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Capacity reliability and rehabilitation of deteriorated water distribution systems

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Abstract

This paper describes an approach to assessing and economically improving the condition of deteriorating water distribution infrastructure. The approach uses a first order reliability method (FORM) to determine the system capacity reliability where capacity reliability is defined as the probability that the system is able to supply the consumer with the specified quantity of potable water at or above a prescribed minimum pressure. The method is capable of recognising uncertainties in both the nodal demands and the pipe capacities. The proposed probabilistic model also provides the facility for computing sensitivity of the reliability to changes in the network parameters. The reliability sensitivity measures are effectively used to prioritize the resource allocation in making rehabilitation decisions to improve the level of service. The performance of the model is illustrated through the application to an example network.

Introduction

The primary objective of water utilities is to provide customers with cost effective, adequate potable water supply delivered on time and at the location of the demand. However, as systems age, components deteriorate and the carrying capacity of water distribution systems often decreases dramatically. The deteriorating components within the systems also become more susceptible to structural failure which may cause service interruptions to the customers. Furthermore the consumer demands imposed on the system generally increase substantially over time. The combination of these conditions may give a rise to deficiencies in performance of the system.

Improvement in the level of service to the customers supplied by an aging system can be accomplished through repair, rehabilitation, replacement or duplication of some of the deteriorated components. However, in the current financial climate budgets available for alternations and improvement to systems are very limited. The issue of how to prioritise the maintenance strategy and to best use the limited resources to ensure an appropriate level of customer service has become a very important part of utility management in the water industry.

Reliability criteria can be used as an objective means for cost effective allocation of resources for maintenance of the aging water distribution networks[1-3]. Over recent years, there has been considerable research activity in the area of development of criteria for assessment of reliability and subsequent improvement for water distribution networks[4-5]. Most of this previous work was directed at analysis and decision process for improving the reliability arising from mechanical failure of system components of water distribution networks[6-9]. Some of this work[10-11] also focused on the incorporation of the uncertainty associated with the carrying capacity of pipe in the maintenance decision making process.

This paper does not consider the effects of mechanical failure of system components on the level of service. Instead the paper focuses on evaluation, through a probabilistic hydraulic model, of the impacts of deterioration in the carrying capacity of aging water mains on the reliability of the system. The proposed probabilistic model also provides the facility for computing the sensitivity of the reliability to changes in the network parameters. This capability enables the priorisation of resources allocation in making rehabilitation decisions to improve the level of service.

First order reliability method (FORM)

FORM was originally developed for structural reliability analysis[12-13] and has recently been applied to the water resources engineering[14-15]. (The reader is referred to Yen et al.[14] and Madsen et al.[12] for a detailed treatment of the first order reliability method.)

Form for capacity reliability analysis and rehabilitation decisions

In capacity reliability analysis of water distribution networks it is necessary to define a performance function, $g(X)$, which defines the reliability in terms of the basic random variables X such that, the hypersurface defined by $g(X)=0$ divides the performance space into 'failure' and 'safe' regions where $g(X)<0$ defines the failure region and $g(X)>0$ defines the safe region and X is a vector of basic random variables, namely, the probabilistic nodal demands and the random values of pipe coefficients. Figure 1 illustrates geometrically the concept of the failure surface for the two-dimensional case. To formulate the performance function, it is first necessary to define what constitutes failure of network performance. In this paper, definition of network performance failure is based on the criterion of insufficient heads under the assumption that the nodal demands are always met. It should be noted that due to the uncertainty in the nodal demands and the values of pipe roughness, the hydraulic head at each individual node is also probabilistic. Accordingly, the capacity reliability at a nodal level is defined as the probability that the nodal demand is met *at or over* the prescribed minimum pressure.

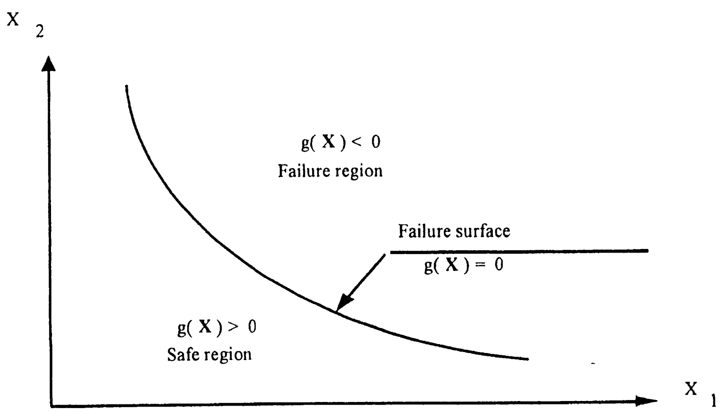

Figure 1 Concept of failure surface in two dimension space

The performance function for capacity reliability analysis for a particular node L can thus be formulated as:

$$Z_L = H_L(\mathbf{X}) - H_L^{\min} = g_L(\mathbf{X}) \tag{1}$$

where H_L is the random nodal head and H_L^{min} is the minimum head specified for demand node L.

Note that H_L is an implicit function of the random nodal demands, pipe roughness and other network parameters, i.e.,

$$\mathbf{F}(\mathbf{H},\mathbf{X}) = 0 \tag{2}$$

where $F(\bullet)$ is a vector of functions representing the mass balance at each node.

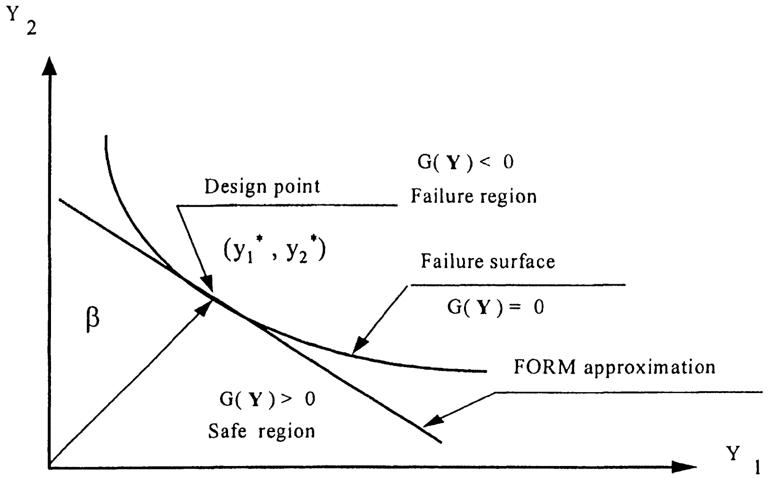

Figure 2 Transformation to standard space and approximation of failure surface in FORM

For reliability computations using FORM, it is convenient to transform the basic random variables X into the standard normal space $Y=\Gamma(X)$, such that the elements of Y are statistically independent and have a standard normal distribution. The exact nature of this transformation depends on the properties of random variables X. The transformation also maps the limit state surface defined by $g(X)=0$ into standard normal space defined by $G(Y)=g(\Gamma^{-1}(\mathbf{Y}))=0$ where the superscript -1 denotes the inverse of the transformation as shown in Figure 2. In this figure, the point (y_1^*, y_2^*) is the design point, i.e., the most probable failure point. The distance of this design point from the origin, namely, $\beta = \left(\mathbf{Y}^T\mathbf{Y}\right)^{1/2} = \sqrt{(y_1^*)^2 + (y_2^*)^2}$, is known as the reliability index (e.g. Madsen et al.[12]).

In implementing FORM for the capacity reliability estimate, an iterative optimisation procedure based on the work of Liu and Der Kiuregian[13] was found to converge on the design point for this problem in

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a few iterations. At each iteration, the value of performance function $G(Y)$ is obtained by the solution of a deterministic network simulation and the gradient of the performance function can be efficiently computed by the chain rule of differentiation, i.e.,

$$\nabla G_L(\mathbf{Y}) = \frac{\partial G_L}{\partial \mathbf{Y}} = \frac{\partial G_L}{\partial H_L}\frac{\partial H_L}{\partial \mathbf{X}}\frac{\partial \mathbf{X}}{\partial \mathbf{Y}} = \frac{\partial H_L}{\partial \mathbf{X}}\mathbf{J}_\Gamma^{-1} \tag{3}$$

where $\mathbf{J}_\Gamma^{-1}$ is the inverse of the Jacobian matrix of the transformation; $\frac{\partial H_L}{\partial \mathbf{X}}$ is the partial derivative of the nodal head at node L with respect to the changes in the values of basic random variables and can be computed by[15]

$$\frac{\partial H_L}{\partial \mathbf{X}} = -\mathbf{J}_L^{-1}\mathbf{J}_\mathbf{X} \tag{4}$$

where $\mathbf{J}_L^{-1}$ is the *i*th row of the inverse of the Jacobian matrix $\boldsymbol{J}$ of system equations; $\boldsymbol{J}_X$ is sensitivity matrix of system equations with respect to changes in the basic random variables.

Since the inverse of the Jacobian matrix is readily available from network simulation, the partial derivative, $\frac{\partial H_L}{\partial \mathbf{X}}$, and consequently the gradient $\nabla G_L(\boldsymbol{Y})$ can be obtained with very little computational effort.

If the capacity reliability of an aging water distribution network, as assessed by FORM, is unsatisfactory, a range of improvement options are possible, e.g., relining or replacement of some of the deteriorated components. The reliability sensitivity measures provide some useful information on how different maintenance policies improve the reliability of supply. These changes can be directly related to the parameters for the probability distribution of random pipe roughness used in FORM. The sensitivity of the reliability index β with respect to the distribution parameters, $\boldsymbol{P}$, of basic random variables can be written following Madsen et al.[12] as

$$\frac{\partial \beta}{\partial \mathbf{P}} = \frac{\mathbf{Y}^{*T}}{\beta}\frac{\partial \Gamma}{\partial \mathbf{P}} \tag{5}$$

where $\partial\Gamma/\partial\mathbf{P}$ is the Jacobian matrix of the transformation functions evaluated at the design point.

Assume that the basic random variables (nodal demands and pipe roughnesses) have mutually independent normal distributions. Each basic random variable can thus be transformed to a standard normal space at the design point by

$$y_i^* = \frac{x_i^* - \bar{x}_i}{\sigma_i} \tag{6}$$

where $\bar{x}_i$ and σ_i is the expected value and standard deviation of the pipe roughnesses.

Substitution of Eqn (6) into Eqn (5) gives the sensitivity of reliability index with respect to the changes $\bar{x}_i$ and σ_i as follows:

$$\frac{\partial \beta}{\partial \bar{x}_i} = -\frac{y_i^*}{\beta \sigma_i} \tag{7}$$

and

$$\frac{\partial \beta}{\partial \sigma_i} = -\frac{y_i^{*2}}{\beta \sigma_i} \tag{8}$$

Provided that the change to the value and uncertainty of the pipe roughness by a particular action, e.g., relining a pipe, is known a priori, the total impact on the capacity reliability of that action can be estimated by the first order approximation:

$$\Delta \beta = \frac{\partial \beta}{\partial \bar{x}_i} \Delta \bar{x}_i + \frac{\partial \beta}{\partial \sigma_i} \Delta \sigma_i = -\frac{y_i^*}{\beta \sigma_i} \Delta \bar{x}_i - \frac{y_i^{*2}}{\beta \sigma_i} \Delta \sigma_i \tag{9}$$

where $\Delta \bar{x}_i$ and $\Delta \sigma_i$ are the improvements in the expected value and standard deviation of the pipe roughnesses by the selected action. This approach can be applied to the range of rehabilitation measures under consideration.

Application

The example network, schematically shown in Figure 1, is used to demonstrate the methodology developed above. The network layout has the same structure as the distribution network used by Alperovits and Shamir[16]. However, the network data, which are listed in Tables 1 and 2, have been modified in this study to model a deteriorating network and to illustrate the decision processes for rehabilitation of such a network. The nodal demands and pipe roughnesses are modelled by independent normal random variables with the expected values given in Tables 1 and 2 respectively. The coefficients of variation (CoV) for nodal demands and values of the pipe roughness coefficients set to 0.3 and 0.1 respectively. In practice, demand data can be estimated from historical data while the pipe roughnesses can be approximately determined by field data.

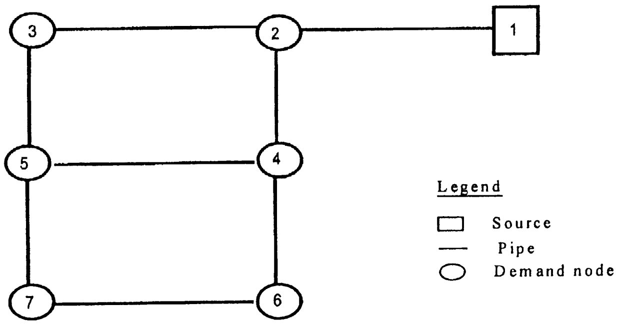

Figure 3 An example network

The initial nodal capacity reliability for the network was determined by FORM as described previously. For example, for node 7 FORM converges to β = 1.3031 in three iterations. At this point the capacity reliability of node 7 is 0.9037.

Table 1 Nodal data for network shown in Figure 3

Node (1)	Ground level (m AOD) (2)	Demand (l/s) (3)	Minimum pressure (m) (4)
1	210*	-	-
2	140	28	25
3	140	28	25
4	140	33	25
5	140	75	25
6	140	92	25
7	140	55	25

* Reservoir node

Consider that a limited budget is available for rehabilitation of some deteriorated pipes to improve the level of service to the customers around node 7 and let rehabilitation be restricted to cleaning and relining the pipes. The cost for relining each pipe is estimated using the formula suggested by Walski[17] and is given in the column 6 of Table 2. It is now necessary to decide which pipe(s) should be relined. The reliability sensitivity analysis provides useful information to how the relining action of a particular pipe will effect the reliability. Assume that relining a pipe will restore the pipe roughness coefficient to 120 and the coefficient of variation of the pipe roughness is also reduced to 0.02. Column 2 of Table 3 gives the predicted changes to the reliability index at node 7 by relining each pipe within the network.

The most cost-effective pipe to be rehabilitated for improving the level of service at node 7 is determined by ranking the ratio of the change in reliability index $\Delta\beta$ and the cost ΔC involved in relining the pipe. Column 4 of Table 3 shows the results of such ranking. In this case, the pipe connecting nodes 2 and 4 is identified as the most cost effective for rehabilitation. The second most effective pipe is that connecting nodes 4 and 6. It is interesting to note that the rehabilitation of the pipe joining nodes 5 and 4 actually has an adverse impact on the capacity reliability at node 7 since the change in the reliability index is negative at the design point.

Table 2 Pipe data for network shown in Figure 3

Pipe (1)	From (2)	To (3)	Length (m) (3)	Diameter (in*) (4)	H-W coeff. (5)	Cost for relining ($/m) (6)
1	1	2	1000	20	110	65
2	2	3	1000	16	80	60
3	2	4	1000	16	75	60
4	3	5	1000	14	80	57
5	4	5	1000	8	85	46
6	4	6	1000	12	95	54
7	5	7	1000	8	75	46
8	6	7	1000	12	90	54

* 1 inch = 25.4 mm

The procedure of using reliability sensitivity measures to rank the maintenance alternatives is illustrated above for improving the level of service at a particular node, i.e., node 7 in this example. It may be argued that the rehabilitation decisions should be made on the basis of overall improvement in the network performance. The proposed method can be easily extended to such cases provided that network reliability is appropriately defined. However, at the moment there is no generally accepted definition of network reliability which would facilitate this extension[5]. Further research into this problem is urgently needed.

Summary of conclusion

This paper has presented a model for assessing and cost effectively improving the hydraulic conditions of aging water distribution networks. The model takes into account the uncertainty in the nodal demands imposed on the system and the carrying capacity of the deteriorated water mains. The first order reliability method (FORM) is used to determine the capacity reliability of the pipe network in the context of these uncertainties. An efficient algorithm using simulation and sensitivity analysis techniques is developed to identify the most probable failure point. A framework for the selection of the most cost effective repair and maintenance strategy to improve the level of service in the aging water distribution systems is developed. The method is based on the concept of ranking the ratio of a reliability sensitivity factor and cost involved for each alternative maintenance strategy. It was shown that the sensitivity of capacity reliability with respect to a range of rehabilitation options can be obtained as an integral part of FORM. A simple example network was used to illustrate the proposed method. Results show that the proposed methodology is a very useful tool for system planners to make informed rehabilitation decisions in management of aging water distribution networks.

Table 3 Ranking of relining options

Pipe (1)	Change in reliability index $\Delta\beta$ (2)	Ratio ($\Delta\beta/\Delta C$) ($\times 10^{-6}$) (3)	Ranking (4)
1	0.1042	1.603	6
2	0.1665	2.775	5
3	0.8537	14.228	1
4	0.2076	3.642	4
5	-0.0050	-0.110	8
6	0.6925	12.824	2
7	0.5426	11.795	3
8	0.0360	0.782	7

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