Computer exercises of basic courses in mathematics at the Helsinki University of Technology
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Abstract

Nowadays computers play an increasingly important role in technology. For this reason the students of engineering sciences should learn to work with mathematical programs that are powerful enough for engineering problems. Computers are effective tools for carrying out mathematical experiments and demonstrations. Mathematical programs such as Matlab, Maple and Mathematica [1, 2, 3] provide good means for demonstrating mathematical phenomena. Thus these programs can be used for supporting students’ learning process.

For more than ten years experience has been gathered about the use of interactive mathematical programs at the Helsinki University of Technology. Various kinds of experiments have been conducted to develop the use of mathematical programs in the teaching of basic courses in mathematics. The aim is to improve computer exercises and to understand what students really learn from using the mathematical programs.

1 Introduction

At the Helsinki University of Technology the first major experiment on using computers in the basic courses in mathematics started in the fall of 1986. Some computer classroom exercises were as a part of one basic course. The programs used were Matlab and muMath [4].

The first experimental course where computers were regularly used throughout the term was started in 1989. Roughly one hundred students
took part in the course on voluntary basis. A two-hour-exercise was held weekly in a computer classroom. The programs used were Matlab and Sim- non [5]; the latter was later replaced by Mathematica. A personal computer and a video projector were used in the lecture room to demonstrate the use of these programs. The course was arranged three times in academic years 1989–1990, 1990–1991 and 1991–1992. [6, 7]

Starting from the fall term of 1992 the largest basic courses in mathematics have been arranged following the ideas and experience of the experimental course. The participants were freshmen from the departments of technical physics and computer science. Since then computers and mathematical programs have been a part of the largest basic courses for the first and second year students.

1.1 The structure of the basic courses in mathematics at the Helsinki University of Technology

Depending on the department, the students of the Helsinki University of Technology have either two, three or four one semester basic courses in mathematics. These courses are taken during the first two years. There are 200–500 students attending each course. Exercise groups have usually 20–50 students.

Each basic course has 6–8 lecture hours and 4–6 hours for traditional homework exercises every week. The largest basic courses have also guided exercises held in computer classrooms: during the first fall term two exercise hours every week and during the first spring term two hours every second week. In the second academic year students use mathematical programs mainly outside of scheduled sessions when doing their homework exercises.

The university has currently two computer classrooms for teaching purposes. Both rooms are equipped with twenty Unix workstations running the X Window System. In the computer exercises two students work together at the same workstation. Many courses use the computer classrooms for guided exercises, but students can use them independently, too.

The first two basic courses consist of analytic geometry, elementary linear algebra, series, differential equations, differential and integral calculus of one variable and vector calculus. The basic courses for the second year students consist of advanced linear algebra, analytic functions, the Fourier transform, systems of ordinary differential equations, partial differential equations and integral equations. Numerical analysis has a significant role in second year courses.
2 The computer exercises of the largest basic courses for the first year students

2.1 The structure of the computer exercises

The mathematical programs used in the computer exercises are Matlab and Mathematica. The first half of the first fall term is devoted to the numerical program Matlab. The second half is devoted to symbolic computation and the program Mathematica. In the spring term both programs are used — sometimes even during the same session.

In the first Matlab and Mathematica sessions the mathematical problems are short and simple. The aim is to familiarize the students with the syntax and the basic commands of the program: algebraic manipulation, differentiation, integration, manipulating equations etc. have been trained. Usually six problems a week are discussed in the computer exercise. A brief help file is written for every computer exercise by the lecturer. Guide books are also used as an introduction to the programs [8, 9].

Problem. Solve the equation

\[ e^{-x} - \sin x = 0 \]

with Mathematica. To this end, plot the graphs of the functions \( y = e^{-x} \) and \( y = \sin x \) in the same figure. Determine the approximate values of the three smallest roots.

The corresponding entry in the help-file is:

`Solve` and `NSolve` can only solve algebraic equations. Try these. Transcendental equation must usually be solved numerically with Newton’s method using the command `FindRoot`. In this case an initial value is needed. The value can be found, for instance, by looking at the graphical representation.

First plot the graphs of each function separately (`Plot`) and then combine these into a single figure (`Show`).

Because the X Window System is used, the students open at least two windows on the screen: one is for the help file and the other is for the program, for the work itself. Students can also view the problem sheet on the screen.

The reason why the first computer exercises contain mainly simple mathematics is that learning the computer program requires extra effort. Gradually the properties and possibilities of the programs are studied more closely and the students can use them for solving more complicated mathematical problems. At this stage, the students first have to outline the structure of the problem, understand what they need to do, and choose the appropriate tools and commands to obtain the results.
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An example of a larger problem:

**Problem.** There are four identical particles which repel each other on the surface of a sphere. The repulsive force is determined by the potential $V(r) = 1/r$, where $r$ is a distance between two particles; the force is in that case of the form $F = -\nabla (1/r) = r/r^3$. The total potential is the sum of all potentials over all pairs of particles:

$$V = \frac{1}{2} \sum_{j \neq k} \frac{1}{r_{jk}}.$$  

According to the variational principle, the system finds its way to the state where the potential has its minimum value. Determine the equilibrium configuration of the particles; solve the problem as a constrained extreme value problem. Repeat the exercise with 5–8 particles on the surface.

### 2.2 Experiments and experiences

The examples and problems of the computer exercises have been intended for studying of mathematics, not only for operating the computer. Practice has shown that the students too often just run the commands without even looking at the results — the mathematical aspects of the problems are lost on them. That is the reason for adding some further questions to every problem. The questions are to guide students' mathematical thinking, after all, the aim is to learn mathematics, not only the programs.

**Problem.** The polynomial of degree $n$ has the value zero at $x_1, x_2, \ldots, x_n$. Determine using Mathematica the coefficients as functions of the zeroes. Solve the problem for $n = 2, 3, 4, 5$.

**Question.** Can you explain how the coefficients of the polynomial depend on its zeroes?

Two kinds of user interfaces of Mathematica have been used: text-based and the notebook. The text-based interface has proved to be easier for the students to understand. With this interface the students have better control of the order of the processed inputs: with the notebook interface the order of the processed inputs is not necessarily the same as the 'visual' order of inputs in the notebook. When the students have become more familiar with Mathematica, the notebook interface is used because it makes it rather easy to get beautiful documents with text and pictures. Also, it is much easier to modify the previous inputs when using the notebook interface. In general, students learn to use notebook interface quite fluently after the first fall term.
The students gain extra credit which affects their grades when they take part in the computer exercises usually for just being present. There have been some experiments during this academic year where the extra points have to be earned by presenting the results of the classwork:

- After some computer exercises students have sent the Mathematica notebook which they have made during the session to their assistant by e-mail. Students have added comments and explanations to the notebook.

- When either Matlab or the text-based Mathematica interface have been used, students have recorded their sessions in a log file. They have edited the file, inserted some comments on solutions and sent the file to their assistant by e-mail.

The purpose of the computer exercises is not only to familiarize the students with syntax of some mathematical programs but also to teach them the philosophy of the programs. That is the reason why the questions of the following kind are used in examinations.

**Problem.** Consider the initial-value problem

\[
\begin{align*}
x'(t) &= x(t) + y(t) - 2z(t), \\
y'(t) &= 2x(t) - 2z(t), \\
z'(t) &= -2x(t) + 2y(t) + z(t); \\
x(0) &= 1, \ y(0) = 2, \ z(0) = 3. 
\end{align*}
\]

Explain how the problem can be solved by using Mathematica according to the eigenvalue theory. It is essential to explain how the solving proceeds by using Mathematica. There is no need to explain the details of the syntax. You do not have to solve the initial-value problem.

**References**


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