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Surface reconstruction from discrete data obtained by a 3D surface measurement system

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Abstract

Two aspects of a surface are of interest: surface form and surface irregularity. The form is the geometry of an object. Superimposed on the form and usually of much finer detail is surface irregularity. The irregularity can be a combination of symmetrical and non-symmetrical deviations from nominal form and conclusions about the manufacturing process or the environmental and operating conditions under which a body has been used can be drawn from it. In this paper the reconstruction of surfaces for computer aided surface assessment (CASA) is discussed. Form characterisation of curved surfaces is outlined and the derivation and application of weighting functions is explicitly described. Form characterisation is the basis for CASA and is essential for dimensional surface assessment as well as for analysis of surface irregularity. Several examples are shown, where surfaces have been measured in three dimensions (3D) with a stylus-based Form Talysurf and then been analysed. The implementation of CASA into an automated system for surface assessment is subject to further research.

1 Introduction

Surface analysis is of interest to many areas of science and technology, giving a better understanding of manufacturing processes as well as functional performance. Often a distinction is made between the analysis of surface form for dimensional assessment and the analysis of surface irregularity.

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1.1 Surface form and surface irregularity

Surface form is the geometry of an object. Superimposed on the form and usually of much finer detail is surface irregularity. Irregularity is defined as a combination of symmetrical and non-symmetrical deviations from the nominal form of a surface. The term irregularity is used instead of waviness and roughness as they are defined ambiguously[1] which may lead to confusion.

1.2 Surface measurement methods

In this paper surface measurement is the simultaneous measurement of surface form and surface irregularity. This can be achieved by different systems, classified into two groups: contact systems and non-contact systems. Contact systems such as stylus instruments measure the surface directly. Non-contact systems on the other hand, obtain the values indirectly by measuring the optical path length or optical path difference and converting it into height values.

The surfaces analysed in a later section have been measured with a stylus instrument. The data, in a Cartesian co-ordinate system, is defined to a resolution of 10nm (1nm = 10^{-9} metre) in the z-axis and is based on a grid with spacing (δx, δy) typically between 20µm and 250µm. Up to 27,000 discrete points are recorded on the area under investigation, usually being not bigger than 1000mm^2.

2 Surface analysis and visualisation

A measured surface is commonly represented through discrete data. For surface analysis often a more compact representation is required, exhibiting only the feature of interest. Surface form can be represented by a best-fit through the discrete data. For the analysis of fine surface details, the surface can be decomposed into error types and so surface irregularity can be classified. Non-symmetric surface irregularities for example can be expressed in amplitude, spatial and hybrid parameters[2]. However, on curved surfaces there is a requirement to remove form before analysis of irregularities is enabled.

2.1 Surface form characterisation

Form characterisation is the reconstruction of a continuous surface from discrete data. This is achieved by minimisation of a residual error, usually defined as the difference between the measured surface and the best-fit surface. Different minimisation norms[3] are used and the two most widely applied are the L_2 and the L_∞-norm. For the L_2-norm, also known as the method of least squares, the sum of the squared deviations between the measured and best-fit surface is minimised. The L_∞-norm requires that the max. and min. deviations are equal and minimum. Depending on the minimisation norm the ideal parameters for a best-fit surface vary. Also, some norms are more sensitive to irregularities than others. The

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dependency of minimisation norm, best-fit parameters and surface irregularity has been investigated for spherical surfaces[4].

2.1.1 Methodology

The simplest data for form characterisation are measurements of plane surfaces. Form characterisation of discrete data from the measurement of spherical geometry is more complicated and many algorithms[5,6,7] have been developed for this task. Sphere fitting does not require evaluation of orientation parameters for rotational alignment. A sphere is symmetric about any axis passing through the centre and hence a sphere is fully described by its position and its form parameter (centre co-ordinates and radius). Aspheric surfaces can be rotationally symmetric, plane symmetric or non-symmetric and therefore do require the evaluation of orientation, position and form parameters for unique characterisation.

On relatively simple aspheric surfaces such as cylinders or cones it is possible to introduce additional parameters to accommodate orientation and position together with the form of the surface. Such an approach has been shown by Forbes[6]. A disadvantage of this method is the inflexibility of the algorithms to be transferred to form characterisation of aspheric surfaces other than cylinders or cones.

Sourlier[8,9] developed an algorithm that is independent of the surface geometry. The independence has been achieved by a separation of geometry from orientation and position. Limitations in Sourliers' approach are given through the restricted choice of algorithms and minimisation norms.

Separation of geometry from orientation and position can be achieved also by pre-processing. Pre-processing is used prior to and independent of form characterisation and aims to identify orientation and position of a surface. Hence, parameter evaluation is split into two tasks: one for evaluation of orientation and position with following alignment and one for characterisation of form. This approach has advantages as it enables the use of iterative or non-iterative algorithms as well as any minimisation norm. An algorithm for pre-processing of rotationally symmetric, aspheric surfaces has been proposed by Jung et al[10]. The methodology of characterisation of aspheric form using a pre-processing algorithm has been described in a previous publication[11].

2.1.2 Calculation of the residual error

Strictly, the residual error is defined as the shortest distance between best-fit surface and measured surface and is therefore taken in normal direction to the best-fit surface. Using explicit equations for fitting, it can be difficult to calculate the residual error and often the inexact definition as shown in eqn.(1) is used.

$$\|d_i\| = \|z_i - f(x_i, y_i, a_j)\| \tag{1}$$

In eqn. (1) d_i is the vertical distance of the measured surface z_i to the best-fit surface $f(x_i, y_i, a_j)$ at the position x_i, y_i. The best-fit parameters that need to be derived are represented by a_j. If a surface is comparatively flat and parallel to the

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xy-plane in a Cartesian co-ordinate system, eqn. (1) is a fair approximation for the residual error. However, if this condition is not satisfied, the residual error is calculated incorrectly and the deviation from the nominal value can be significant at places. The dilemma is related to what has been described before as a transformation error[11] due to data shift into a different domain (in that case a logarithmic domain). The transformation error has been compensated by introduction of a weighting function ($1/L_i$) and the same method is applicable for improvement of the incorrectly calculated residual error. Therefore eqn. (1) changes to:

$$\|d_i\| = \left\| \frac{1}{L_i} \cdot \left[z_i - f(x_i, y_i, a_j) \right] \right\| \tag{2}$$

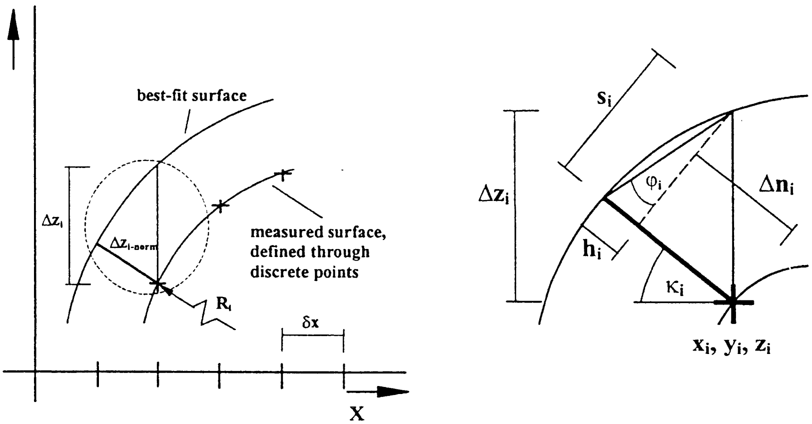

Figure 1: Derivation of a weighting function ($1/L_i$) for the improvement of an incorrectly calculated residual error Δz_i.

A weighting function can be derived through a trigonometric relationship. In *Figure 1* Δz_i is the incorrectly calculated residual error in the vertical direction and $\Delta z_{i\text{-norm}}$ is the corrected residual error in the normal direction to the best-fit surface. The elevation of the normal vector relative to the xy-plane is κ_i. The following trigonometric relationship can be derived between Δz_i, $\Delta z_{i\text{-norm}}$ and κ_i:

$$\Delta z_{i-norm} \approx \Delta n_i = \frac{1}{L_i} \cdot \Delta z_i = |\sin(\kappa_i)| \cdot \Delta z_i \qquad hence: \frac{1}{L_i} = |\sin(\kappa_i)| \tag{3}$$

The weighting function as proposed in eqn. (3) has been reported in literature[8,12] in this or in similar form (e.g. in vector form) and can be used satisfactory for calculation of the residual error on curved surfaces that do not exhibit a steep rise. However, from *Figure 1* it can be seen that $\Delta z_{i\text{-norm}}$ as defined

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in eqn. (3) is inaccurate by a distance h_i on curved surfaces, due to linearisation. This can cause Δn_i to underestimate $\Delta z_{i\text{-norm}}$, as shown in *Figure 1*, or in other cases it can cause an overestimation of $\Delta z_{i\text{-norm}}$. If higher accuracy is required it will be necessary to compensate for the error introduced by the linearisation, i.e. h_i must be added or subtracted to Δn_i and hence $\Delta z_{i\text{-norm}} = \Delta n_i \pm h_i$. A change in the weighting function allows to take h_i into account.

$$\frac{1}{L_i} = \left|\sin(\kappa_i)\right| \pm \frac{s_i \cdot \tan(\varphi_i)}{\Delta z_i} = \left|\sin(\kappa_i)\right| \pm \frac{\sqrt{\Delta z_i^2 \cdot \cos^2(\kappa_i)} \cdot \tan(\varphi_i)}{\Delta z_i}$$
$$\frac{1}{L_i} = \left|\sin(\kappa_i)\right| \pm \left|\cos(\kappa_i)\right| \cdot \tan(\varphi_i) \qquad (4)$$

The usage of eqn. (4) for form characterisation is difficult and non-trivial, especially if linear fitting techniques such as linear least squares are to be used. Initially the best-fit surface is not known. If the strict definition is applied that κ_i is defined through the normal vector to the best-fit surface, the fitting problem becomes in most cases non-linear. However, often it can be assumed that the measured surface is approximately parallel to the best-fit surface and hence the normal vector to the measured surface can be used as an alternative to define κ_i. The gradient and hence the normal vector to the measured surface can be calculated with standard methods, e.g. the central difference approximation[13]. A much bigger problem is the evaluation of φ_i. Assuming that a small region of a curved surface can be approximated by a best-fit spherical section, φ_i can be defined as shown in eqn. (5), where R_i is the radius of curvature of the measured surface at the point $P_i = (x_i, y_i, z_i)$.

$$2 \cdot \varphi_i = \arctan\left(\frac{s_i}{R_i \pm \Delta n_i}\right) = \arctan\left(\frac{\sqrt{\Delta z_i^2 - \Delta n_i^2}}{R_i \pm \Delta n_i}\right) = \arctan\left(\frac{\sqrt{\Delta z_i^2 \cdot \cos^2(\kappa_i)}}{R_i \pm \left|\sin(\kappa_i)\right| \cdot \Delta z_i}\right) \qquad (5)$$

Provided that the irregularities on the measured surface are comparatively small in magnitude, R_i can be derived by sphere fitting[4] of the points in the direct neighbourhood of P_i. Nevertheless, this still leaves two unsolved problems, the calculation of Δz_i and related to it the decision whether the positive or the negative sign in the equations is valid.

Despite those problems the improved weighting function can be used for form characterisation. With iterative fitting methods, the surface defined by the parameter estimates at the start of each iteration provides a reference to which Δz_i and hence φ_i can be calculated. Linear fitting methods can be modified to become "semi-iterative". A "quasi best-fit" surface is derived with the residual error as defined in eqn. (3) and this surface is used as a reference. In a second step this "quasi best-fit" surface is improved using eqn. (4).

For profile deviation measurement in co-ordinate measuring technology, the improved weighting function as in eqn. (4) might be valuable too and it is directly applicable since a reference surface for the calculation of Δz_i is available. Zhang and Gong[12] who derived a similar weighting function describe its value for the analysis of profile deviation measurements.

2.2 Surface decomposition into error types

Once a best-fit through the measurement of a curved surface has been derived it is possible to remove the underlying form and the analysis of the surface irregularity is enabled. The classification of the irregularity into error types provides feedback for the manufacture of a surface or allows detailed study of the operating and environmental conditions under which a surface has been used. Examples are given in a later section.

The error types can be a distinction between symmetrical and non-symmetrical irregularity or become more detailed. For optical surfaces for example the symmetrical irregularity can be separated into two parts, an aspherical and a spherical. The later is required for the calculation of the sagitta error[14]. The non-symmetrical surface irregularity can also be sub-classified for specialist assessment. On very smooth surfaces long scratches are identifiable and can be classified by length, width and number of scratches on the surface[15]. Similar procedures are possible for classification of coating errors (optical or painted surfaces), holes (woven reinforcement fibres) or other imperfections. The degree to which it is meaningful to characterise surface irregularity into error types might be different from application to application. Existing standards can easily be incorporated. The methodology of decomposition of surface irregularities into error types is discussed in more detail in another paper[16].

2.3 Intelligent system for automated surface assessment

Surface form characterisation involves the selection of an algorithm and surface function and also the selection of an appropriate minimisation norm and weighting function. The best-fit surface can be quite different depending on the choices made. Hence, many surfaces can be computed each being in their own definition a best-fit surface to the same set of discrete data. This is not an ideal situation and makes comparisons of best-fit surfaces difficult or meaningless. The dilemma has been identified also by other researchers. Brown[17] recommends the development of standardised inspection techniques and Varghese et al[18] concludes that surface metrology would gain from a system that can automatically select the required analysis tools, whenever a surface needs to be assessed.

It has been shown for spherical surfaces[4] that a spectral analysis of an initially computed surface irregularity can be used for the selection of the minimisation norm. Algorithm and weighting function can be selected from the surface form that needs to be fitted. An intelligent system automatically selects the required analysis tools by such criteria. To complete the system, form and surface error characterisation need to be combined with data manipulation routines such as slicing of surfaces, area and volume calculations, alignment of surfaces for comparison, etc.. Furthermore, visualisation of the measured data

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and the surface error types, and standardised test protocol generation is essential for communication between different parties, i.e. manufacturer and customer.

3 Computer aided surface assessment (CASA)

CASA is the analysis of a surface, defined through discrete points, in a computer software package. In the following sections three examples are presented from different fields of engineering science. The surface measurements have been taken on a stylus based Form Talysurf. The analysis and generation of figures has been carried out using a surface analysis software[19] developed at the University of Southampton, Department of Mechanical Engineering.

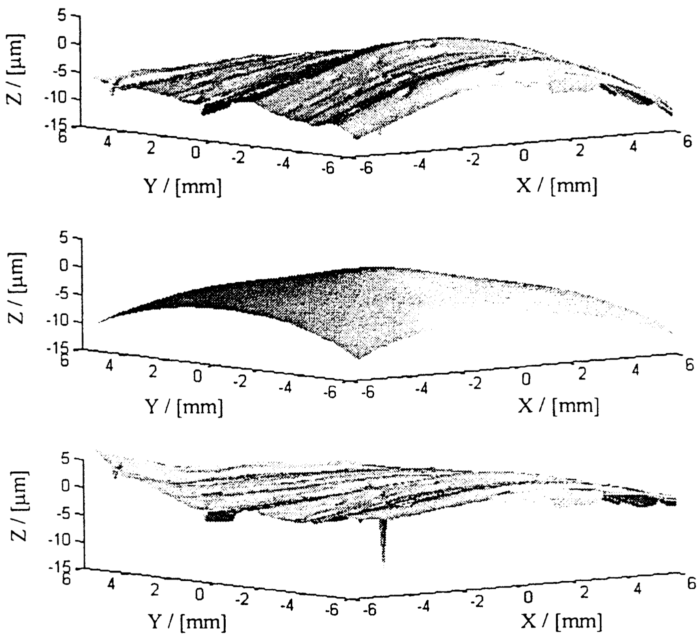

Figure 2: Classification of the total surface irregularity (top) of an aspheric lens into symmetrical (centre) and non-symmetrical (bottom) surface irregularity.

3.1 Aspheric lens surface

In *Figure 2* the surface irregularity of a diamond turned aspheric lens is shown. The irregularity has been obtained by removal of the best-fit form and is further

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decomposed. The symmetric irregularity often can be related to wrong tool settings and be useful for the calculation of a tool path correction. The non-symmetric irregularity on the other hand shows fine surface damage in form of scratches and craters.

The surfaces as shown in *Figure 2* are defined on a grid of 64 by 88 points in the xy-plane. Grid spacing in is $\delta x = 197\mu m$ and $\delta y = 140\mu m$. The asymmetric irregularity has a standard deviation of $\sigma = 1.35\mu m$.

3.2 Eroded electrical contact

Arc erosion in electrical switching contacts affects the reliability of the contact interface and hence the reliability of the switch itself. The mass change together with a 3D profile of the surface irregularity can be used to describe the condition of a contact. On curved contact surfaces there is a requirement to remove the form in order to evaluate the mass change[20]. *Figure* 3 shows the surface measurement of an electrical switching contact (cathode) after 4000 break only operations, switching a 14.7A current (240V, 50Hz). Opening velocity of the switch has been 0.1m/s and opening occurred at 1ms relative to the current zero. Also shown is the surface irregularity after removal of the best-fit sphere.

The surfaces as shown in *Figure 3* are defined on a grid of 90 by 90 points in the xy-plane. Grid spacing is $\delta x = \delta y = 25\mu m$. Form characterisation by minimum zone sphere fitting evaluated a radius of $R = 2.213mm$ to be used for form removal.

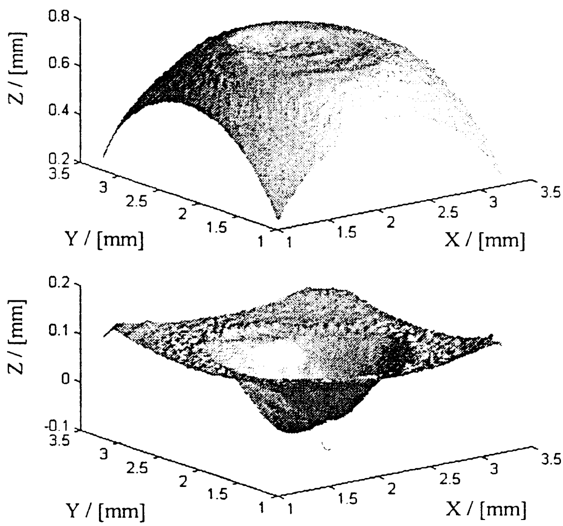

Figure 3: Measured surface of an eroded electrical switching contact (top). Form fitting followed by form removal enables analysis on the surface irregularity (bottom).

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3.3 Woven reinforcement fibre

Analysing a surface measurement of woven reinforcement fibre is different to the other two examples. The spatial distribution of woven reinforcement fibres can be assessed without need for form characterisation. The spatial distribution, which is affected by the style of weave, is important in the manufacture of composite materials to estimate the resin flow rate and for the prediction of the mechanical properties of the cured composite material. A contour analysis of a measured surface can be used to determine the proportion of resin or fibre at different levels along the vertical axis (z-axis). Numerical integration methods allow the calculation of resin or fibre volume and numerical differentiation methods can be applied for the calculation of surface areas. CASA can therefore simplify the assessment of a surface as shown in *Figure 4*.

The surfaces in *Figure 4* are defined on a grid of 164 by 164 points in the xy-plane. Grid spacing is $\delta x = \delta y = 125\mu m$.

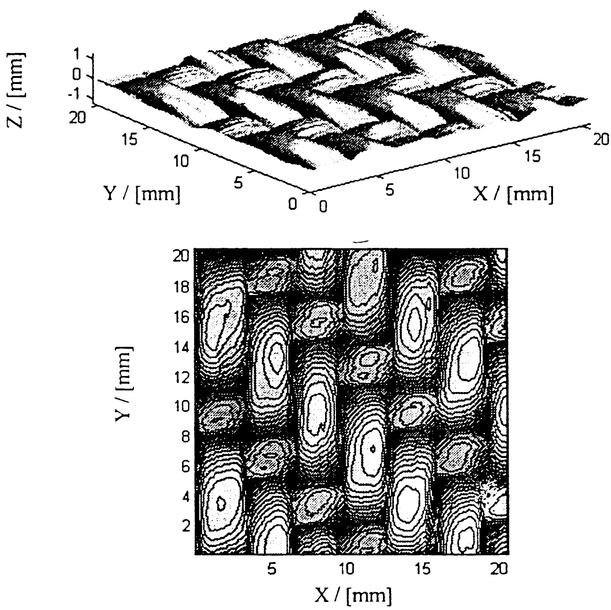

Figure 4: Surface measurement of a woven reinforcement fibre (top). Contour analysis of the surface (bottom) can be used to determine the proportion of resin or fibre at different levels along the vertical axis (z-axis).

4 Conclusions

In this paper the characterisation of surface form has been described. Characterisation of surface form is important for dimensional assessment of surface geometry and for form removal on curved surface measurements, enabling the characterisation and analysis of surface irregularity. Sometimes weighting functions are used for form characterisation. The derivation of a weighting function for the improvement of an incorrectly calculated residual error is described in detail. Such a weighting function can in many cases simplify form characterisation with explicit surface functions.

Computer aided surface assessment (CASA), to which surface form and surface irregularity characterisation is the groundwork, has been introduced. The next step in the development of CASA is the upgrade to an intelligent and standardised system for automated surface assessment. Such a system would ensure comparable results. Thoughts for its realisation have been presented, e.g. the use of spectral analysis of the surface irregularity for selection of an appropriate minimisation norm. However, the full development is subject to further research.

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