Isolating and quantifying tooth deformation in worm gear systems operating under a torque load

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Abstract

The positioning error value at any point of contact in a gear set is known as the transmission error. Changes in transmission error during the engagement cycle have been identified as the source of vibration in an operating system. To predict the extent of these vibrations for a given application it is necessary to model the various effects which define contact conditions. It is possible to acquire data to model some effects by direct measurement. Gear tooth deformation due to a torque load has been difficult to quantify under dynamic conditions since the harsh operating environment excludes the mounting of delicate equipment used in data acquisition such as strain gauges. Also, a substantial quantity of data must be gathered simultaneously to carry out a full analysis. This paper describes the monitoring, recording and post processing techniques used to isolate individual components of the composite transmission error signal. A test rig has been developed to collect transmission error data from a worm gear system operating under load. Measurements from several test gear sets are used to determine the tooth deformation due to a torque load and the wave form generated through an engagement cycle.

1 Introduction

Transmission error is a continuous graphical output of gear set positioning accuracy during a meshing cycle. Noise and vibration characteristics derived from this\(^{[1]}\) are often assumed to exhibit complex or random behaviour in
industrial gear box installations due to simultaneous contributions to contact from several different sources. This paper describes how the dynamic behaviour of a system can be isolated if all the constituent influences are monitored. It further describes how experimental results have been applied to develop a model simulating dynamic behaviour under an applied load to aid the design and development of future worm gear applications.

2 Factors Determining The Output Error Signal

2.1 Worm Thread Geometry And Wheel Cutting Tool Design

Conjugate gear design is used to determine gear set capacity and performance characteristics. This assumes an ideal contact in which the gearing ratio is maintained throughout the engagement cycle and every potential point of contact on the component surfaces is utilised. The conjugate worm thread and wheel tooth surface are easily calculable using standard equations defining geometry and the ideal contact relation. In industrial practice the profile of the worm gear wheel cutting tool is modified based upon small variations in the worm thread design parameters. Further changes can be induced through the setting of the wheel cutting tool axis and changes to the nominal centre distance. Wildhaber[2] stated that the changes are known collectively as ease-off or mismatch. These design methods deviate from the ideal relation creating non-contacting points on the component flanks with an associated clearance value. This clearance permits small increments of movement which induce position error during contact of the mismatch surfaces. Recent literature from several independent sources[3][4][5] has demonstrated independently that the dynamic contact relation of the mismatch surfaces can be calculated. The contact position established by the modified geometry is related to the conjugate contact position to determine a position error. A series of position error values through the engagement cycle generates a transmission error signal.

2.2 Surface Contact Error

Machine tools now permit the technique of using a digital control system to grind a specified profile directly onto the worm flank or wheel cutting tool profile which is independent of the basic geometric form. The modified profile alters the thread or tooth form and consequently the new dynamic relation can create further variations in contact characteristics.

Surface generation inaccuracy caused by worm flank represent the equivalent of additional profile modification and create clearance variation accordingly. Wheel cutter profile errors are transferred directly to the wheel tooth surface and therefore create clearance in the same way.
2.3 Engagement interval

Lead errors in the flank of the worm and wheel tooth pitch spacing errors are generated by transmission error within the manufacturing machinery used to generate the component. These sources determine errors in the point of tooth pair engagement which can advance or retard the wheel position contributing directly to the position error value. Though the development of CNC machine tool technology has reduced the errors to a few microns, they are still of the same order of magnitude as transmission error magnitude in precision worm gear systems and should therefore be considered relevant.

2.4 Axis Alignment

Misalignment can be created during assembly by machining tolerance limits in the reference surfaces of the gear set components. Operation under load can create a similar effect through movement due to compression of the bearings or distortion in the housing. Each of these contributes to centre distance and height variation which change clearance values between the engaging flanks independently of the theoretical form or generation errors. Changes in the alignment of the gear set in the box can cause small changes in transmission error value as this redefines points of contact between the surfaces during engagement. Consequent changes in clearance influence the lubrication conditions and surface contact area which determine operating potential.

2.5 Tooth Deflection Due To Load

There are two elements which combine to produce deflection due to an applied load. The worm thread and gear tooth will undergo bending and elastic deformation due to surface compliance. Both effects move the point of contact at each position of the meshing cycle and allow the wheel to move against the direction of drive. By convention this is defined as a negative error value and is directly additive to the position error established during engagement under no load. Munro[6] illustrates that the composite transmission error signal is analogous to an electrical signal output. The tooth to tooth transmission error created by contact clearance can be considered an AC signal, whereas the mean deformation and deflection from a datum position is constant for a given load and is considered the DC level.

3 A Test Rig To Record Transmission Error Under Load

A test rig was constructed to examine dynamic behaviour under load. This comprised of a 4kW motor with a 0-350 rpm input shaft speed driving a test box. The output shaft of the box was connected to a helical step-up gear system and a gear pump circulating oil which acted as a hydraulic braking mechanism. Load could be varied over a 0-2400Nm range using a restrictive pressure valve within
the oil circulating system. Several test gear sets comprising of a steel worm and phosphor bronze wheel were manufactured based on a 50:1 ratio to operate at 6" centres. The components were manufactured to BS721 Class A standard with profile and lead generation errors to within 4μm. These test gear sets were mounted in a test gear box as in Figure 1.

![Diagram of probe points monitoring eccentricity in the assembly and test box movement under load.](image)

Digital encoders were mounted on the input and output shafts of the test box and configured to give a resolution of 0.3 arc sec wheel position equating to 0.2μm of tangential worm profile translation. Linear inductive probes were used to determine axis alignment deviations. In Figure 1 probes 1 and 2 detect movement in the worm reference bands, probe 3 detected equivalent movement in the wheel reference band. Probe 4 monitored the absolute movement in the worm shaft axially, probe 5 indicated the movement of the worm housing while probe 6 monitors the position of the wheel housing. A similar bank of probes perpendicular to these were positioned to monitor lateral shift in the assembly.

4 Processing The Composite Transmission Error Signal

Using the test rig the position error, e, can be recorded continuously over one forward and reverse cycle of the wheel to generate the composite backlash transmission error signal. The raw data signal recorded by the encoders is in fact an amalgam of several effects. It is necessary to remove various contributions in order to isolate the error due to tooth contact only.
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Error:
- Due to surface contact $e_c$
- Due to backlash $e_B$
- Due to axial displacement of the worm $e_u$

Total composite transmission error at torque load $M$:

$$e = e_{c(M)} + e_{c(0)} + (e_{B(M)} - e_{B(0)})/2 + (e_{u(M)} - e_{u(0)})/2$$

Some values can be measured directly while others are calculated based on data from the encoder readings. Though the axial position can be varied by assembly tolerances, by defining the $e_{u(0)}$ value as the datum axial position this is automatically removed from the equation. The $e_{u(M)}$ value can be derived by probe measurement readings. The net worm movement can be calculated by subtracting the absolute worm movement from the relative movement in the housing. This must further be related to any shift in the datum position of the wheel axis. It is possible to determine $e_{B(0)}$ using encoder data recorded while the gear set is under no load by detaching the test box from the brake mechanism. An equivalent record of the output signal under load $e_{B(M)}$ is used to quantify the movement from the backlash datum due to tooth deformation. Also, the $0$ frequency contribution in a Fourier analysis spectrum of the signal represents the $e_{B(M)}$ value. Removing the known quantities from the raw measurement data leaves the signal contribution from tooth contact:

$$e_{c(M)} + e_{c(0)} = e + (e_{B(0)} - e_{B(M)})/2 + (e_{u(0)} - e_{u(M)})/2$$

The graphs in Figure 2 show examples of this for wave forms recorded by the experimental test rig driving forward and reverse under loads of 0, 1000, and 2000 Nm produced in this way.

5 Fourier Analysis Of The Transmission Error Signal

A frequency spectrum of the transmission error signal can prove a powerful analysis tool in the investigation manufacturing capability or operating errors such as noise and vibration in industrial applications[7]. An example of this can be demonstrated using the experimental results shown in Figure 2b. A significant interference frequency effect was observed in the higher load level measurements. The frequency spectrum of the 2000Nm signal recorded by the test rig is shown in Figure 3. This indicated a peak at a frequency of 49 cycles per wheel revolution and associated harmonics. The gear pump producing the braking torque was known to be operating at a frequency of 49.07 relative to the test wheel rotation. Force variations in the applied torque load were suspected as producing the effect. Closer examination confirmed the presence of small axial position variations of the worm induced by the gear pump at a frequency of approximately 49 cycles per revolution.
Figure 2a: The test rig results of transmission error due to contact only recorded at several load intervals with wheel driving in the forward direction.
Figure 2b: The test rig results of transmission error due to contact only recorded at several load intervals with wheel driving in the reverse direction.
Simulating the Tooth Contact Transmission Error Signal

Measurements from a control gear set were recorded under load. Shaft movement and the interference from the braking system were removed. Figure 4 shows the 2000Nm load signal before and after mathematical filtering the 49 cycles per revolution frequency and subsequent harmonics identified in Section 5.

![Original and Filtered Transmission Error Wave Signals](Image)

**Figure 4**: The original and filtered transmission error wave signals recorded for the control gear set under 2000Nm load.

With all external influences removed the total error in the data due to load is the sum of $e_{c(0)}$ and $e_{c(1)}$. The resulting target wave forms can then be used to develop a model simulating the transmission error signal. Research has shown that the $e_{c(0)}$ characteristics for a specified mismatch design can be modelled to a high degree of accuracy by including manufacturing error and alignment error[8]. Work on spur and helical gearing has confirmed that deformation due to load can be simulated by a simple linear stiffness model[9][10]. This technique suggests that clearance changes at each point on the contacting flanks caused by deformation are equivalent to a series of springs of constant stiffness producing a total resistive force equal to the applied load. Tooth dimensions such as height and width can be used to modify the basic linear function.
It possible to apply the same principles to worm gearing. A constant stiffness value function, $e_{\phi|\theta}$, was determined which optimised the correlation of calculated and measured $e_{B|\theta}$ value identified in Section 5 while retaining the characteristics of the target wave form at each load interval. Tests were performed on several gear sets of differing designs operating under various speeds, loads and axis alignment. In each case the contact model of transmission error under load was found to simulate actual mean deflection to 5\(\mu\)m and wave form to within 2-3\(\mu\)m. A comparison of synthesised and measured transmission error derived from an analysis of a gear set design with profile modification is shown in Figure 5.

![Figure 5](image_url)

Figure 5: A comparison of synthesised and measured transmission error for a test gear set with gear tooth profile modification operating under 0-2000 Nm load in 500 Nm intervals.

MS-DOS based software was developed to perform the calculations operating on an analysis cycle of approximately 20 seconds on a PC equipped with a *Pentium* I processor and 16Mb RAM. This allows gear designers to optimise contact conditions for a specific application by establishing the associated design parameters and machine settings before manufacture.
7 Conclusions

When considering the analysis of tooth deformation effects it is important to quantify all potential influences on error signal. When the transmission error signal due to this component is isolated it can be a significant investigation and development. A frequency spectrum of the transmission error signal identifies potential manufacturing and operating errors. Dominant frequencies from tooth contact under no load will be present in any dynamic system incorporating the gear set as a multiple of the operating speed. Experimental results show that it is necessary to also consider input interference from the loading mechanism to accurately simulate the noise and vibration effects in the system.

It is possible to accurately model the transmission error due to gear tooth contact and deformation using a linear stiffness model. Calculations can be carried out fast enough to permit optimised design for a specific application by iterative use. Analysis could be used to improve positioning accuracy under various operating conditions. Further, results could be used as one element of a more complex model containing a series of mechanical systems[7].

8 Acknowledgements

This paper has been produced with funding and resources from the following academic and industrial establishments:

Department of Trade and Industry, Gear Research Foundation, Engineering and Physical Sciences Research Council, Holroyd, David Brown (Radicon) Ltd, Renold Gears Ltd, Highfield Gears Ltd, Express Evans Ltd.

9 References


