The fiber/matrix interfacial stress on and beneath the surface of long-fiber-reinforced composites

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ABSTRACT

The numerical and analytical calculation of the fiber/matrix interfacial stress was carried out for a long-fiber-reinforced composite. By using the plane stress and plane strain conditions, the interfacial shear stress distribution was studied on the surface and in the interior of a tensile specimen. It was found that the interfacial shear stress increases with the fiber aspect ratio and decreases with the fiber volume fraction. For small fiber volume fraction, the interfacial debonding will occur preferentially inside the composite. For large fiber volume fraction, the result is reversed. The comparison between numerical results and analytical ones show that neglecting the fiber end stress concentration reduces the interfacial shear stress.

INTRODUCTION

For unidirectional composites reinforced with long fibers, one of the main mechanisms of fracture is the fiber fragmentation during loading in the fiber direction. In order to study this phenomenon and the ensuing interface debonding, a monofilament specimen is often used, with which one can measure the fiber fragment length. Using micromechanical models [1, 2], the average shear strength of fiber/matrix interface could be found.

To take into account the interactions between the stress fields around the fibers such as they exist in real composites, Wagner et al. [3, 4] have achieved a new approach by using a "micro-composite" in which the fibers are placed parallely on a given distance. Their results are much closer to the reality of genuine composites.

However, measuring the length of fiber fragments is impossible for real composites during mechanical tests. One can estimate the fiber/matrix interface strength from the pull-out length of fibers after specimen rupture [5].
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The failure mechanisms during loading is unknown in that case. By in situ tests, it is possible to follow continuously the damage mechanisms on the surface of the composites. However, the stress distribution around the fibers at the surface where a plane stress condition should prevail is considered to be different from the one inside, which can result in different failure processes. Indeed, Choi and al. [6] showed this effect by finite element computations.

In this paper, the stress distribution at the surface and in the interior of unidirectional composites are analyzed in a similar way by a finite element method to find if different failure mechanisms prevail, complementing those previous results [6] by an extension of the parameters studied: namely the fibers volume fraction. A model is also used for a comparison between numerical and analytical results.

FINITE ELEMENT CALCULATION MODEL

We are interested particularly in the debonding of fiber/matrix interface after fiber fragmentation. In Fig. 1 are shown elementary volumes for the surface and the interior of composites loaded in the fiber direction. The elementary volume of Fig. 1a is assumed to be in a plane strain condition, the stress state inside the composites. Figure 1b represents the stress state of the surface which is assumed to be in a plane stress condition. In this last case, the stress perpendicular to the surface (σ33) is zero.

The two-dimensional finite element model and its boundary conditions used are shown in Fig. 2. The finite element model represents only one-half of the composite volume element, in view of the geometric symmetry. A crack was introduced between two fiber fragments to represent the real condition of a composite damaged by fiber fragmentation. The model can be used both for the plane stress and for the plane strain elementary volumes. A periodic condition is imposed, which means that fibers and cracks distributions are periodic. The elastic moduli of fiber and matrix are also given in Fig. 2.

An important parameter is the fibers volume fraction which is equal to the ratio (r/R)² in the plane strain model and to r/R in the plane stress one, r being the radius as the half width of the fiber and R the radius as half width of the volume element.

PIGGOTT’S MICROMECHANICAL MODEL

Piggott’s model [7] was used for describing the stress fields in the composites reinforced with short fibers, especially the interfacial stress. According to this model, when a tensile strain ε₀ is applied in the fiber direction, the interfacial shear stress τᵢ is given by following expression:

\[ \tau_i = n E_f \varepsilon_0 \frac{\sinh(nx/r)}{2 \cosh(ns)} \]

\[ n^2 = \frac{2E_m}{E_f (1 + \nu_m) \ln(P_f / V_f)} \]
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Where \( s = L/2r \), \( L \) is the fiber length. \( P_f \) is the packing factor of fibers, \( \ln(P_f/V_f) = \ln(R/r) \) for hexagonal packing. \( V_f \) is the fiber volume fraction.

Note that the model is approximate. The stress transfer across the fiber ends and the stress concentration there are neglected. In general, the stress concentration increases greatly the interfacial shear stress at the fiber ends.

NUMERICAL AND ANALYTICAL RESULTS

Shear stress at the fiber/matrix interface
Along a line near and parallel to the fiber/matrix interface, the variation of the shear stress is plotted in Fig. 3. The distance between this line and the interface is 0.13r (2.64\( \mu \)m). This distance is chosen so as to avoid the stress singularity along the interface, and still keep representative results.

Several remarks can be made:
- the shear stress in plane stress is greater in the vicinity of the crack. Far from the crack, the two numerical curves coincide.
- near the crack, the stress obtained by the analytical model is less than those obtained by numerical calculations.
- the analytical model yields a stress transfer length at the interface much longer than the numerical ones.
- after the numerical calculations, the shear stress reaches its maximum value near the fiber end, but not at the end point, which is confirmed by other authors [8, 9]. This is different from the analytical results which give the maximum shear stress at the fiber end.

Thus, it appears that the interface debonding will take place, if possible, preferentially at the surface of composites.

Fiber aspect ratio and content effects
(a) Fiber aspect ratio \( L/d \) The variation of the maximum shear stress as a function of \( L/d \) is shown in Fig. 4 after both the numerical calculations and the theoretical analyses. It is found that the maximum shear stress increases with the fiber aspect ratio \( L/d \), and stabilizes when \( L/d \) is great (about 100). The analytical model gives a constant stress in the domain of \( L/d \) chosen. In the case of \( R/r = 2 \), the maximum shear stress obtained in plane stress is always larger than in plane strain.

(b) Fiber content Until now, all the results presented were obtained with the ratio \( R/r = 2 \), i.e. the fiber volume fraction is 0.25 in plane strain model. The interactions between the stress fields could affect the stress distributions in the vicinity of the crack. In Fig. 5 is shown the variation of the maximum shear stress as a function of \( R/r \). It is found that greater is the fiber content, smaller is the interfacial shear stress.

An important remark is that it exists a change of rupture mode. The two numerical curves intersect at the value of \( R/r = 3 \). So, if \( R/r > 3 \), the interface debonding will take place in the interior of the composite. If \( R/r < 3 \), the result is reversed. This critical ratio \( R/r \) should depend on the fiber content, the aspect ratio and the properties of fiber and matrix.
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In the case of short-fiber-reinforced composites, Choi et al. [6] found that the shear stress inside the composites is always greater than on the surface. But, for long-fiber-reinforced composites, the situation is not the same as that for short-fiber-reinforced ones, because a crack is introduced between two fiber fragments in the first case.

Longitudinal modulus of damaged composites
The Young's modulus is certainly diminished by the fiber fragmentation. In Fig. 6, the variation of the modulus of damaged composites is plotted as a function of the aspect ratio \( L/d \). The numerical results by finite element in plane strain are compared with the rule of mixtures. It is found that the modulus decreases greatly when \( L/d \) is small. This means that the influence of cracks on the modulus is important when the fiber fragments are short. With the increase of the fiber fragment length, the modulus obtained by numerical calculations approaches more and more to that obtained by the rule of mixtures. It appears that the influence of fiber fragmentation on the modulus is not important when the fiber fragment is long. Note that the rule of mixtures fails to describe the influence of fiber length on the modulus of composites.

Comparison in the case of same fiber volume fraction
Until now, the comparison is made for the given composite which has different fiber volume fraction on the surface and in the interior. If the fiber volume fraction is the same, we can obtain the results showed in Fig. 7. The interfacial shear stress is always more great in the plane stress model than in the plane strain one.

DISCUSSIONS

For the finite element calculations in plane stress, it was assumed that the fiber on the surface was rectangular object. In reality, the fiber on the surface has a round or half round cross-section and it faces the matrix on the inner side of the composite. The model is reasonable, however, in that the fiber and the matrix at the surface are largely governed by the plane stress conditions as compared with the case in the interior.

we can analyze the different stress transfer at the fiber/matrix interface. The cross section area for the plane strain model is \( \pi R^2 \), whereas that for the plane stress model is \( 2\pi R \) where \( t \) is the thickness of the model. The ratio of the interfacial area to the cross section of model is then \( S_i = \frac{2\pi L}{\pi R^2} = \frac{2tL}{R^2} \) for the plane strain model and \( S_S = \frac{2tL}{2\pi R} = \frac{L}{R} \) for the plane stress model. The ratio of interfacial areas for both models is \( \frac{S_i}{S_S} = \frac{2t}{R} \). So, if \( R = 2t \), the stress transfer along the interface for the plane strain model must be different from the plane stress model.

If \( R = 2r \), one should have the same stress transfer at the interface for both plane strain model and plane stress model. So, the damage mechanisms will be changed from the case where \( R < 2r \) to the case where \( R > 2r \). In our case, we found that the critical value of matrix radius is \( R = 3r \). This result is different from the above analyses. We think that this difference comes from the stress concentration in the front of crack which delays the change of rupture mode.
In the case of same fiber volume fraction on the surface and in the interior, we can obtain the relation of $S_i = 2S_s$. So, the interfacial shear stress obtained in plane stress model is always greater.

**CONCLUSIONS**

Stress fields and failure mechanisms have been investigated in long-fiber-reinforced composites damaged with fiber fragmentation.

- The interface debonding should occur in the interior of composite for small fiber volume fractions. Otherwise, interface debonding should begin at the surface.
- The interfacial shear stress increases with the fiber length. Thus, interface shear failures occur preferentially for longer fibers.
- The longitudinal modulus of damaged composites decreases greatly when the fiber fragments are short.
- Neglecting the stress concentration in the front of the crack reduces the shear stress evaluated at fiber ends.

**REFERENCES**


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Figure 1: Stress state in uniaxial tension

(a) Inside-plane strain, $\sigma_{33} \neq 0$

(b) Surface-plane stress $\sigma_{33} = 0$

Figure 2: Finite element model and the boundary conditions

- **E-glass fiber**: Young's modulus, $E_f = 74\text{GPa}$
  Poisson's ratio, $\nu_f = 0.22$
  diameter, $d = 20\mu\text{m}$

- **Epoxy matrix**: Young's modulus, $E_m = 2.8\text{GPa}$, Poisson's ratio, $\nu_m = 0.35$

$\delta$ is the imposed displacement
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Figure 3: Distribution of $\tau_i$ along the fiber length ($L/d = 50, R/r = 2$)

Figure 4: Maximum shear stress of $\tau_i$ as a function of fiber aspect ratio $L/d$ ($R/r = 2$)
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Figure 5: Maximum stress of $\tau_i$ as a function of $R/r$ (L/d = 50)

Figure 6: Longitudinal modulus as a function of L/d (R/r = 2)

Figure 7: Comparison for the same fiber volume fraction