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Lumped damage models for oligocyclic fatigue in RC frames

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ABSTRACT

This paper presents a simplified damage theory for the nonlinear analysis of frames based on the lumped plasticity models and continuum damage mechanics. Based on this formulation, a particular model for RC frames under oligocyclic fatigue is proposed.

INTRODUCTION

Classic damage mechanics has been developed within the framework of continuum mechanics, but this is not the most suitable framework for the analysis of many civil engineering structures. These are often modeled as trusses or frames because continuum models can be used only for relatively simple structures. Plasticity theories have been successfully adapted to frame analysis through the notion of "lumped plasticity models" which consists in assuming that plastic effects are concentrated in some special locations called "plastic hinges". In this paper we present a formulation that generalizes these lumped plasticity models to include damage effects. This formulation can be considered as a simplified damage or fracture mechanics for frames. A time-independent model for reinforced concrete (RC) structures under monotonic loading is then presented. Based on this model a damage evolution law for frames under oligocyclic fatigue (i.e. loading that produce failure at a low number of cycles) is proposed. These simplified models can be used, for instance, in the numerical simulation of the collapse of civil engineering structures, off-shore platforms, etc. under seismic loading, waves and so on. In these models, damage is assumed to concentrate in inelastic hinges, so localization problem such as defined in continuum mechanics, does not arise. For a given set of inelastic hinges it is expected that only a finite number of solutions for the first rate problem can appear. It can be noticed that the location of inelastic hinges in frames can be intuitively stated in many engineering problems.

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SIMPLIFIED DAMAGE MODELS FOR FRAMES

For the sake of simplicity, let us consider a member of a planar frame in the small deformations case (see Flórez [1] for a more general presentation). Generalized stresses and deformations of the member are denoted respectively by $\{M\}^t=(M_i,M_j,N)$ and $\{\Phi\}^t=(\Phi_i,\Phi_j,\delta)$ (see figure 1).
Under severe overloads, the member undergoes damage, plasticity and other energy-dissipation phenomena. In order to characterize these inelastic effects, we considered the "lumped dissipation model" of the member shown in figure 1c.

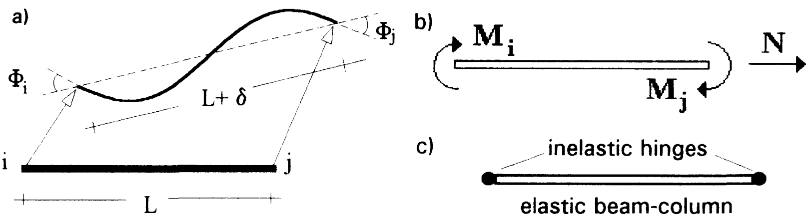

Figure 1: a)Generalized deformations b)Generalized stresses c) Lumped dissipation model of a member

The member is represented as the assemblage of an elastic beam-column and two zero-length inelastic hinges. All inelastic effects are assumed to concentrate in the hinges while beam-column behavior remains always elastic. This representation is similar to that used in the construction of plastic theories for frames.
We postulate the existence of two sets of internal variables, the plastic deformations $\{\Phi^p\}^t=(\Phi^p_i,\Phi^p_j,\delta^p)$ and the damage $\{D\}^t=(d_i,d_j,d_n)$, so that the potential energy of a member can be expressed as (see references [1,2] for a justification of equation (1) based in continuum damage mechanics):

$$U(\Phi,\Phi^p,D)=\frac{1}{2}\{\Phi-\Phi^p\}^t[S(D)]\{\Phi-\Phi^p\} \qquad (1)$$

where the [S(D)] is the stiffness matrix of a damaged member whose coefficients are given by:

$$S_{11}=\frac{(1-d_i)(d_jS^o_{12}S^o_{12}-S^o_{11}S^o_{22})S^o_{11}}{d_id_jS^o_{12}S^o_{12}-S^o_{11}S^o_{22}}; \qquad S_{12}=\frac{(1-d_i)(d_j-1)S^o_{11}S^o_{12}S^o_{22}}{d_id_jS^o_{12}S^o_{12}-S^o_{11}S^o_{22}}$$

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$$S_{22} = \frac{(1-d_j)(d_i S^o_{12} S^o_{12} - S^o_{11} S^o_{22}) S^o_{22}}{d_i d_j S^o_{12} S^o_{12} - S^o_{11} S^o_{22}}; \qquad S_{33} = (1-d_n) S^o_{33}; \qquad S_{13} = S_{23} = 0$$

S^o_{ij} are the coefficients of the stiffness matrix of the elastic beam-column.

The first internal variable contains the plastic rotations and permanent elongation of the inelastic hinges such as defined in standard plastic theories for frames. The last internal variable has three parameters that measure respectively the state of damage of hinges "i" and "j" under flexural actions and the state of damage of the member under axial effects. These parameters can take values in the interval [0,1] as in the classic continuum damage theory. If a flexion parameter takes the value zero (no damage), we have a plastic hinge as that of standard plastic models. If it takes the value one (totally damaged) the hinge has the same behavior of an internal hinge in an elastic frame.
The use of the potential energy as thermodynamic potential allows to determine the stress-deformation relationship and the definition of the thermodynamic forces conjugated to the internal variables:

$$\{M\} = \left\{\frac{\partial U}{\partial \Phi}\right\} = -\left\{\frac{\partial U}{\partial \Phi^p}\right\} = [S(D)]\{\Phi - \Phi^p\} \qquad \{G\} = -\left\{\frac{\partial U}{\partial D}\right\} \tag{2}$$

Forces $\{G\}$ are the equivalent of the energy release rate introduced in fracture mechanics and continuum damage mechanics. The constitutive model consists of the state laws (2) and the evolution laws of the internal variables. In the next section a time-independent model for monotonic loading is presented. In this model the damage evolution law is expressed as a function of the thermodynamic force $\{G\}$. This model was first proposed in [3] and is fully described in [4]. This model can be included in the library of finite elements of standard structural analysis software (see [1,4]). Based on this model an evolution law for oligocyclic fatigue in RC frames is proposed in the last section.

A TIME-INDEPENDENT DAMAGE MODEL FOR RC FRAMES

Two sets of inelastic (plastic and damage) functions: $\{f\}=\{f(M,\Phi^p D)\}=(f_i,\ f_j)$ and $\{g\}=\{g(G,\Phi^p D)\}=(g_i, g_j)$ are introduced. Each set have two functions, one for every hinge. Internal variables evolution laws are now defined in the following way:

$$\dot{\Phi}_i^p = \dot{\lambda}_i^p \frac{\partial f_i}{\partial M_i} \qquad \dot{\Phi}_j^p = \dot{\lambda}_j^p \frac{\partial f_j}{\partial M_j} \qquad \dot{\delta}^p = \dot{\lambda}_i^p \frac{\partial f_i}{\partial N} + \dot{\lambda}_j^p \frac{\partial f_j}{\partial N}$$

$$\dot{\lambda}^p \begin{cases} =0 & \text{if} \quad f<0 \quad \text{or} \quad \dot{f}<0 \quad \text{(no plasticity)} \\ \neq 0 & \text{if} \quad f=0 \quad \text{and} \quad \dot{f}=0 \quad \text{(plastic increment)} \end{cases} \tag{3}$$

$$\dot{d}_i = \dot{\lambda}_i^d \frac{\partial g_i}{\partial G_i} \qquad \dot{d}_j = \dot{\lambda}_j^d \frac{\partial g_j}{\partial G_j} \qquad \dot{d}_n = \dot{\lambda}_i^d \frac{\partial g_i}{\partial G_n} + \dot{\lambda}_j^d \frac{\partial g_j}{\partial G_n}$$

$$\dot{\lambda}^d \begin{cases} =0 & \text{if} \quad g<0 \quad \text{or} \quad \dot{g}<0 \quad \text{(no damage)} \\ >0 & \text{if} \quad g=0 \quad \text{and} \quad \dot{g}=0 \quad \text{(damage increment)} \end{cases}$$

where the "plastic" ($\dot{\lambda}^p$) and "damage" ($\dot{\lambda}^d$) multiplicators are calculated by the consistence condition.

In equation (3) only inelastic functions {f} and {g} must be identified from experimental results. This identification can be performed with civil engineering standard tests on beam-column joints.

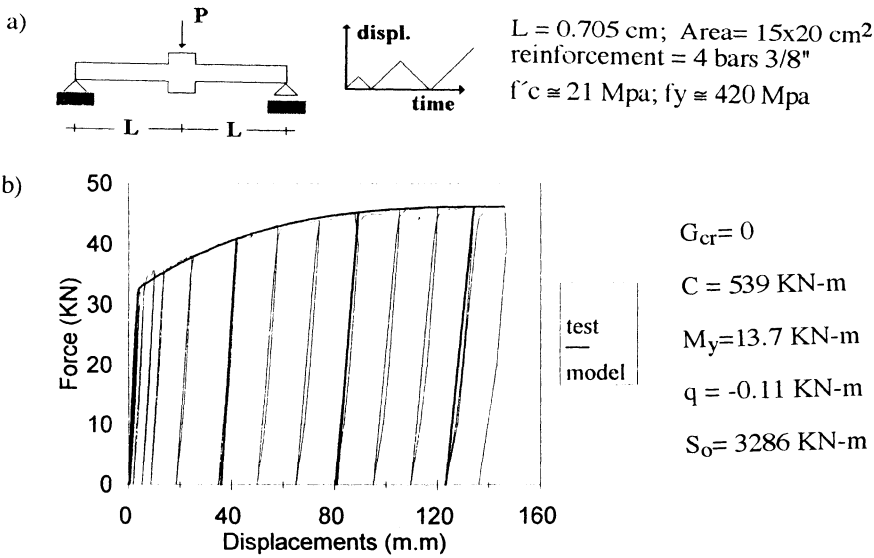

Figure 2 Test on a beam-column joint. a) specimen and loading b) experiment and numerical simulations.

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The expression of the inelastic functions for the particular case of negligible axial effects (i.e. $\delta^p = 0$ and $d_n = 0$) is proposed in references [3,4]. These expressions are:

$$f = \left| M - \left(\frac{1-d}{4-d}\right) c\Phi^p \right| + 4\left(\frac{1-d}{4-d}\right) M_y \qquad g = G - \left(G_{cr} + q\frac{\ln(1-d)}{(1-d)} \right) \tag{4}$$

where "c", "M_y", "G_{cr}", and "q" are constants that can be calculated as function of the cracking moment, the yield or plastic moment, the ultimate moment and the plastic deformation at the ultimate moment (see [4]). These properties of the member can be determined from classic reinforced concrete theory. Figure 2(b) shows the experimental results of a test performed in the specimen indicated in figure 2 (a). A numerical simulation of the test is indicated in figure 2(b).

MODELING OF OLIGOCYCLIC FATIGUE IN RC FRAMES

From equations (3-4), it can be noticed that a damage parameter "d" can evolve if and only if the actual value of the corresponding thermodynamic force is also the maximum value of "G" at this time of the loading (see [5] for a proof of this statement in the context of continuum damage models):

$$\dot{d}(t) > 0 \quad \text{only if} \quad G(t) = \sup_{-\infty \leq \tau \leq t}(G(\tau)) \tag{5}$$

where G(t) represents the actual value of the thermodynamic moment and $G(\tau)$ is the value of the same variable at a past time "τ". This means that in the model, it is assumed that damage is a function of the "maximum loading" represented by the Sup($G(\tau)$). This hypothesis is not valid if the behavior of the structure is studied during a long span of time (time-dependent effects, fatigue) or under a short cyclic loading where the maximum solicitation of the cycles is very close to the ultimate moment of the inelastic hinge (oligocyclic fatigue). In the latter case, the experimental evidence indicates that non-negligible damage and plastic increments occur before the actual value of "G" reach the Sup of "G" (see figure 3 and other experimental results reported in [6]).

The purpose of this section is the proposition of a model for oligocyclic fatigue based on the time-independent model presented in the previous section.

The new model is obtained assuming that: a) No additional internal variables are needed to describe oligocyclic fatigue. b) Plastic behavior is governed by the same evolution law and yield function of the previous model. c) Damage increment is zero during elastic unloadings d) Damage increments are possible during the loading phases even if the actual value of "G" is inferior to the Sup of "G".

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The new damage evolution law must be modified taking into account assumptions "c" and "d" but must give the same results of the previous model in the case of monotonic loading. This can be done following the procedure presented in references [7,2] that is briefly described in this section:
During damage evolution, the rate of damage has the following expression in the previous model:

$$\dot{d} = -\frac{1}{\partial g / \partial d}\dot{G} \tag{6}$$

During an elastic unloading this equations is, of course, no valid since it would imply a negative rate of damage. Relation (6) could be modified substituting $\dot{G}$ by the positive part of $\dot{G}$: $<\dot{G}>$. In this way, we obtain an evolution law that check the assumptions and conditions previously listed. However, it overestimates the damage rate for low values of the moment. Therefore the following damage evolution law for oligocyclic fatigue is proposed:

$$\dot{d} = -\frac{(A(G,d))^p}{\partial g / \partial d} <\dot{G}> \quad \text{if} \quad G \geq G_{cr}; \qquad \dot{d} = 0 \quad \text{otherwise} \tag{7}$$

$$\text{where } A(G,d) = \frac{G}{B(d)}; \qquad \text{and} \qquad B(d) = G_{cr} + q\frac{\ln(1-d)}{(1-d)}$$

The terms "G_{cr}" and "q" are the same parameters that were defined in the time-independent model described in the previous section. A new parameter "p" is included in the model in order to control the increment of damage during each cycle. It can be notice that the function A=A(G,d) derives from the inelastic function g(G,d) = G-B(d) defined in the previous model. As stated in the preceding section, the function "g" is always negative, therefore the function "A" can take values only in the interval [0,1]. This is why damage evolution (7) tends to model (3-4) when parameter "p" tends to infinite, i.e.:

$$\text{when } p \rightarrow \infty; \quad \dot{d} \cong 0 \quad \text{if} \quad G(t) < \text{Sup}\, G(\tau); \quad \dot{d} = -\frac{\dot{G}}{\partial g / \partial d} \quad \text{if } G(t) = \text{Sup}\, G(\tau)$$

The rate of damage in the phase of loading where G(t) < SupG(τ) increases when parameter "p" decreases. This parameter must be chosen to fit experimental results. Whatever the value of "p", there is no difference in the expression of the damage rate in both models during the phases of loading when G(t) = SupG(τ). Comparison between a numerical simulation with the oligocyclic fatigue model and experimental results is shown in figure 3. It must be underlined that all parameters of the new model, except "p", can be calculated for RC members of any shape or reinforcement since they are functions of the cracking, yield and ultimate moment; and the ultimate plastic rotation which can be calculated with the standard RC theory. The model can be extended to include unilateral effects due to the change of sign of the loading by the addition of a new set of damage parameters (see [1]).

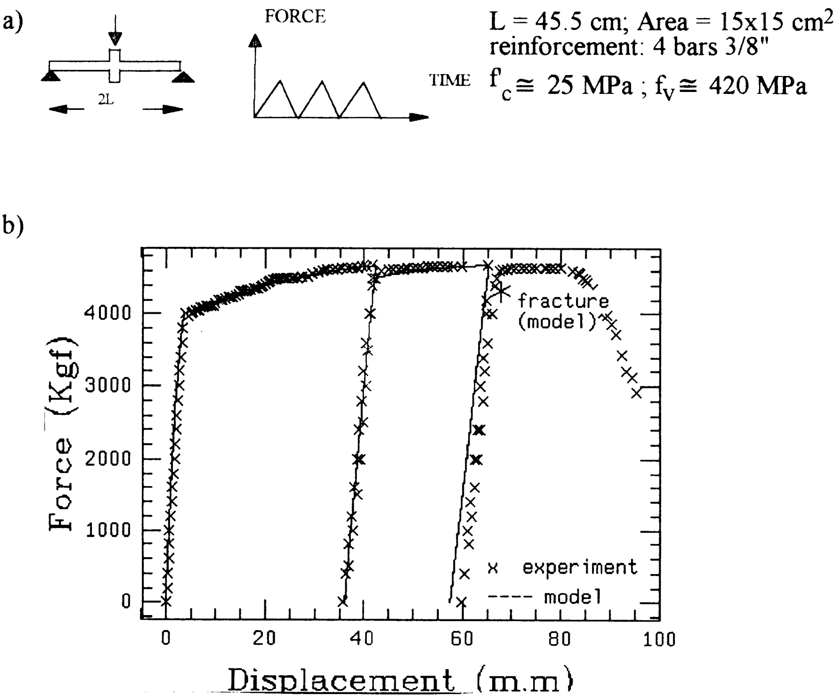

Figure 3 Oligocyclic fatigue test on a beam-column joint. a) specimen and loading b) experiment and numerical simulations.

CONCLUSIONS AND FINAL REMARKS

The framework presented in this paper introduces the methods of continuum damage and fracture mechanics in the analysis of nonlinear frames. The new model for oligocyclic fatigue reproduces qualitative and quantitatively the experimental behavior of RC members under oligocyclic fatigue. Most of the parameters of the model can be calculated from the standard theory of RC structures and this is very important in the practical applications. This model can be easily included as a finite element in the library of standard structural analysis programs.

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