Simulation and optimisation of the waste water flow in urban sewer network
M. Boyarshinov¹, P. Trusov¹, A. Kozlinskikh² & A. Lepikhin²
¹ Perm State Technical University
² Institute of Ecology and Genetics of Microorganisms
29-A, Komsomolsky Ave., Perm, 614600, Russia
E-mail: bmg@matmod.pstu.ac.ru

Abstract

The irregular loading of waste water treatment systems is determined by the non-uniformity of dwelling and industrial sewers. Only industrial sewers can be controlled to regulate the total amount of waste waters received in the treatment system. To carry out such regulation it is necessary to take into account the enterprises’ locations, pipe parameters and sewer system configurations. The computer model of the uniaxial liquid flow is based on a system of non-linear equations in partial derivatives with corresponding initial and boundary conditions. To solve this problem the finite difference method is applied here. Two approaches are compared: the linearized system of the nonlinear equations and Newton’s method. Areas of stability and convergence of both numerical solutions are estimated. The problem of modelling the dynamics of a liquid flow in a system of pipelines is also formulated. The objective function is determined as the remainder between the highest and lowest wastewater levels, i.e. non-uniformity of the loading of waste water treatment systems. The defined optimum solution shows that the non-uniformity of the fluid inflow in the cleaning structures can be reduced by one half.

Introduction

Recent interest in problems of water resource protection and forecasting of water quality has greatly increased. One of the basic causes of water pollution are waste waters from industrial enterprises, which are
transferred to the water drainage systems of biological water cleaning station, and then are spilled into natural bodies of water. In connection with this process a major problem becomes regulation of the polluting substances spilling through water drainage systems from cleaning structures and into receivers of waste waters. Solving the problem of optimum control by flows requires construction of the mathematical model of the movement of a liquid flow containing dissolved impurities at the given flow rates and speeds of a liquid in points of drains.

**Mathematical formulation of problem**

In the present work the following basic hypothesises are accepted. The liquid is considered as ideal and incompressible; the concentration of impurities are small and do not influence the properties of a liquid and character of the current; the dissolved substances do not react with each other, do not drop out in a deposit and do not evaporate from the surface of a flow. The speeds of all particles of a liquid in a plane of cross section are identical and are directed along the axis of a pipe, that allows it to proceed from a multiaxial to a uniaxial model of movement; the current of a liquid changes slowly, that is the characteristics of a flow are rather smooth functions of time and coordinate. The walls of a pipe are absolutely rigid and motionless; their influence on the current is taken into account with the help of roughness factors, describing the material and condition of the pipes and physical properties of a liquid. At a merger, two or more flows (in the sewer network junctions) vectors of liquid particle speed are turned instantly without change of modules; the mix of flows occurs instantly in the whole section. A sewer network is simulated as a set of rectilinear sites of pipes, connected to each other. With specified assumptions to determine distribution of the flow speeds and depths for each rectilinear site, it is necessary to integrate the system of differential equations in partial derivatives by Chertousov [1]:

\[-\text{continuity} \quad \frac{\partial \omega}{\partial t} + \frac{\partial Q}{\partial s} = 0,\]

\[-\text{impulse} \quad \frac{\partial h}{\partial s} + \frac{1}{g} \frac{\partial v}{\partial t} + \frac{v}{g} \frac{\partial v}{\partial s} = i - i_{fr},\]

with boundary \( h(t,0) = h_{0}(t), \quad v(t,0) = V_{0}(t) \)

and initial conditions \( h(0,s) = \mathcal{h}(s), \quad v(0,s) = \mathcal{V}(s). \)
Is here designated:

\( \nu(t, s) \) - speed of a liquid current;
\( h(t, s) \) - depth of a flow;
\( \omega(h) \) - area of a flow cross section;
\( Q(h, v) \) - flow rate of a liquid;
\( i \) - pipe tilt, determined by a sine of a corner of an inclination of its axis in relation to a horizontal plane;

\( i_{fr} \) - friction tilt, \( i_{fr} = \frac{v^2}{C^2(h)R(h)} \);

\( C(h) \) - special factor, \( C(h) = \frac{1}{n_r}R^y(h)(h) \);
\( R(h) \) - hydraulic radius of pipe;
\( y(h) \) - experimentally established dependence; at realization of accounts the Pavlovsky formula was used:

\[ y(h) = 2.5\sqrt{n_r} - 0.75\sqrt{R(h)(\sqrt{n_r} - 0.10)} - 0.13; \]

\( n_r \) - roughness factor of pipes;
\( t \) - time;
\( s \) - longitudinal coordinate.

At record of boundary conditions (3) it is supposed, that the liquid enters the pipe from the left edge (\( s=0 \)) and goes to the right (\( s = L \)).
During construction of the solution, the network as a whole is taken into account. Speeds and the depths of liquid flows at the exit of each separate site (with number \( j \)) become boundary conditions at the entrance of the following section of a pipe (with number \( j + 1 \)):

\[ \nu_j(t, L_j) = \nu_{j+1}(t,0), \quad h_j(t, L_j) = h_{j+1}(t,0). \]  

At the merger of two flows (with numbers \( p \) and \( q \)) into one common flow (with number \( r \)), conforming to the above stated assumptions and using the laws of mass and impulse preservations, the following relationship take place:

\[ Q_p + Q_q = Q_r, \quad \nu_p Q_p + \nu_q Q_q = \nu_r Q_r. \]  

**Method of analysis**

For the numerical solution of the nonlinear non-uniform differential eqns (1) - (2) in partial derivatives with boundary conditions (3) and initial conditions (4) finite-difference schemes are used
where $\Delta s$ - coordinate step, $\Delta t$ - increment of time; the volumes $h$ and $\hat{v}$ are related to the next moment of time ($t + \Delta t$). Schemes (7) and (8) are implicit on construction, but can be calculated as explicit. Newton’s method is applied to solve the constructed system of the nonlinear eqns (7) and (8).

At the replacement of the differential equations with difference analogues, the question of determining parity between steps of integration of time and coordinate, which ensures stability of the computing process, becomes extremely important. By virtue of nonlinearity of the mentioned problem, the analytical research of the difference scheme properties is complicated. Therefore, computer research of computing stability on the following test sample was carried out: a pipe of length $L = 10$ m, diameter $D = 1.0$ m, tilt 10 degrees, roughness factor $n_r = 0.013$ (old concrete pipes) has the initial filling $h(s) = 0.3$ m; the initial liquid speed is calculated from a condition of steady-state current: $v(0,s) = C(s)\sqrt{i \cdot R(h)}$. During 5 seconds on the left end of a pipe the level of a flow grows according to the sine law (1/4 period) with 0.3 m up to 0.8 m and is fixed at this volume. Distribution of a wave without impurities was considered. On fig. 1 area of stability of the settlement circuit is specified.

![Figure 1. Area of the difference scheme stability (horizontal axis - $\Delta t$, s; vertical axis - $\Delta s$, m); $\Delta$ - the solution is steady; $O$ - the solution misses](image-url)
Calculations have shown that the difference schemes (7) and (8) used in combination with the Newton method have good stability only in a small range of steps on time and coordinate (accordingly, $\Delta t=0.1...0.3$ s and $\Delta s=1...3$ m). This greatly limits productivity of the model accounting for large systems.

To increase efficiency it is appropriate to linearize the difference eqns (7) and (8). For this purpose the required nodal values of speed and the depth of a flow are written down as

$$\hat{h}(s,t) = h(s,t) + \Delta h(s), \quad \hat{v}(s,t) = v(s,t) + \Delta v(s),$$

where $\Delta h(s)$, $\Delta v(s)$ are increments of the respective values (in a considered point for time interval $\Delta t$), which are now subject to definition. Taking it into account and also neglecting magnitudes of $\Delta h(s) \cdot \Delta v(s)$, $\Delta v^2(s)$, it is possible to write the eqns (7), (8) down as

$$\Delta h_j \cdot \left( \frac{\Delta s}{\Delta t} + v_j \right) \cdot \frac{\partial \omega}{\partial h} + \Delta v_j \cdot \left( \frac{\partial \omega}{\partial h} \cdot (h_j - \hat{h}_{j-1}) + \omega \right) = \omega \cdot (\hat{v}_{j-1} - v_j) - \frac{\partial \omega}{\partial h} \cdot v_j \cdot (h_j - \hat{h}_{j-1}),$$

$$\Delta h_j \cdot g + \Delta v_j \cdot \left( \frac{\Delta s}{\Delta t} + 2 \cdot v_j - \hat{v}_{j-1} \right) = (i - i_f) \cdot \Delta s \cdot g + v_j \cdot (\hat{v}_{j-1} - v_j) - (h_j - \hat{h}_{j-1}) \cdot g.$$ 

Thus, a system of two linear algebraic equations with unknown values $\Delta v_j$ and $\Delta h_j$ is achieved.

For comparison of accuracy of the results received by two methods, a series of numerical research for the above problem is carried out. The solutions (nonlinear and linearized), with steps consistently reduced twice in time and coordinate, were compared. The distinction them was estimated using the formulas

$$\varepsilon_v = \max_{t,s} \frac{|v^{(k+1)} - v^{(k)}|}{v^{(k)}}, \quad \varepsilon_h = \max_{t,s} \frac{|h^{(k+1)} - h^{(k)}|}{h^{(k)}},$$

where the indexes $(k)$ and $(k+1)$ indicate the solutions with consistently reduced steps.

The obtained outcomes allow us to make the conclusion that the Newton method gives a converging sequence of solutions at a uniform reduction of steps. It may be assumed that at $\Delta t \to 0$ and $\Delta s \to 0$, the numerical solutions converge to exactly one. The linearized equations give consecutive solutions, converging to consecutive results of the
nonlinearized system, and this gives us the basis to suggest that the linearized decision converges to an exact solution. If the solution of the nonlinear problem at \( \Delta s = 0.125 \) m and \( \Delta t = 0.0125 \) s is accepted as exact, a 10% error in the linearized result occurs at \( \Delta s = 5 \) m and \( \Delta t = 0.0125 \) s, which reduces expenses of the computing experiments.

Table 1. Values of time and coordinate steps

<table>
<thead>
<tr>
<th>Variant</th>
<th>Time step, s</th>
<th>Coordinate step, m</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.0</td>
<td>0.2</td>
</tr>
<tr>
<td>2</td>
<td>1.0</td>
<td>0.1</td>
</tr>
<tr>
<td>3</td>
<td>0.5</td>
<td>0.05</td>
</tr>
<tr>
<td>4</td>
<td>0.25</td>
<td>0.025</td>
</tr>
<tr>
<td>5</td>
<td>0.125</td>
<td>0.0125</td>
</tr>
</tbody>
</table>

Figure 2. Estimations \( \varepsilon_h \) (left) and \( \varepsilon_v \) (right) convergence of the results; 
- O - use of Newton’s method; □ - linearized in comparison with appropriate nonlinear; Δ - linearized in comparison with the more exact nonlinear solution

Formulation of optimum control problem

We shall now consider the problem of drain management of industrial enterprises with the purpose of reduction of non-uniform currents of liquid flow in the sewer network of the city Perm. Great attention in modern literature is given to methods of solving optimization problems of designing and operating sewer networks, e. g. Nikaev [2], Kuhar, Zaitsev & Suhorukov [3], Evdokimov, Tevyashev & Dubrovsky [4], Ermolin & Skryabin [5]. The large extent and branching of sewer networks of industrial enterprises and residential areas in the city of Perm causes prolonged times of waste water flows to biological cleaning station from distant regions and industrial enterprises. Absence
of time regulation and waste water volume by the industrial enterprises in urban area water drains in adverse confluence circumstances results in emergencies. As a consequence, waste waters flow directly into the Kama river.

According to Ermolin & Skryabin [5], the waste waters flow rate of residential areas are distributed rather non-uniformly during the day, which fig. 3 illustrates.

![Figure 3. The residential areas waste water flow rate (conditional units) depending on time of days](image)

On fig. 4 the Perm sewer network scheme is shown with the indication of residential areas and enterprises. In table 2 the numbers of enterprises incorporated on the circuit in industrial regions, the daily volumes of waste water spills of each enterprise and the total volumes for the entire conditional industrial platform, are specified.

Use of the developed model of waste water flows in the Perm sewer network have allowed us to estimate the level of non-uniformity of a liquid in a biological cleaning system (BCS) (fig. 5, the bar graph). It was accepted that spills of all industrial enterprises come in regular intervals during the day, as the authentic information on volumes and times of actual liquids spills is absent.

It was expedient to re-distribute time and spill volumes of waste liquids of the separate industrial enterprises for reducing the total flow rate irregularity caused by non-uniform flow of waste waters from residential areas. For statement of the problem of rational modes search of waters by the enterprises of city, we shall accept the following condition: for each enterprise a 24-hour mode of waste water spills, conditionally shared on 8 periods by 3 hours each is established; for each such period a constant standard expense allowance of a liquid is established, so that general
daily spills volume coincides with those really existing. In this case for each enterprise 8 managing parameters are defined.

Table 2. Daily volumes of waste waters spills

<table>
<thead>
<tr>
<th>Number of region</th>
<th>Total volume of drains, m³/day</th>
<th>Number of region</th>
<th>Total volume of drains, m³/day</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>107340</td>
<td>12</td>
<td>111946</td>
</tr>
<tr>
<td>2</td>
<td>19384</td>
<td>13</td>
<td>707640</td>
</tr>
<tr>
<td>3</td>
<td>20526</td>
<td>14</td>
<td>299360</td>
</tr>
<tr>
<td>4</td>
<td>52250</td>
<td>15</td>
<td>4917</td>
</tr>
<tr>
<td>5</td>
<td>229997</td>
<td>16</td>
<td>284590</td>
</tr>
<tr>
<td>6</td>
<td>6700</td>
<td>17</td>
<td>30573</td>
</tr>
<tr>
<td>7</td>
<td>6600</td>
<td>18</td>
<td>7106</td>
</tr>
<tr>
<td>8</td>
<td>714749</td>
<td>19</td>
<td>5200</td>
</tr>
<tr>
<td>9</td>
<td>60847</td>
<td>20</td>
<td>12880</td>
</tr>
<tr>
<td>10</td>
<td>13343</td>
<td>21</td>
<td>153402</td>
</tr>
<tr>
<td>11</td>
<td>96414</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Owing to the large number of enterprises (more than 120) and residential areas, the direct solution of a problem of optimization is represented inexpedient by virtue of excessive parameters of optimization, which exceed 900.

For simplification of the solution (which requires reducing the number of parameters) we shall accept that the industrial enterprises and the residential areas are incorporated in separate sites. It can be inferred that
reduction of non-uniform waste waters received from each of allocated sites will result in the reduction of general irregularity of the loading of biological cleaning structures. Thus, solving the problem of the search for optimum modes of waste water spills by an industry of the city of Perm is shown to be determined by rational modes of waste water spills of the enterprises of separate sites.

Let \( q_i \in \mathbb{R}, \ i = 1,k \) - required flow rates of the enterprises, incorporated in industrial platforms (\( k = 8 \times m, \ m \) - number of three-hour intervals, \( m \) - quantity of platforms) of the given site. We shall determine criterion function (at a considered site) as

\[
J(q_1, \ldots, q_k) = \max_{t \in [0,T]} Q(t, q_1, \ldots, q_k) - \min_{t \in [0,T]} Q(t, q_1, \ldots, q_k),
\]

where \( T \) - researched period, is equal to one day; \( Q \) - flow rate of waste waters at an exit of a considered site. Expression (9) defines non-uniformity of the flow rate of a liquid at the final point in the sewer system of an optimized site. It is obvious, that the criterion function has the minimum value, which is equal 0.

Thus, the formulated search of the rational modes problem of a waste water’s flow rate by the enterprises of Perm can be now set as the following: to determine for each site \( n \) of sewer system such meanings of parameters \( q_i^*, \ i = 1,k \), for which

\[
J(q_1^*, \ldots, q_k^*) \leq J(q_1, \ldots, q_k) \quad \forall q_i \in \mathbb{R}, \ i = 1,k
\]

With restrictions as a system of the eqns (1) - (4), describing liquid flow in the system of pipelines, and also condition \( \sum_{i=1}^{k} q_i = Q_n^c \), where \( Q_n^c \) is the given daily average charge of total \( n \) site. In this case, for each enterprise only 7 managing parameters are defined.

For the solution of an optimum problem, the Nelder & Meed’s method was used. As initial approach of required parameters values of the enterprises spills, appropriate constant of waste water daily spills were accepted. Rational modes of the enterprises spills in the sewer system of the city are given in fig. 5.

Results of the developed mathematical model have shown that the initial non-uniformity of the liquid charge makes 8.88 - 5.26 = 3.62 m³/s. Accounts with use of results of the optimum solution have allowed to receive meaning of non-uniformity 7.58 - 5.77 = 1.81 m³/s, so the irregularity of the fluid inflow in the cleaning structures can be reduced by one half.
Figure 5. The waste waters flow rate on an sewer system exit, m³/s

Conclusion

The process of waste water flow in sewer network pipes is considered. The problem of reconciling industrial enterprises' waste water spills in Perm with reduction of non-uniform water in biological cleaning structures, caused by a large sewer network and the absence of waste water regulation, is formulated. The numerical solution of an optimum problem has allowed us to define a possible mode of polluted waters spills, allowing to reduce the non-uniformity of the loading of cleaning structures by one half. It confirms a real possibility to define the conditions of a waste water dumping which will help to reduce a loading of the biological cleaning structures.

References