Modelling and simulation of the turning process

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Abstract

A block-diagram model of the cutting process in the turning operation has been developed through an analytical approach in order to predict the cutting forces. This gives a fundamental idea about cutting forces acting on the axis-drive and spindle-drive systems that can be used in machine tool virtual prototyping. The aim of this paper is to analyse the process in physical terms and hence to develop a cutting process model that relates the feed rate and spindle speed to the cutting forces generated. The model presented here permits a better understanding of the cutting process and its interaction with the machine tool, and can be used as a part of an integrated computer-aided engineering design tool. Model parameters were measured experimentally so that the model can be simulated. The results of the simulation were compared with data obtained during cutting operations.

1 Introduction

Knowledge of cutting forces in turning process is essential in the computer modelling and design of the lathe machines. The computer aided analysis capable of generating the cutting forces has been developed by many researchers in the past but these models are either based on mechanistic approaches which are highly complicated and labour intensive or are based on statistical methods which are not useful in understanding of the dynamics of the system. There is clearly a need for a simplistic approach, which simulates the dynamic response of the cutting process. The output of the model then can be used as inputs to other CAD packages such as Finite element or Multi-body dynamic for structural analysis of the machine. The overall model then can result in a virtual prototyping environment of design optimisation.
The non-linear nature of cutting process makes it impossible to be represented as a simple mathematical function in these CAD packages, therefore the effort in this paper is focused in generation of a model which can be easily incorporated with other systems.

The analysis of the cutting forces can reveal valuable information about the machine tool structure. This is because the cutting process is the one relationship of the whole machine tool dynamics which closes the loop between the axis feed subsystem and the spindle subsystem [1]. Also it is the cutting process that affects the control of the servomechanism by generating the disturbance torque on the feed and the spindle motors.

![Diagram showing the interaction between cutting process dynamics and machine structure dynamics.](image)

Figure: 1 Closed loop interaction of the machine tool and the cutting process.

Many researchers have investigated the turning process. Ee et al. [2] presented a new methodology for decomposing and distributing the resultant force generated in cutting process with a groove tool within the three major tool-wear regions. El Baradie [3] discussed a closed loop system with two main elements representing the machine tool structure and the cutting process as shown in figure 1 to develop a generalized statistical theory of chatter to predict a threshold of stability in terms of mean values with confidence level. Balkrishna and Yung [4] presented a model of the dynamic cutting force process for the three-dimensional or oblique turning operation. They linked the mechanistic force model to a tool-workpiece vibration model to predict the dynamic cutting forces.

### 2 Theoretical model of cutting force in turning

The cutting force, \( F \), acting on the tool is generated by the engaged part of the cutting edge including the main cutting edge, nose radius, and a part of the secondary cutting edge [2]. This force is composed into three components as shown in figure 2, feed force component \( F_f \), which is acting as a direct load on the feed drive and in an opposite direction with the feed, \( f \), the radial force \( F_r \), which results in a deflection in the tool and consequently affecting on the machined surface, and the tangential force \( F_t \), which is in the same direction of the spindle speed, \( N \) and obtains the required cutting power. There are many variables affecting the cutting force in turning, some of these variables are tool geometry such as, depth of cut, feed, cutting speed, work piece hardness and...
lubrication state, and tool wear. These variables are summarised in the standard equation for $i^{th}$ cutting force component as follow [5].

$$F_i = K_{si} d_n f^a$$  \hspace{1cm} (1)

Where $i^{th}$ cutting force components are tangential force, feed force, and radial force. This model when logarithmically transformed becomes:

$$\ln F_i = \ln K_{si} + b_1 \ln d + a_1 \ln f$$  \hspace{1cm} (2)

Figure 2: Cutting force components in turning operation

If the error term, $\varepsilon$ is included, the cutting force model can be written as:

$$Y_{Fi} = c + px_1 + mx_2 + \varepsilon$$  \hspace{1cm} (3)

Therefore, the linear mathematical model can be formulated as follows:

$$\eta = \beta_0 + \beta_1 x_1 + \beta_2 x_2$$  \hspace{1cm} (4)

Where, $\eta$ is the true value of the $i^{th}$ cutting force component value on a logarithmic transformation and $\beta_0, \beta_1$ and $\beta_2$ are the parameters to be estimated corresponding to $c, p,$ and $m$ respectively. Due to the experimental error, the true responses are:

$$\eta = Y_{Fi} - \varepsilon$$

Then, equation (4) can be rewritten as:

$$\hat{Y}_{Fi} = b_0 + b_1 x_1 + b_2 x_2$$  \hspace{1cm} (5)

Multiple regression analysis estimates these three parameters for the first-order model in equation (5). If there was any statistical evidence of lack of fit a second-order or third-order model can be developed. The third-order mathematical model developed will be in the following form.

$$\hat{Y}_{Fi} = b_0 + b_1 x_1 + b_2 x_2 + b_3 x_1^2 + b_4 x_2^2 + b_5 x_1^3 + b_6 x_2^3 + b_7 x_1 x_2$$  \hspace{1cm} (6)

A second or third-order mathematical model is essential when the true response (cutting force) function is non-linear or unknown.
3 Experimentation

The experiments were performed on a Colchester (Combi K4) CNC lathe using a tool holder PCLNR 2020 K12 supporting coated carbide inserts (Sandvik Coromant CNMG 12 04 12-PM) of nose radius \( r_n = 1.2 \) mm with chip breaker. Cutting tests were carried out on BS970 080A42 (EN8D) Steel work pieces. The work piece geometry is a cylindrical bar with 60-70 mm diameter and 300 mm length. The tool geometry is given in table 1.

At each test, the cutting edge has been changed by a new one to avoid the wear effect. To investigate the effect of the cutting speed on the cutting forces, 16 tests were carried out at two different cutting speeds. The cutting force components were measured using a piezoelectric transducer (Kistler 9257A). For each of the three measured cutting force components a proportional electrical charge is set up in the measuring platform. These charges are fed into three charge amplifiers (KIAG SWISS type 5001), where they are converted into proportional voltages. The output of the charger amplifier is connected to a data acquisition card (DAQ), which converts the analogue signal into digital one. The acquired data from the NI DAQ Card can be saved, plotted, and analysed using MATLAB software. The duration time of measurement was 10 sec with sampling rate 100 sample/sec. To identify the cutting model parameters 32 tests were conducted for two cutting speed of \((200 \text{ m/min}, 300 \text{ m/min})\). For each cutting speed there are four different feeds \((0.125, 0.25, 0.375 \text{ and } 0.5 \text{ mm/rev})\) and four different cutting depths \((0.5, 1.0, 1.5 \text{ and } 2.0 \text{ mm})\).

### Table 1: Tool geometry

<table>
<thead>
<tr>
<th>Nose angle</th>
<th>Clearance angle</th>
<th>Thickness</th>
<th>Cutting length edge</th>
<th>Normal rake angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>80°</td>
<td>0°</td>
<td>4.76 mm</td>
<td>12 mm</td>
<td>6°</td>
</tr>
</tbody>
</table>

4 Identification of cutting model parameters

MINITAB statistical package software was used to identify the cutting model parameters. Figure 3 shows the procedure of regression analysis. These results were fed into the MINITAB software and transferred into natural logarithmic values.

An analysis of variance was made to obtain the cutting speed significance. Table 2 show ANOVA for the tangential, feed and radial force components versus cutting speed, \( V_C \) respectively.

This table shows that the cutting speed is not significant since \( P \) value is high. \( P \) value is defined as the smallest level of significance that would lead to rejection of the null hypothesis \( H_0: \mu_1 = \mu_2 \), i.e. lower value of \( P \) means more significance of the investigated factor. Edward [6] indicated that the cutting speed during the range of experimentation \((201 - 300 \text{ m/min})\) has no effect on the cutting force components.
Table 2: ANOVA for different force components versus cutting speed, $V_C$

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Feed Force Component $F_t$</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$V_C$</td>
<td>1</td>
<td>2312</td>
<td>2312</td>
<td>0.01</td>
<td>0.917</td>
</tr>
<tr>
<td>Error</td>
<td>30</td>
<td>6261134</td>
<td>208704</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>31</td>
<td>626344</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Tangential Force Component $F_t$</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$V_C$</td>
<td>1</td>
<td>113</td>
<td>113</td>
<td>0.00</td>
<td>0.949</td>
</tr>
<tr>
<td>Error</td>
<td>30</td>
<td>817347</td>
<td>27245</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>31</td>
<td>817460</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Radial Force Component $F_t$</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$V_C$</td>
<td>1</td>
<td>98</td>
<td>98</td>
<td>0.01</td>
<td>0.924</td>
</tr>
<tr>
<td>Error</td>
<td>30</td>
<td>313620</td>
<td>10454</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>31</td>
<td>313718</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The response factor (dependent variable) is the tangential force $F_t$. The predictors (the independent variables) are $x_1$, $x_2$, $x_1^2$, $x_2^2$, $x_1^3$, $x_2^3$, and $x_1 \times x_2$. Table 3 shows the tangential force model parameters (coefficients).

Table 3: Tangential force model parameters

<table>
<thead>
<tr>
<th>Predictor</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>T value</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>8.2858</td>
<td>0.2944</td>
<td>28.14</td>
<td>0.000</td>
</tr>
<tr>
<td>$x_1$</td>
<td>1.34161</td>
<td>0.07611</td>
<td>17.63</td>
<td>0.000</td>
</tr>
<tr>
<td>$x_2$</td>
<td>3.5819</td>
<td>0.7604</td>
<td>4.71</td>
<td>0.000</td>
</tr>
<tr>
<td>$x_1^2$</td>
<td>-0.02923</td>
<td>0.03743</td>
<td>-0.78</td>
<td>0.443</td>
</tr>
<tr>
<td>$x_2^2$</td>
<td>2.2287</td>
<td>0.6040</td>
<td>3.69</td>
<td>0.001</td>
</tr>
<tr>
<td>$x_1^3$</td>
<td>0.0934</td>
<td>0.1477</td>
<td>0.63</td>
<td>0.533</td>
</tr>
<tr>
<td>$x_2^3$</td>
<td>0.5329</td>
<td>0.1477</td>
<td>3.61</td>
<td>0.001</td>
</tr>
<tr>
<td>$x_1 \times x_2$</td>
<td>0.29442</td>
<td>0.02709</td>
<td>10.87</td>
<td>0.000</td>
</tr>
<tr>
<td><strong>Model summary</strong></td>
<td>S = 0.04153</td>
<td>$R^2$ = 99.7%</td>
<td>$R^2_{adj}$ = 99.6%</td>
<td></td>
</tr>
</tbody>
</table>
By investigation table 3, the predictors $x_1^2$ and $x_1^3$ can be neglected since the P value for each predictor is relatively high compared with the other predictors. Table 4 reports the coefficient of multiple determinations $R^2$, which is a measure of the amount of reduction in the variability of the response factor obtained by using the predictors in the model. However, a large value of $R^2$ doesn’t reveal that the regression model is a good one. Adding a variable to the model will always increase $R^2$, regardless of whether the additional variable is statistically significant or not. Thus it is possible for models that have a large value of $R^2$ to yield poor predictions of new observations or estimates of the mean response. So, adjusted $R^2$ statistic $R^2_{adj}$ can be used as a statistical tool to indicate the model is poor or not. Adjusted $R^2$ statistic defined as

$$R^2_{adj} = 1 - \left(\frac{n-1}{n-p}\right)(1-R^2)$$

(7)

In general, the adjusted $R^2$ statistic will not always increase as variables are added to the model. Table 5 shows that $R^2_{adj} (0.996)$ is very close to $R^2$. Thus the model is strong and significant. The analysis of variance and lack of fit test of the tangential force model shows that the regression model is significant since $P$ is 0.000. A further regression has been done after eliminating the second and third order of the natural logarithmic value of depth of cut $x_1^2$ and $x_1^3$.

Figure 4 and 5 show the measured and predicted tangential force at the cutting speed 201 and 300 m/min, respectively.

The identification of the feed force $F_f$ model is the same as the tangential force model. The response factor is the feed force $F_f$. The predictors are $x_1$, $x_2$, $x_1^2$, $x_2^2$, $x_1^3$, $x_2^3$, and $x_1x_2$. Table 3 illustrates the feed force model parameters.

![Figure 4: Measured and predicted tangential force at cutting speed 201 m/min](image-url)
5 Simulation and results

Once the cutting model’s parameters are identified the overall block diagram model of the machine tool and cutting process is constructed. Figure 6 shows the model of the cutting forces in SIMULINK. The inputs to the cutting process are the tool feed rate and the spindle speed and the outputs from the cutting process are the $F_x$, $F_y$, and $F_C$ acting on drive and spindle system. The simulation result is shown in figure 7. The depth of cut was entered to the model as a constant value while the feed was developed from the outputs of the axis and spindle drive subsystems [7].

The cutting force model subsystems were incorporated with the axis and spindle drive model subsystems to develop the SIMULINK block diagram of the Combi
K4 Colchester machine tool. The feed rate \( f_x \) is the X-axis drive subsystem output, which is divided by the spindle drive model subsystem output, the spindle speed \( N_s \), to obtain the feed \( f \). The feed force component affects on the X-axis drive system as a load on the ball-screw while the tangential force applies a cutting torque on the spindle axis. The feed force affects on the X-axis drive system and leads to an increasing in the X-axis motor current, as shown in figure 8. The tangential force acts as a torque on the spindle axis and increases the spindle motor current.

6 Conclusions

In this paper a mathematical model of cutting for turning process was identified and simulated. The process of developing the model from the physical block diagram model is discussed. In the model simulation stage the drive system parameters were not presented here. Firstly, the model was divided into three subsystems, the X-axis drive, the spindle drive, and the cutting force subsystems. The first two subsystems were simulated individually. The X-axis and spindle drive subsystems can be linked together with the cutting force model to develop the model of the turning machine tool. The model can be used in machine tool design and tool condition monitoring.
Figure 8: Simulated spindle drive motor current in cutting

**Notation**

- $b_0, b_1, b_2$: Parameters used to estimate $\beta_0, \beta_1$ and $\beta_2$ respectively
- $d$: Depth of cut
- $e_i$: Constant
- $f$: Feed (mm/rev)
- $F$: Ratio in regression model
- $F_f$: Feed force component (N)
- $F_r$: Radial force component (N)
- $F_t$: Tangential force component (N)
- $g_i$: Constant
- $K_{li}$: State of lubrication factor
- $K_m$: Material hardness
- $K_{si}$: Specific cutting force (N)
- $K_{di}$: Related to tool geometry
- $N$: Spindle speed
- $P$: Smallest level of significance
- $r_n$: Nose radius of carbide insert
- $V_c$: Cutting speed (mm/min)
- $W_1$: Wear land width
- $x_1, x_2$: Natural logarithmic of cut depth and feed depth
- $Y_{F_i}$: Natural logarithmic of cutting force component
- $\beta_0, \beta_1, \beta_2$: Estimated parameters corresponding to $c$, $p$ and $m$ respectively
- $\varepsilon$: Experimental error
- $\eta_i$: The $i^{th}$ cutting force component true value on logarithmic transformation.
References