



# **Kinematic errors in precision worm gears**

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## **Abstract**

This paper describes the measurement of error sources in a worm gear set and assesses their effects when developing a computer model to synthesise behaviour. To do this several test gears have been made and the process monitored by controlled measurement at several stages using some of the most sensitive and accurate equipment available. A computer program has been written to model the characteristics of these worm and wheel gear pairs under a range of conditions. The predictions have been validated using a test rig measuring rotary accuracy of the input and output shafts under a torque load.

## **1 Introduction**

Precision worm gears are widely used in machine tools, rotary tables and robots. Normal positioning accuracy is around 7-15 arc secs. but this may be influenced by design, production and operating conditions.

The geometry of this type of gearing demands many hundreds of calculations for even a basic analysis. When combined with the sensitivity of the errors relative to working tolerances this has made even subtle changes in design difficult to predict, frequently causing precision worm and wheel sets to be produced as a unique pair. Previous analysis of worm gear design has been carried out in the first half of this century by Merritt[1], Tuplin[2], and Buckingham[3] and more recently by Dudley & Poritsky[4] but performance has been assessed on an empirical basis with results frequently undocumented.

Improvements in computing power and tooling technology within the last decade have allowed a more controlled study to be carried out. A program of research has been established at the University of Huddersfield to take advantage of these developments. The aims are to understand the influence of design on worm gear characteristics, identify and quantify potential error

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sources, and to create a computer model to predict behaviour under a given set of conditions.

### 2 Transmission Error

Assessment of a worm gear set is generally made by inspection of the contact marking pattern indicating that entry and exit conditions are sufficient for oil lubrication of the surfaces. For high accuracy applications this method is not sensitive enough since satisfactory conditions can be achieved several ways with different accuracy on each occasion.

The concept and measurement of transmission error was developed by Harris and Munro[5][6] in the 1950's to 1960's for spur gears but can be applied to any form of gearing. It is defined as the error in output position of a gear shaft for a defined position of the input shaft with respect to the gearing ratio. Measurement of this can be achieved by mounting encoder gratings to the input and output shafts of a test gear box and linking these to a computer. A rotary accuracy of 0.3 arc secs. is currently achievable by this method.

Though transmission error is a measure of the positioning accuracy of a gear set research has shown that this positioning inaccuracy can be linked directly to noise and vibration [7]. In order for improvements to be made in any of these areas it is important to establish the contributions from the individual elements of worm gearing.

### 3 Sources Of Error

#### 3.1 Design

Design is based around modification of the gear wheel cutter known as mismatching. Lead, cutter radius and involute profile relative to the worm dimensions are all parameters which can be changed, while the cutting axis is slightly inclined relative to the operating axis. If no mismatch were to be applied, the cutting tool would be a copy of the worm thread and the contact between worm and wheel defined as conjugate. In practice a controlled design mismatch is essential to allow oil lubrication of the contact surfaces, but in doing this any worm gear design induces transmission error.

For analysis purposes the worm gear set is reduced into several sections of equivalent rack and pinion as shown in Figure 1. For each rack section the difference in conjugate and mismatch thread position along the path of contact can be calculated. This is the relief value and represents the amount of metal which would be removed relative to a conjugate gear tooth as shown in Figure 1 for a typical section for an arbitrary angular position. Calculating relief values in each rack section at a series of angular positions through the cutting cycle a topological relief map can be compiled over the gear tooth as in Figure 2.

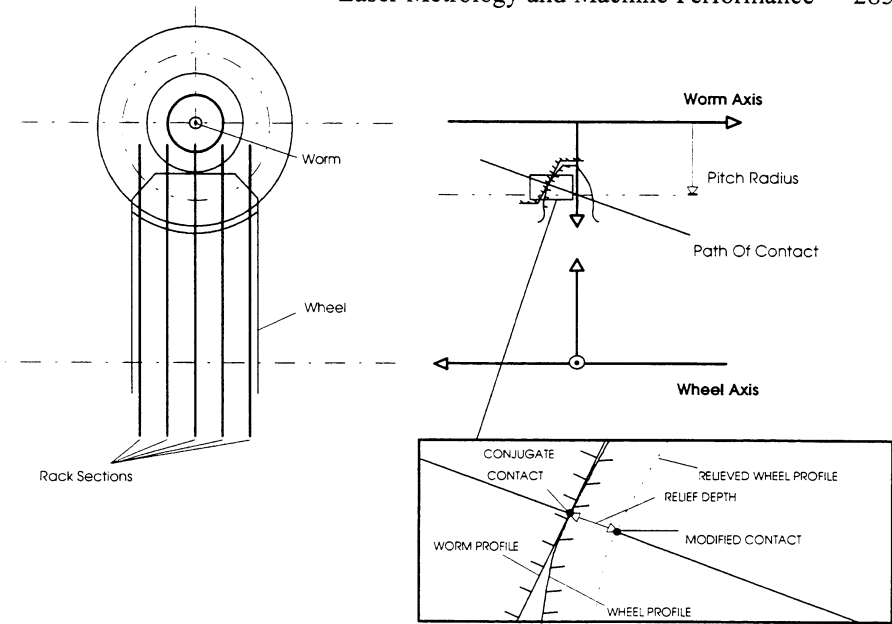


Figure 1 : A diagram showing typical rack sections through a worm gear set and the effect of relief in a typical section.

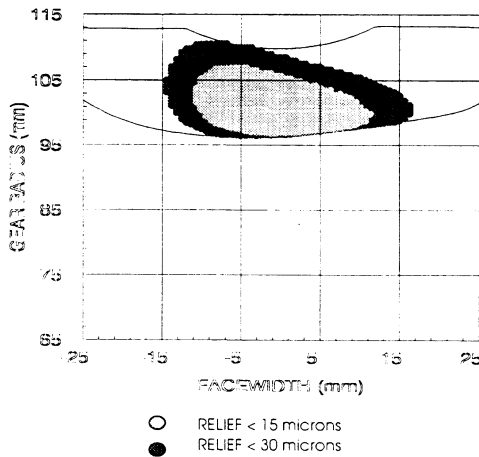


Figure 2 : An example of a topological relief map with two contour depths for a specimen gear design.

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At each angular position the rack section with the minimum relief will be in contact and the associated modified contact position becomes the transmission error. By calculating this effect at a series of angular positions through the tooth engagement a transmission error curve can be compiled.

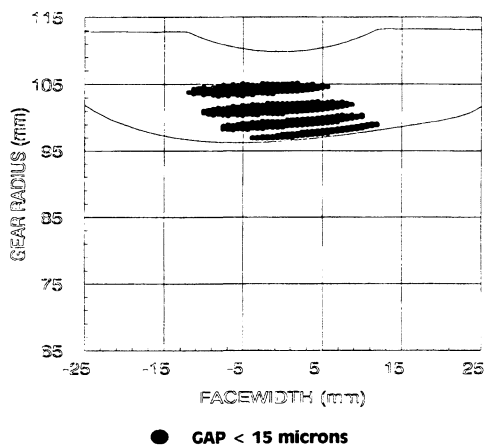


Figure 3 : An example of worm and gear flank gap calculations in four angular positions for a specimen gear design.

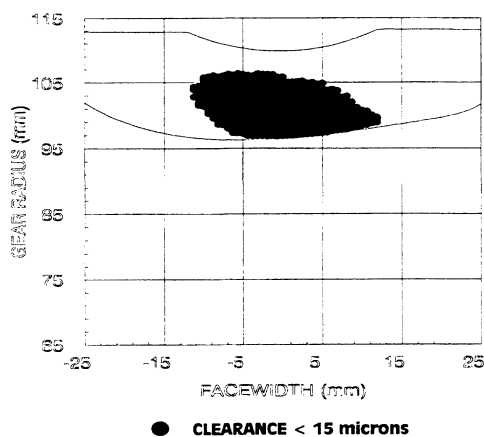


Figure 4 : An example of a clearance diagram for a specimen gear design.

Once the transmission error has been established the contact conditions can be examined. For any angular position the gap between the mating flanks can be calculated. An example of this is shown in Figure 3 with the gap calculations performed in four angular positions.

By compiling a series of these calculations through the meshing cycle the minimum gap, defined as the clearance, can be found at each point on the gear tooth surface. Clearance diagrams show areas of the gear tooth with a specified value of clearance to assess the lubrication potential. An example is shown in Figure 4. This is the equivalent of performing an accurate contact marking test. Varying the mismatch parameters alters the extent and position of the relief and consequently the contact. The intention is to concentrate contact in a particular area of the gear tooth.

Production of precision worm gear sets is a series of compromises between adequate lubrication conditions and rotary accuracy. Recent work by Janninck[8] and Colbourne[9] has examined the way in which contact can be altered for a particular application. By varying the mismatch parameters theoretical transmission error amplitudes of 1-2 microns can be achieved while still allowing satisfactory contact conditions.

### 3.2 Manufacturing

The final accuracy of the gear set depends upon the ability of the machine tools to reproduce the theoretical design.

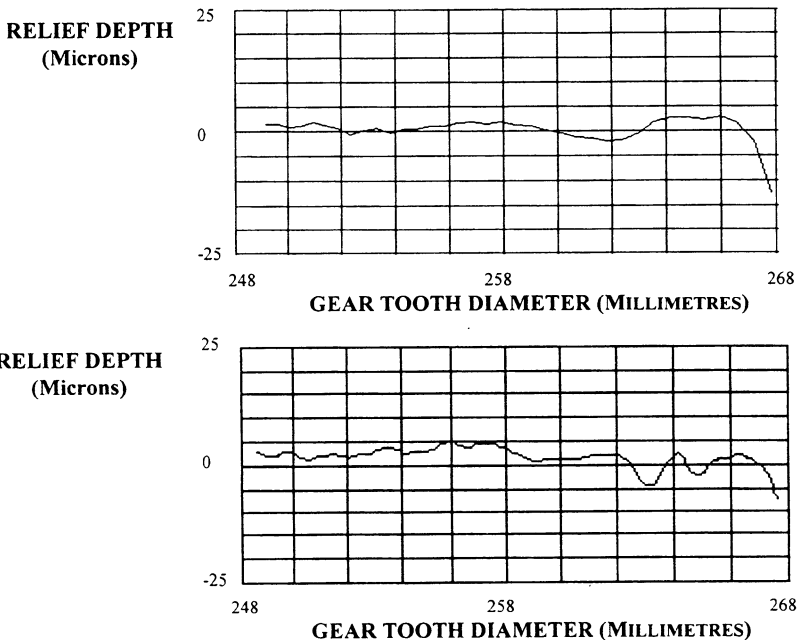


Figure 5 : A comparison of measured and synthesised gear wheel profile errors from a cutting tool produced using a standard cutter grinder machine.

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An example of this is the profile error in the gear cutting tool. In Figure 5 synthesised gear tooth profile errors (A), calculated using measured gear cutting tool profiles, are compared with the profile errors from a completed gear wheel cut with this tool (B). This shows that these are transferred directly onto the wheel profile to within 1-2 microns. In extreme cases profile errors can dominate the contact in turn causing transmission error. Clearly a more accurate profile is preferred. A standard cutter grinding machine can produce these profiles to within 10 microns of the original form. CNC machine tools have been able to improve on this producing profile generations to within 2 microns. In Figure 6 examples of profile errors on cutting tools produced using the cutter grinder and a CNC machine are shown.

Other main error sources are in the worm lead which determines the helical shape of the worm, and wheel tooth pitch which defines the angular spacing. These are often caused by errors from the machine tool gearing which determine the feed ratios. The worm form is generally a ground finish allowing high profile accuracy to be achieved with the lead error potentially held to within 4 microns per 100 mm of thread. Pitch errors can be held to within 3 microns between adjacent teeth.

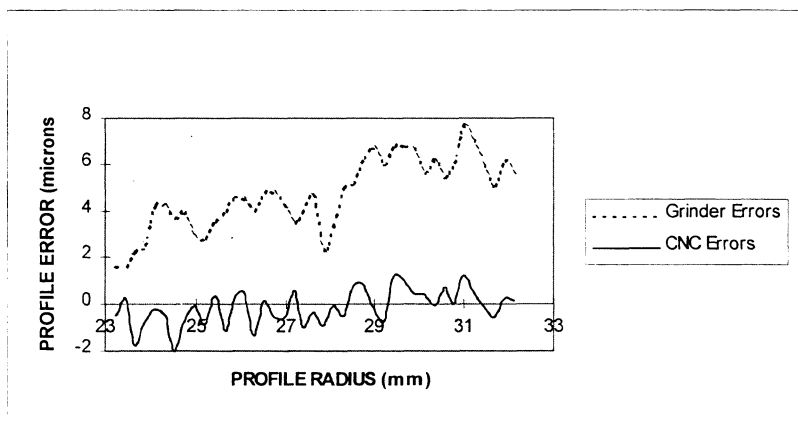


Figure 6 : A plot of profile errors from gear cutting tools produced using a standard cutter grinder machine technique and a CNC machine tool.

### 3.3 Assembly

The alignment of the gear set in the box can cause small changes in transmission error value, but more critically it is influential over the inlet and outlet contact conditions. Misalignment is caused by machining tolerances in

the reference surfaces, or by radial movement in the bearings causing eccentricity in the machined reference bands of the worm and wheel components. Linear inductive probes can be used to determine this effect. In Figure 7 probes 1 and 2 detect movements in the worm reference bands, probe 3 detects movement in the wheel reference band. A similar bank of probes perpendicular to these monitor lateral shift in the assembly.

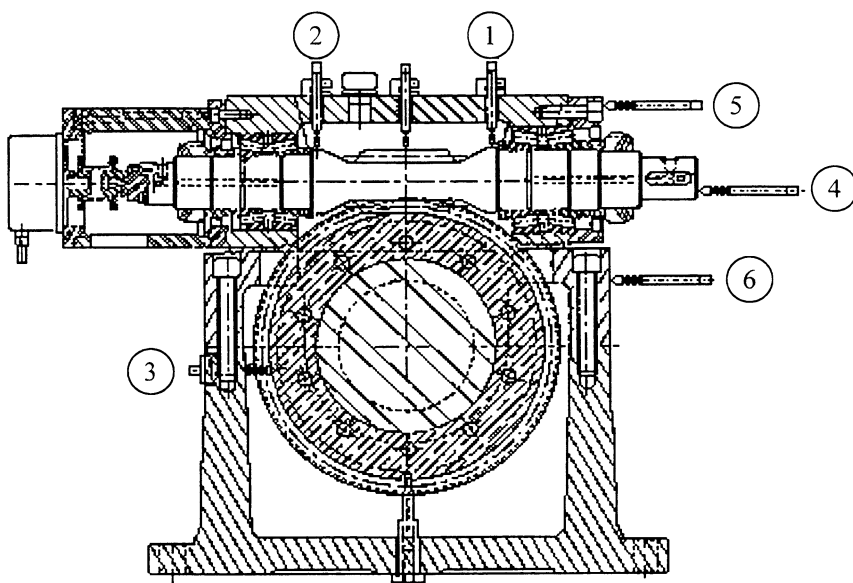


Figure 7: A diagram of probe points monitoring eccentricity in the assembly and test box movement under load.

### 3.4 Deflection

Tooth deformation by compression and bending occurs when a torque load is applied to the gear system. This modified tooth form alters the contact point and induces transmission error directly. Movement also occurs axially in the bearings and in the gear box structure. The shift in position of the worm with respect to the wheel contributes directly to the transmission error detected by the measuring encoders, however probe readings can be used to evaluate this effect. In Figure 7, probe 4 monitors the movement in the worm shaft axially, probe 5 indicates the movement of the worm housing while probe 6 monitors the position of the wheel housing. The net worm movement can be calculated from these readings, and when subtracted from the total transmission error isolates the tooth deformation effects.

## 4 Composite Model

A computer program was developed to calculate the effects of parameter variation on worm gear design. This was then extended to allow analysis including error sources. It was then used to predict the characteristics of several production gear sets all based upon a 152 mm centre and 50:1 ratio design.

The graph shown in Figure 8 is the computer prediction of theoretical transmission error based upon the calculations from one design specification.

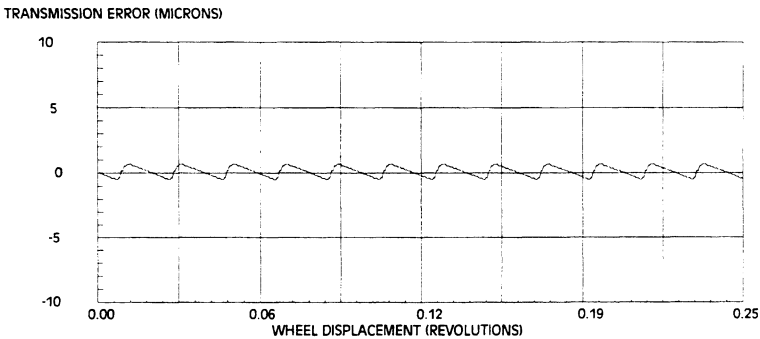


Figure 8 : Theoretical transmission error for a test worm gear design.

A gear set was then manufactured using this specification. Several error sources such as gear cutting tool profile errors of, 7 microns, eccentricity in the worm and wheel, 5 microns, and misalignment of the operating centre distance, 5 microns, were recorded. These were added to the computer program and a prediction of synthesised transmission error was calculated. The resulting graph is shown in Figure 9.

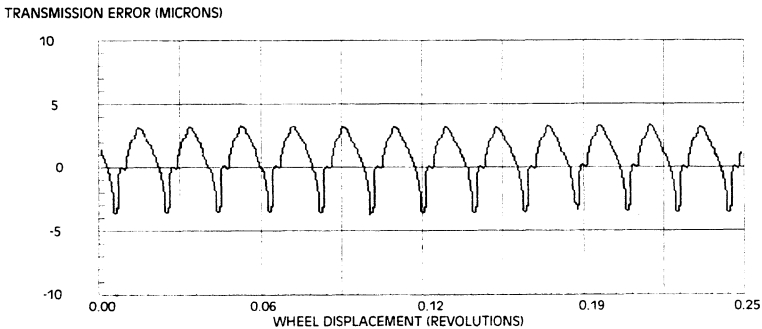


Figure 9 : Synthesised transmission error for test worm gear set.

A test was then performed on the completed gear set to obtain a measured transmission error plot. The result is shown in Figure 10.

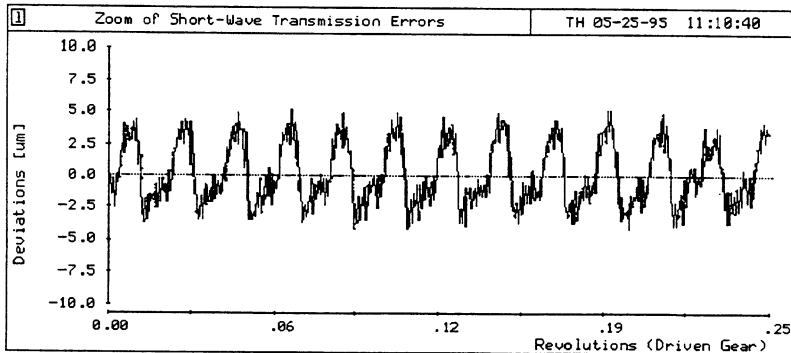


Figure 10 : Measured transmission error from a production worm gear set.

## 5 Conclusions

From the diagrams shown in section 4 several conclusions can be drawn. The results of Figure 8 show that theoretical designs can be produced with a relatively high accuracy. Comparing this with the synthesised results of Figure 9 indicates that even small inaccuracies can amplify the transmission error. Despite this however, if enough error sources are known then a computer model can be used to synthesise worm gear characteristic errors to within a micron as shown by comparing Figure 9 with Figure 10.

The good correlation in these results allow the computer model to be regarded as a tool. It can be used to investigate the effects of modifying design or isolate which error sources are most influential in determining the error without the need for expensive production costs.

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