Use of measured and interpolated cross-sections in hydraulic river modelling
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Abstract

Recently, a benchmarking test study of several hydraulic river models has been undertaken by the authors [1]. One of the tests being investigated was the performance of hydraulic river models in using measured and interpolated cross-sections for unsteady flows. Details of the interpolation algorithm for obtaining interpolated cross-sections are described in the paper. It is shown that the appropriate use of the interpolated cross-sections can improve the performance and accuracy of hydraulic river models. Comparisons of results obtained by interpolating hydraulic parameters and interpolating geometric cross-sections have been made. It is demonstrated that the interpolation of cross-sections is superior to the interpolation of hydraulic parameters.

1 Introduction

River flooding may occur following excessive rainfall on the contributing catchments to natural rivers. Numerical river models are increasingly used by hydraulic and river engineers for water level prediction, flood protection design and river management purposes.
Using cross-section interpolation based upon a weighted average of conveyance for water surface profile calculations of gradually varied steady state flow has been found to improve the accuracy [2]. The main objective of this paper is to report the investigations of the effects of using cross-sectional interpolations for unsteady river flows and compare two different approaches of interpolating hydraulic parameters and interpolating geometric cross-sections.

In order to test the performance of hydraulic river models in using measured and interpolated cross-sections for unsteady flows, a section of River Blythe, UK, was modelled, with the modelled river reach being 13.6 km long and the mean longitudinal bed slope being about 0.0009. There are 5 relatively steep regions located approximately at chainage 4.5 km, 5.8 km, 9.3 km, 10.7 km and 12.7 km, with the steepest bed slope being 0.0033. Three different combinations of cross-sectional data were used, i.e. case (A) 39 measured cross-sections only; case (B) 39 measured and 26 interpolated cross-sections; and case (C) 39 measured cross-sections and 30 interpolated cross-sections at different locations from (B), generally at reaches of steep gradients. The locations of the measured and interpolated cross-sections are illustrated in Figure 1 for these three cases. A 55 hour unsteady inflow hydrograph was specified at upstream boundary, together with a downstream water level boundary and two tributary inflows at chainage 4.2 km and 6.6 km. The Manning resistance coefficient used for each cross-section throughout the river is 0.03 for the main channel and 0.06 for floodplains.

![Figure 1 Locations of measured and interpolated cross-sections](image)
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Two widely used commercial hydraulic models were tested for using interpolated cross-sections, including ISIS (version 1.0) by HR Wallingford Ltd. and Halcrow, UK, and MIKE 11 (version 3.11) by Danish Hydraulic Institute, Denmark. The well-known Saint Venant equations were used by both models as the governing equations, with the finite difference method being used for solutions. The time step used was 15 second for all the model runs. For case (A), MIKE 11 produced smooth water surface profiles, whereas ISIS had difficulty in producing converged results and suggested cross-sectional interpolations.

Two cross-sectional interpolation approaches were investigated for cases (B) and (C) to obtain additional information between measured cross-sections, with one being the interpolation of hydraulic parameters, as cases (B1) and (C1), and the other the interpolation of cross-sections, as cases (B2) and (C2).

2 Interpolation of hydraulic parameters

Interpolated cross-sections can be specified between any two measured cross-sections, without providing actual geometry to the interpolated cross-sections. Instead of directly calculating hydraulic parameters from geometric data the parameters for the interpolated cross-sections are interpolated from the corresponding values of the adjacent measured cross-sections. Such an approach is referred as internal approach and needs to be programmed into the models by software developers.

Although the two models tested were both capable of interpolating the hydraulic parameters internally between measured cross-sections, ISIS model offered a better user control in defining the locations of the interpolated cross-sections at varying chainage. ISIS produced converged and smooth results for case (C1) but had difficulty in producing stable results for case (B1). On the other hand, MIKE 11 can only locate the internally interpolated cross-sections at equal distance and is not suitable to the data sets of cases (B1) and (C1) since specific locations of varying chainage for the interpolated cross-sections are required.

3 Interpolation of cross-sections

Externally generated interpolated cross-sectional geometry can be provided at desired locations. These interpolated cross-sections can be
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treated in exactly the same way as the measured cross-sections once the interpolated geometric data have been obtained. Such an approach is referred as external approach. The difficulty in this approach lies in its three-dimensional nature in interpolation.

A tri-linear interpolation algorithm to obtain the cross-sectional geometry at a specified location between two measured cross-sections has been developed. Each measured cross-section is divided into 4 portions, consisting of left floodplain, left main channel, right main channel and right floodplain. Therefore the interpolation algorithm described herewith is limited to river cross-sections having only one main channel. The corresponding widths of each portion for the upstream and downstream measured cross-sections are $UB_i$ ($i=1$ to 4) and $DB_i$ ($i=1$ to 4) respectively. Non-existence of any portion will have a zero width for that portion. The boundaries between the floodplains and the main channel can be determined by the sharp changes in lateral slopes. The main channel can be divided into two parts by the deepest point. The cross-sectional geometry is then interpolated linearly within the same portion between the two measured cross-sections. An interpolated cross-section at mid-distance between two measured cross-sections is shown in Figure 2 as an example.

This interpolation algorithm has been successfully used in this study to obtain interpolated cross-sectional geometry for cases (B2) and (C2). Both ISIS and MIKE 11 models produced converged and smooth water surface profiles for these two cases.

![Figure 2](https://example.com/figure2.png)
4 Comparisons of interpolation approaches

The instantaneous water surface profiles at $t = 25$ hour (peak flow) and $t = 55$ hour are illustrated in Figures 3 and 4 respectively for each model. It can be seen in Figure 3 that consistent results were obtained by ISIS using the internal (C1) and the external (C2) approaches with the same interpolation locations. The maximum water elevations at chainage 0 km and 7.06 km after 55 hours simulation are compared in Table 1. It can be seen that these results are consistent between the two models.

![Figure 3 Water surface profiles by ISIS model](image-url)
Figure 4 Water surface profiles by MIKE 11

Table 1 Comparisons of maximum water elevations (m)

<table>
<thead>
<tr>
<th>Case</th>
<th>Chainage</th>
<th>A</th>
<th>B2</th>
<th>C1</th>
<th>C2</th>
<th>A</th>
<th>B2</th>
<th>C1</th>
<th>C2</th>
</tr>
</thead>
<tbody>
<tr>
<td>ISIS</td>
<td>0 km</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MIKE 11</td>
<td>2.51 km</td>
<td>84.963</td>
<td>84.908</td>
<td></td>
<td>84.954</td>
<td>82.886</td>
<td>82.908</td>
<td></td>
<td>82.928</td>
</tr>
<tr>
<td>ISIS</td>
<td>9.58 km</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MIKE 11</td>
<td>77.072</td>
<td>79.458</td>
<td>79.474</td>
<td></td>
<td>79.465</td>
<td>77.119</td>
<td>77.112</td>
<td></td>
<td>77.072</td>
</tr>
</tbody>
</table>
Water elevation variations at selected locations throughout the simulation period are shown in Figures 5 and 6 respectively for each model. It can be seen from Figure 5 that higher water levels were predicted for case B2 by ISIS. There is some degree of instability in the water levels at 2.51 km during the early and later stages of the simulation for case (C1) and the results had been improved using the external approach (C2). Furthermore, the fact that ISIS can only produce results for case (B2) but not for case (B1) indicates that the interpolation of cross-sections is superior to the interpolation of hydraulic parameters.

Figure 5  Water elevation variations with time by ISIS model
It can be seen from Figure 6 that MIKE 11 produced very consistent results for cases (A), (B2) and (C2) with little variation in the results for different data sets. However, no direct comparison can be made with regard to the two interpolation approaches by MIKE 11. Nevertheless, a simple analysis on a rectangular prismatic channel can be conducted to investigate the internal approach. Two measured cross-sections of widths $W_1$ and $W_2$ and depths $h_1$ and $h_2$ may be assumed, with an interpolated
cross-section being located in the middle. The interpolated hydraulic parameters, including width $W$, depth $h$, wetted perimeter $P$, flow area $A$ and hydraulic radius $R$, with a subscript $m$ indicating the value by definition and an over-bar indicating the mean value, for the interpolated middle section may be written as:

$$\overline{W} = 0.5(W_1 + W_2) = W_m$$
$$\overline{h} = 0.5(h_1 + h_2) = h_m$$
$$\overline{P} = 0.5(P_1 + P_2) = 0.5(W_1 + W_2) + (h_1 + h_2) = W_m + 2h_m = P_m$$
$$\overline{A} = 0.5(A_1 + A_2) = 0.5(W_1h_1 + W_2h_2) \neq W_mh_m = A_m$$
$$\overline{R} = 0.5(R_1 + R_2) = 0.5\left(\frac{W_1h_1}{P_1} + \frac{W_2h_2}{P_2}\right) \neq \frac{W_mh_m}{P_m} = R_m$$

where inconsistencies in flow area and hydraulic radius interpolations are clearly demonstrated. Assuming $W_1 = W_2(1 + \alpha)$ for a general case, it is obvious that $W_1 = W_2$ when $\alpha = 0$, then

$$A_m = W_mh_m = \frac{W_1 + W_2}{2} \frac{h_1 + h_2}{2} = \overline{A} + \alpha \left(\frac{A_2}{4} - \frac{A_1}{1 + \alpha}\right)$$

It can be seen that in general $A_m \neq \overline{A}$ and $R_m \neq \overline{R}$.

The above simple analysis confirm again that the interpolation of cross-section geometry is a better approach.

5 Conclusions

The investigation results in this paper showed that it is always preferable to use measured cross-sections for all locations for any hydraulic model study. It would be useful for hydraulic models to incorporate a cross-section interpolation facility in order to overcome non-convergence problems in modelling simulations and thereby to give a better guidance with regard to where more measured cross-sections should be located. It has been demonstrated that the interpolation of cross-sections is superior to the interpolation of hydraulic parameters. A tri-linear interpolation algorithm to obtain the cross-sectional geometry at a specified location between two measured cross-sections has been presented in the paper.
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References
