{a}{\rho}+1}+(0.5+\nu)\frac{a}{\rho}+(1+\nu)\left(\sqrt{\frac{a}{\rho}+1}+1\right)\right\}$ $N=\frac{a}{\rho}+2\nu\sqrt{\frac{a}{\rho}+1}+2$	Eqn (4) m=2.8 (6 %)	Eqn (6) (1%)
Problem (b)	$K_{ts}=K_{tV}$ (Eqn (5))	$K_{td}=\frac{1}{N}\frac{3}{4}\left(\sqrt{\frac{a}{\rho}+1}+1\right)\left\{3\frac{a}{\rho}-(1-2\nu)\sqrt{\frac{a}{\rho}+1}+4+\nu\right\}$ $N=3\left(\frac{a}{\rho}+1\right)+(1+4\nu)\sqrt{\frac{a}{\rho}+1}+(1+\nu)\Big/\left(1+\sqrt{\frac{a}{\rho}+1}\right)$	Eqn (4) m=2.8 (5 %)	Eqn (7) (1%)
Problem (c)	$K_{ts}=1+\sqrt{\frac{t}{\rho}}$	$K_{td}=\frac{3\left(1+\sqrt{a/\rho+1}\right)^2}{4\left(1+2\sqrt{a/\rho+1}\right)}$	Eqn (4) m=2.8 (6 %)	Eqn (8) (1%)
Problem (d)	$K_{ts}=K_{tV}$ (Eqn (5))	$K_{td}=\frac{2(a/\rho+1)\sqrt{a/\rho}}{(a/\rho+1)\tan^{-1}\sqrt{a/\rho}+\sqrt{a/\rho}}$	Eqn (4) m=2.8 (7 %)	Eqn (9) (1%)
Problem (e)	$K_{ts}=K_{tV}$ (Eqn (5))	$K_{td}=\frac{4a/\rho\times\sqrt{a/\rho}}{3\left\{\sqrt{a/\rho}+(a/\rho-1)\tan^{-1}\sqrt{a/\rho}\right\}}$	Eqn (4) m=2.8 (5 %)	Eqn (10) (1%)

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