Investigation of the cutting force coefficients in ball-end milling

X.-W. Liu¹, K. Cheng¹, D. Webb¹, A.P. Longstaff², M.H. Widiyanto², X.-Q. Jiang², L. Blunt² & D.G. Ford²
¹ School of Engineering, Leeds Metropolitan University, Leeds, UK
² Ultra Precision Engineering Centre, University of Huddersfield, Huddersfield, UK

Abstract

This paper investigates the cutting force coefficients embodied in the cutting force model of ball-end milling. A series of slot milling tests was performed with a one-tooth helical ball-end mill for the calibration of the cutting force coefficients. The recorded dynamic cutting forces are averaged to fit the theoretical model and yield the force coefficients. The reasonable agreement between the measured and simulated dynamic cutting forces demonstrates the effectiveness of the calibration method. The results are compared with those obtained from a series of slot milling tests using a one-tooth helical end mill under similar cutting conditions. The two sets of coefficients, especially the axial force ratios, differ from each other significantly. In a further investigation of the coefficients, another series of cutting tests including plane milling, down-hill and up-hill slope milling with a two-edge helical ball-end mill was undertaken, and the results obtained indicate that the axial force ratio is greater when the engaged part of the cutting edge moves away from the tip point of the cutter, and it approaches zero as the contact part moves closer to the tip point.

1 Introduction

Ball-end milling has been widely used in the manufacture of free form surfaces such as those embodied in dies and moulds, turbine engine blades and aircraft structural components. The dimensional accuracy of the machined surfaces in these operations heavily depends on the tool and workpiece deflections caused by the forces present during the cutting process. In many cases, the tool
deflection can create significant dimensional errors in the shape of the machined surface and these should be taken into account as a first priority in the machining process planning.

An accurate prediction of the tool deflection depends on a reliable cutting force model. The concept of equivalent orthogonal cutting [1] is used widely to generate a cutting force model of ball-end milling [2]. The cutting edge of a ball-end mill can be considered to be a series of infinitesimal cutting elements, and the oblique cutting process occurring in this small element is analysed as an orthogonal cutting process. The differential cutting forces applied to each element are calculated based on the concept of the equivalent orthogonal conditions and the total cutting forces obtained by summing up the differential forces of all elements of the engaged teeth. In the cutting force model, three vital coefficients, the so-called cutting force coefficients, must be experimentally evaluated or calibrated.

Traditionally, two different methods are used to evaluate the cutting force coefficients. The first is by direct calibration: using the dynamic cutting forces measured during the cutting process to calibrate the cutting force coefficients. A series of slot milling is generally established, and the dynamic cutting forces are recorded on-line to calibrate the cutting force model. The second method is by indirect evaluation or prediction: using the orthogonal cutting data to predict the cutting force coefficients of ball-end milling. This method was adopted by Altintas, who presented a dynamic cutting force model [3] for ball-end milling, in which the cutting force coefficients were obtained from an orthogonal cutting database [4, 5] and applied to ball-end milling using an oblique cutting transformation model [6].

Based on the geometric analysis of ball-end milling [7] and the orthogonal cutting principle [1], and also taking into account the size effect of undeformed chip thickness and the influence of the effective rake angle [1], the authors formulated an improved theoretical dynamic cutting force model (not yet published). The cutting force coefficients in the model are experimentally calibrated first by a series of slot milling tests with a one-tooth helical ball-end mill. Unlike the previous methods [4,5], the recorded dynamic cutting forces are averaged to fit the theoretical model and determine the cutting force coefficients.

In order to compare the cutting force coefficients of peripheral milling, a series of slot milling tests with a one-tooth helical end mill and similar cutting conditions was carried out, and a set of cutting force coefficients for peripheral milling was obtained. The results show a considerable difference between the two sets of cutting force coefficients.

In further investigation of the cutting force coefficients of ball-end milling, a series of cutting tests including plane milling, down-hill and up-hill slope milling with a two-edge helical ball-end mill was performed, and the results indicate that there is a significant difference between down-hill and up-hill slope milling.

2 Cutting force model

Figures 1(a) and 1(b) show the geometric models of a helical end mill and a helical ball-end mill, which can be visualised as the combination of a number of slices taken along the z-direction. Within each slice, the cutting action for an individual
tooth can be modelled as for single point oblique cutting, and the differential
tangential, normal and axial cutting forces at any point on the rake face can be
obtained from the oblique cutting theory [1], as shown in Figure 1(c),

\[
dF_{ti} (\phi_i) = K_s t_i (\phi_i) R_0 \cot \beta_0 \, d\phi \\

dF_{ni} (\phi_i) = c_1 dF_{ti} (\phi_i) \\

dF_{ai} (\phi_i) = c_2 dF_{ti} (\phi_i)
\]

where \( t_i (\phi_i) \) is the undeformed chip thickness of the cutting point on the \( i \)th
helical flute at position angle \( \phi_i \).

In the model, \( K_s \) is the tangential cutting force coefficient, and \( c_1 \) and \( c_2 \)
are the ratios of radial and axial cutting force components respectively. All the
three coefficients are supposed to be variable, depending on the cutter material
and geometry, work material, and cutting conditions.

Resolving the differential cutting forces of Eqs. (1)-(3) into the X, Y and Z
components and summing up in each direction gives the differential cutting
forces \( dF_{tx}, dF_{ty} \) and \( dF_{tz} \). Integrating them along the whole cutting edge
yields the total cutting force applied to the edge.

(a) End mill geometry  (b) Ball-end mill geometry  (c) Differential cutting forces

Figure 1: Differential cutting force models of peripheral and ball-end milling
\[
\begin{aligned}
F_{ix} &= \int_{\phi_s}^{\phi_e} dF_{ix} d\phi_i \\
F_{iy} &= \int_{\phi_s}^{\phi_e} dF_{iy} d\phi_i \\
F_{iz} &= \int_{\phi_s}^{\phi_e} dF_{iz} d\phi_i 
\end{aligned}
\]

Summing up the cutting forces acting on all the \( m \) helical flutes of the cutter gives the total force applied to the whole cutter:

\[
\begin{aligned}
F_x &= \sum_{i=1}^{m} F_{ix} \\
F_y &= \sum_{i=1}^{m} F_{iy} \\
F_z &= \sum_{i=1}^{m} F_{iz}
\end{aligned}
\] (5)

For the evaluation of the cutting force coefficients, the easiest way (and probably the best way) is to use the average cutting forces measured during the cutting process. The theoretical average cutting force components for peripheral milling can be written as:

\[
\begin{aligned}
\bar{F}_x &= u_0 \sum_{i=1}^{m} \bar{f}_{ix1} + u_0 c_1 \sum_{i=1}^{m} \bar{f}_{ix2} \\
\bar{F}_y &= u_0 \sum_{i=1}^{m} \bar{f}_{iy1} + u_0 c_1 \sum_{i=1}^{m} \bar{f}_{iy2} \\
\bar{F}_z &= u_0 c_2 \sum_{i=1}^{m} \bar{f}_{iz}
\end{aligned}
\] (6)

and for ball-end milling:

\[
\begin{aligned}
\bar{F}_x &= u_0 \sum_{i=1}^{m} \bar{f}_{ix1} + u_0 c_1 \sum_{i=1}^{m} \bar{f}_{ix2} + u_0 c_2 \sum_{i=1}^{m} \bar{f}_{ix3} \\
\bar{F}_y &= u_0 \sum_{i=1}^{m} \bar{f}_{iy1} + u_0 c_1 \sum_{i=1}^{m} \bar{f}_{iy2} + u_0 c_2 \sum_{i=1}^{m} \bar{f}_{iy3} \\
\bar{F}_z &= u_0 c_1 \sum_{i=1}^{m} \bar{f}_{iz1} + u_0 c_2 \sum_{i=1}^{m} \bar{f}_{iz2}
\end{aligned}
\] (7)
where $\overline{F}_x$, $\overline{F}_y$ and $\overline{F}_z$ are the average cutting forces measured, $\bar{f}_{bij}$, $\bar{f}_{bij}$ and $\bar{f}_{ij}$ are the geometric factors in the cutting process, which can be derived from the cutting force model, and $u_0$, $c_1$ and $c_2$ are the cutting force coefficients to be evaluated.

3 Experimental investigation

All the cutting tests were performed on a three axis vertical CNC machine centre, Cincinnati Arrow2-500. The 3D dynamic cutting forces were recorded by a Kistler table dynamometer, mounted on the worktable of the machine tool. The workpiece was fixed using the Microloc fixture system, Kit-75.

3.1 Slot milling tests with ball-end mill

In order to evaluate the cutting force coefficients for ball-end milling, a series of slot milling tests was carried out with the following cutting conditions and parameters:

Cutter: solid carbide helical ball-end mill, $m = 1$, $R_o = 10$ mm, $\beta = 30^\circ$, $\alpha = 5^\circ$

Work material: carbon steel EN8

Cutting condition: with fluid

Cutting parameters: spindle speed $n = 1114$ rpm (cutting speed $v = 70$ m/min)

The feed rate $f_t$ and axial depth of cut $b_a$ are listed in Table 1.

<table>
<thead>
<tr>
<th>$f_t$ (mm/tooth)</th>
<th>0.0197</th>
<th>0.0296</th>
<th>0.0494</th>
<th>0.0691</th>
<th>0.0790</th>
<th>0.0889</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_a$ (mm)</td>
<td>9.95</td>
<td>9.95</td>
<td>9.95</td>
<td>9.95</td>
<td>9.95</td>
<td>9.95</td>
</tr>
</tbody>
</table>

Table 1: Cutting parameter (slot milling with ball-end mill)

Table 2: Experimental calibrated cutting force coefficients (by slot milling with helical ball-end mill)

<table>
<thead>
<tr>
<th>$f_t$ (mm/tooth)</th>
<th>0.0197</th>
<th>0.0296</th>
<th>0.0494</th>
<th>0.0691</th>
<th>0.0790</th>
<th>0.0889</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_0$ (G J/m^3)</td>
<td>2.5215</td>
<td>2.4860</td>
<td>2.3759</td>
<td>2.3075</td>
<td>2.3045</td>
<td>2.2972</td>
</tr>
<tr>
<td>$c_1$</td>
<td>0.6155</td>
<td>0.5842</td>
<td>0.5422</td>
<td>0.5515</td>
<td>0.5443</td>
<td>0.5293</td>
</tr>
<tr>
<td>$c_2$</td>
<td>0.0464</td>
<td>0.0453</td>
<td>0.0561</td>
<td>0.0999</td>
<td>0.1009</td>
<td>0.0957</td>
</tr>
</tbody>
</table>

Averaging the cutting forces from the measured dynamic ones and substituting them in Eq. (7) yields the cutting force coefficients $u_0$, $c_1$ and $c_2$. Table 2 and Figure 2 show the calibrated cutting force coefficients, in which $u_0$...
varies between $2.2972 \times 10^9$ and $2.5215 \times 10^9$ J/m$^3$, $c_1$ between $0.5293$ and $0.6155$, and $c_2$ between $0.0453$ and $0.1009$.

Figure 2: Experimental calibrated cutting force coefficients (by slot milling with helical ball-end mill).

Figure 3: Measured and predicted dynamic cutting forces.
Figure 3 shows the measured and predicted dynamic cutting force components in the x, y and z directions, in which the predicted values are obtained from the cutting force model using the calibrated force coefficients. The reasonable agreement between the measured and simulated dynamic cutting forces demonstrates the effectiveness of the calibration method.

3.2 Slot milling tests with end mill

Also, in order to evaluate the cutting force coefficients for peripheral milling, a series of slot milling tests was performed with the following cutting conditions and parameters:

- Cutter: solid carbide helical end mill, \( m = 1, R_o = 10 \text{ mm}, \beta = 45^\circ, \alpha_i = 5^\circ \)
- Work material: carbon steel EN8
- Cutting condition: with fluid
- Cutting parameters: spindle speed \( n = 1114 \text{ rpm} \) (cutting speed \( v = 70 \text{ m/min} \))
- The feed rate \( f \) and axial depth of cut \( b_a \) are listed in Table 3.

| \( f \) | 0.0197 | 0.0296 | 0.0395 | 0.0494 | 0.0592 | 0.0691 | 0.0790 | 0.0889 |
| \( b_a \) | 10.545 | 10.545 | 10.545 | 10.450 | 10.390 | 10.450 | 10.390 | 10.390 |

Averaging the cutting forces from the measured dynamic ones and substituting them in Eq. (6) yields the cutting force coefficients \( u_0, c_1 \) and \( c_2 \). Table 4 and Figure 4 show the evaluated cutting force coefficients, in which \( u_0 \) varies between \( 2.3851 \times 10^9 \) and \( 2.5184 \times 10^9 \text{ J/m}^3 \), \( c_1 \) between 0.3457 and 0.5114, and \( c_2 \) between 0.2617 and 0.3599. These results agree well with the data provided by Shaw [1].

| \( f \) | 0.0197 | 0.0296 | 0.0395 | 0.0494 | 0.0592 | 0.0691 | 0.0790 | 0.0889 |
| \( u_0 \) | 2.4067 | 2.4433 | 2.3851 | 2.4532 | 2.3891 | 2.5146 | 2.4429 | 2.5184 |
| \( c_1 \) | 0.5114 | 0.4832 | 0.4776 | 0.3889 | 0.3457 | 0.4052 | 0.3992 | 0.4171 |
| \( c_2 \) | 0.2617 | 0.2871 | 0.3191 | 0.3529 | 0.3525 | 0.3596 | 0.3570 | 0.3599 |

The results of the two sets of cutting tests shown in Figures 2 and 3 reveal us a significant difference between the two sets of cutting force coefficients. The coefficient \( u_0 \) keeps reasonable constant, \( c_1 \) for ball-end slot milling is a little
smaller than for slot milling with the end mill, $c_2$ for ball-end slot milling, which values between 0.0453 and 0.1009, is much smaller than that for slot milling with the end mill, which is between 0.2617 and 0.3599.

### 3.3 Milling tests with ball-end mill

To investigate further the cutting force coefficients of ball-end milling, a series of cutting tests including plane milling, down-hill and up-hill slope milling was performed with the following cutting conditions and parameters:
- Cutter: solid carbide helical ball end mill, $m = 2$, $R_0 = 5$ mm, $\beta = 30^\circ$, $\alpha_r = 5^\circ$
- Work material: carbon steel EN8
- Cutting condition: with fluid
- Cutting parameters: spindle speed $n = 2228$ rpm (cutting speed $v = 70$ m/min), normal depth of cut $b_n = 2$ mm

The feed rate $f_t$ and the corresponding cutting method are listed in Table 5, and Figure 5 shows the examples of down-hill and up-hill slope milling.
Table 5: Cutting method and parameter (ball-end milling)

<table>
<thead>
<tr>
<th>Series No.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_t$</td>
<td>0.035</td>
<td>0.035</td>
<td>0.0494</td>
<td>0.0592</td>
<td>0.0494</td>
<td>0.0592</td>
</tr>
<tr>
<td>Method</td>
<td>Plane milling</td>
<td>Down-hill slope milling</td>
<td>Up-hill slope milling</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Averaging the cutting forces from the measured dynamic ones and substituting them in Eq. (7) yields the cutting force coefficients $u_0$, $c_1$ and $c_2$. Table 6 lists the evaluated cutting force coefficients.

Table 6: Experimentally evaluated cutting force coefficients (ball-end milling)

<table>
<thead>
<tr>
<th>Series No.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_0$ (G J/m$^3$)</td>
<td>2.7194</td>
<td>2.4219</td>
<td>2.3555</td>
<td>2.5985</td>
<td>2.7649</td>
<td>2.6634</td>
</tr>
<tr>
<td>$c_1$</td>
<td>0.74</td>
<td>0.5687</td>
<td>0.649</td>
<td>0.6251</td>
<td>0.4977</td>
<td>0.5343</td>
</tr>
<tr>
<td>$c_2$</td>
<td>0.008</td>
<td>0.022</td>
<td>0.0019</td>
<td>0.0038</td>
<td>0.1654</td>
<td>0.1586</td>
</tr>
</tbody>
</table>

The results listed in Table 6 show that the coefficients $u_0$ and $c_1$ stay reasonably constant for ball-end milling. However, the values of $c_2$ for plane milling and down-hill slope milling are very small, much smaller than for ball-end slot milling, and the values of $c_2$ for up-hill slope milling are much greater than for ball-end slot milling. This is because the tip of the ball-end mill is not engaged in cutting in the up-hill slope milling process, as shown in Figure 4(b), and it is more like peripheral milling. But for down-hill slope milling, only the tip of the ball-end mill is engaged in cutting, as shown in Figure 4(a). This means...
the coefficient $c_2$ for the tip of the ball-end mill is very small, almost equal to zero, but for the upper part its value approaches that for peripheral milling.

4 Conclusions

The experimental results presented above demonstrate that the cutting force coefficient $c_2$ for ball-end milling varies in relation to the nature of the contact between the cutting edge and workpiece. If contact is near to the tip of the cutter, the coefficient is very small (smaller than 0.01). When contact moves away from the tip, the value of the coefficient increases. Its maximum value approaches the corresponding value for peripheral milling when the contact part of the cutting edge is close to the straight part of the cutter. This result seems reasonable. So it is not practical to evaluate the cutting force coefficients for ball-end milling directly from the orthogonal cutting database, even though the coefficients $u_0$ and $c_1$ are almost the same values as those for peripheral milling. For an accurate prediction of the dynamic cutting forces, it is necessary to calibrate the cutting force coefficients before the cutting force model is applied.

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References


