



# Stress-dependent permeability in earth dam alluvial foundation

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## Abstract

The influence of stress-dependent permeability on the seepage in the foundation of an earth dam is presented, with an application in the case of Stratos dam in Greece. The objectives include the definition of the stress field and the effect on the permeability distribution in the foundation of the earth dam separately for the effective stress, the pore pressure and the total stress. The implication of the findings is that the mean square error in evaluating the total head potential at the position of the piezometers remains small, assuming that the vertical permeability in the foundation is equal to the horizontal times a factor  $K = (\text{stress in m})^{\nu}$ , where  $\nu$  is approximately -0.5 for the effective or total stress fields and approximately -1 for the pore pressure field.

## 1 Introduction

Irregular particles deposited in a gravitational field exhibit anisotropy in permeability in the horizontal and vertical directions. This anisotropy in large alluvial deposits is present even if there is no variability of the grain size, the shape distribution and the direction of deposition of the particles. The in-situ permeability tests usually give good results for the horizontal permeability, while the vertical permeability is almost unknown, especially at great depths.

The effect of stress in the permeability of the alluvium is examined in this paper. The stress field is examined separately as effective stress, pore pressure and total stress. The area of seepage is the one within the earth dam and the foundation of the dam. The assumption of the variation of the horizontal and vertical permeability with stress was made for a series of cases.

The finite differences method is used to calculate the total head potential at the nodes of a rectangular mesh placed over the cross section of the dam and its foundation. The total head potential at selected nodes of the mesh is compared with average values of piezometer readings in the foundation of the dam. The sum of squares of the difference of calculated values of the total head potential minus the average values of piezometer readings, gives the mean square error for



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each case examined. Conclusions are drawn on the effect of the stress field to the permeability distribution in the alluvial foundation.

The procedures of the study require the definition of the stress field and the testing of assumption on the permeability distribution in the foundation of the earth dam, separately for the effective stress, the pore pressure and the total stress.

## 2 Mathematical model

The total head potential  $H$  of the ground water flow within the earth dam and the alluvial foundation is a continuous scalar function of position, which in the steady state case is expressed by using the divergence operator  $\text{div}$  as

$$\text{div}(\text{grad } H) = 0. \quad \dots\dots\dots (1)$$

The total head potential  $H$  is the sum of the pressure head  $p/\rho g$ , the potential head  $y$ , and the kinetic energy head  $v^2/(2g)$ . The latter is very small and almost equal to zero. Then equation (1) is written as

$$\text{div}(\text{grad} (p/\rho g + y)) = 0. \quad \dots\dots\dots (2)$$

In the 2-dimensional space, considering the plane  $(x,y)$ , where  $x$  is the horizontal and  $y$  is the vertical, the quantity  $\text{grad}(H)$  is a vector in the direction  $x_i+y_j$  at the position  $(x,y)$ , and is expressed as

$$\text{grad}(H) = (x_i+y_j) / (x^2+y^2)^{0.5}, \quad \dots\dots\dots (3)$$

which means that the vector is in the direction of maximum rate of increase and its magnitude is equal to the rate of increase in that direction (Schey, 1973).

Equation (1) is also satisfied by the flow potential function  $Q$ , which is a conjugate harmonic function of the total head potential  $H$ . Hence, the following equation is also valid

$$\text{div}(\text{grad } Q) = 0. \quad \dots\dots\dots (4)$$

The quantity  $\text{grad } H$  analyzed in the two orthogonal directions  $x$  and  $y$  and multiplied by the permeabilities  $k_x$  and  $k_y$  of the medium, gives the velocities in the  $x$  and  $y$  directions according to the expressions

$$v_x = k_x (\text{grad } H)_x, \quad v_y = k_y (\text{grad } H)_y. \quad \dots\dots\dots (5)$$

This expression is valid if  $k_x$  and  $k_y$  are the principal components of the permeability ellipse.

The effective stress field  $\sigma$  and the pore pressure field  $\theta$ , as well as the total stress field  $\sigma+\theta$ , affect the distribution of permeabilities in the foundation of the dam in an unknown way. It is argued here, that the horizontal permeability  $k_x$  and the vertical permeability  $k_y$  in the foundation of the dam, will vary according to the stress fields  $\sigma$ ,  $\theta$ , and  $\sigma+\theta$  by a value of  $K = \sigma^\nu, \theta^\nu$ , and  $(\sigma+\theta)^\nu$ , where  $\nu$  is an exponent that will give the smallest mean square error of the calculated total head potential values minus the recorded values at the positions of the piezometers in steady state seepage.

The justification of the proposed functions  $K = \sigma^v$ ,  $\theta^v$ , and  $(\sigma+\theta)^v$  is that the stress field affects proportionally but inversely the horizontal and vertical permeability values, particularly in the areas beneath the reservoir and the earth dam. This means that the stress field affects the vertical movement of the water flow. This study defines the function of  $k$  and the value of  $v$ , which gives at the position of the piezometers total head potential values closest to the readings of the piezometers.

The reduction of pore size due to compaction or consolidation is certainly giving rise to a reduction of the vertical permeability. The alluvial foundation of an earth dam would not be considered such as highly compressive soil with variation of  $k$  due to vertical compaction, since it already has a state of stress with depth before the placement of the earth dam.

### 3 Example case history

The Stratos earth dam on the Acheloos river near the town of Agrinion, in Western Greece, is considered as the case history to evaluate the theory. The Stratos earth dam is associated with the Stratos hydropower installation, commissioned in 1989. The reservoir serves hydropower and at the same time irrigation interests. The Stratos installation includes an underground powerplant with capacity 2X75 MW in the right abutment of the dam and a small outdoor powerplant with capacity 2X3.35 MW on the left abutment of the dam.

The Stratos reservoir impounded in May 1989 has a capacity of  $80 \times 10^6 \text{ m}^3$  at a maximum operating level at el. 68.6 m. The variation of the Stratos reservoir level is usually between el. 67 m and el. 68.6 m. The maximum flood level at el. 69 m is 4 m below the crest of the earth dam, which is at el. 73 m. The geometry of the Stratos earth dam is shown in the cross section of Fig. 1.

The Stratos earth dam is a zoned dam which consists of sand gravel shells, two filter zones and a central clay core. The height of the dam above the river bed is 26 m, and the length of the dam is 1,900 m. The width at the crest is 9 m, and the width at the base is approximately 100 m. The total volume of the dam is equal to  $2.8 \times 10^6 \text{ m}^3$ .

A cutoff wall 20m-deep made of cement-bentonite slurry is located below the clay core to control seepage in the foundation of the dam. The specific cross section at station 1+450 m, close to the left abutment of the dam, was chosen as having the largest thickness of alluvium to the impermeable bedrock (approximately 45 m). The cutoff wall, the alluvial foundation and the bedrock at station 1+450 m of the Stratos earth dam are also shown in Fig. 1.

The seepage within the body of the dam and the foundation is monitored with a series of pneumatic, hydraulic and electric piezometers. The piezometer readings employed in this study refer to the pneumatic piezometers placed in the foundation of the dam. Readings taken twice a month in the period July 1989 to June 1991 are used in this analysis.

Twelve pneumatic piezometers were placed in the foundation of the cross section at station 1+450 m of the Stratos earth dam. From these the 10 piezometers are shown in the cross section of Fig. 1. The names and positions of the piezometers in the foundation of the dam are: Pf-40 and Pf-44 (5 m upstream), Pf-41, Pf-45 and Pf-48 (5m downstream), Pf-43, Pf-46 and Pf-64 (15 m downstream) and Pf-42 and Pf-49 (30 m downstream), where the indication upstream or downstream refers to the cutoff wall. The two piezometers not included in Fig. 1 are piezometers Pf-62 and Pf-63, which indicated almost the

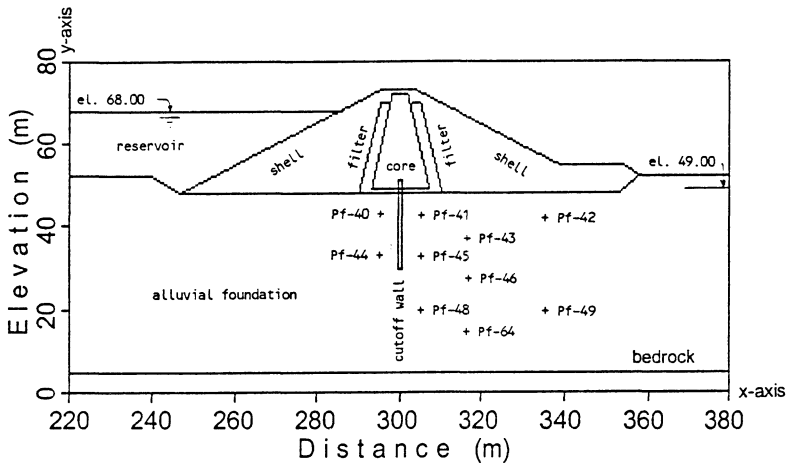


Fig. 1. Cross section of the earth dam.

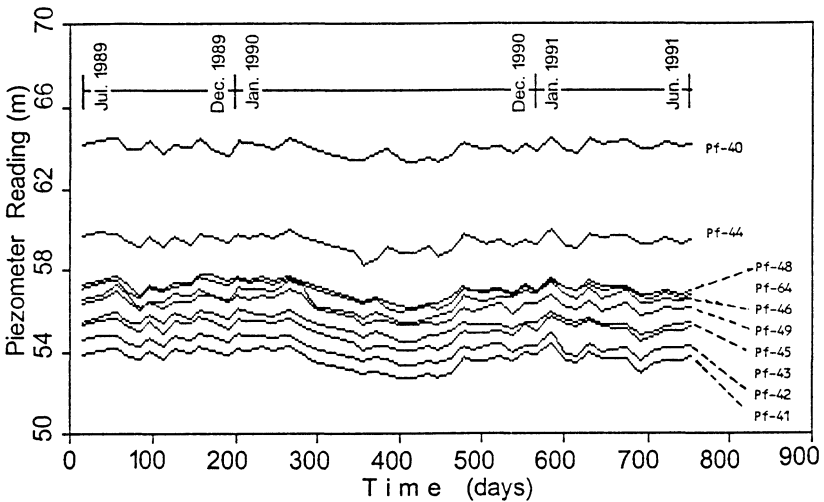


Fig. 2. Variation of piezometer readings versus time.

same readings as piezometers Pf-48 and Pf-64 respectively and were located at the position of Pf-48 and Pf-64 respectively.

## 4 Piezometer readings

The variation of the reservoir level upstream from the dam over the period examined indicated a minimum at el. 67.5 m and a maximum at el. 68.5 m, with an average value at el. 68 m. The water level of the re-regulating reservoir, 200 m downstream from the dam axis, which re-regulates the flow from the small



powerplant for irrigation purposes, is at el. 51 m in the period April to September. In the non-irrigation period October to March the water level is at el. 49 m.

The readings of the 10 piezometers in the foundation of the earth dam are plotted versus time (days) in Fig. 2, within the total period examined. A seasonal variation of the piezometer readings with lower values in the summer months and higher values in the winter months is indicated by the diagrams of Fig. 2. Although over the summer months the re-regulating reservoir is at el. 51 m and over the winter months at el. 49 m, high piezometer readings are present in the winter months and not in the summer months. The explanation to this may be attributed to conditions prevailing in the upstream sediment of the reservoir, as it happens in examining seepage in concrete dams (Kalkani, 1992).

The point of showing the annual variation in the hydraulic head of the piezometers is to make the reader understand the small variation between summer and winter readings, and as well understand the statistics (mean values and standard deviations) of Table 1. The actual flow net from the field data, which if drawn by hand would assist the reader in understanding how good the simulations are, was not drawn because of the discontinuity in total head potential at the cutoff wall.

The distance of the piezometers as plus (downstream) or minus (upstream) in meters from the cutoff wall and the exact elevation of the piezometers in meters are shown in Table 1. Also, the average values of the piezometer readings in meters and the standard deviation in meters for the total period and separately for the winter periods for each of the 10 piezometers of Fig. 2 are shown in Table 1. The winter average values were used to define the square error of their difference from the total head potential values calculated by using the numerical model.

Upstream from the cutoff wall the average piezometer values are in the order of 59-64 m, while downstream they are in the order of 53-56 m. The presence of the cutoff wall results in 3-11 m drop of the total head potential (pressure head plus potential head) from upstream to downstream. The total head potential values calculated at the position of the piezometers for some of the cases examined are shown in Table 1, along with the square error calculated as the squared difference of the total head potential minus the winter average piezometer value.

## 5 Assumptions of the model

The distribution of the total head potential in the earth dam and the foundation is computed by applying the finite differences method (Rushton and Redshaw, 1978) which solves the Laplace equation in the (x,y) plane. Equation (2) is solved numerically to give the distribution of the total head potential over a rectangular mesh. The coordinate axes at the modelled region are shown in Fig. 1. The x-axis in the model is from zero to 600 m (in Fig. 1 the region from 220 m to 380 m is shown) and the y-axis is from zero to 70 m. At the intersections of the horizontal to vertical lines of the rectangular mesh the solution of the differential equations is performed. The mesh consists of 48 horizontal lines and 65 vertical lines, which cover the earth dam and the alluvial foundation of the cross section examined.

The Laplace equation for anisotropic media is not presented here, since the anisotropy is represented by permeability values at each node, which are assigned during the Laplace equation solution. At the upstream slope of the dam and at the bottom of the reservoir a constant total head potential is considered equal to



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68 m (average reservoir level). Also, at the riverbed downstream of the dam and at a distance of 200 m from the centerline of the dam a constant total head potential equal to 49 m is considered (level of the re-regulating reservoir in the winter months). In the specific case of the earth dam examined, the solution of the equations of the steady state seepage considers the impermeable boundary at a depth of 45 m under the base of the dam on the rock surface. The constant head potentials and the impermeable rock boundary are shown in Fig. 1.

The effective stress field is calculated as the sum of the weights at the nodes of the rectangular mesh of the dam and the foundation, from top to bottom in the earth material. The computer (written by the author) was verified by comparing the results to analytical solutions and it has been used in situations of studying seepage under concrete dams (Kalkani, 1992).

The permeabilities considered applicable to the materials of the zones of the dam and the foundation are given here: a) permeability of the core of the dam (laboratory tests) and the cement-bentonite slurry cutoff wall (specifications)  $1 \times 10^{-8}$  m/s, b) permeability of the filter zones and shells (laboratory tests)  $1 \times 10^{-5}$  m/s, and c) permeability of the alluvial foundation (field tests)  $1 \times 10^{-4}$  m/s.

Information regarding the methods or number of laboratory and field tests conducted was not available. Hence, no summary of statistics relating to laboratory permeability results is presented here.

The attention of this paper is concentrated to the effect of the stress field, while the best available data are the permeabilities in the horizontal direction. Sensitivity runs with permeability values higher and lower than the ones assumed gave the same results on the total head potential distribution but higher or lower total flow values respectively.

Regarding the permeability in the alluvial foundation, the in-situ permeability of  $1 \times 10^{-4}$  m/s was used mostly as the horizontal permeability  $k_x$  in the analysis. The vertical permeability  $k_y$  was assumed varying in the different cases examined as  $k_y = K k_x$ . The value of  $K$  was considered having either constant values  $10^v$  or values depending on the effective stress field  $\sigma^v$ , the pore pressure field  $\theta^v$ , and the total stress field  $(\sigma+\theta)^v$ . The values of  $v$  vary from 0 to -1.5.

## 6 Application of the model

The calculations of the total head potential are made in consecutive iterations, assuming that at each mesh point the total head potential is equal to the elevation of the mesh point (potential head) plus the pressure head (the zero potential head horizon is assumed at elevation zero). The total head potential calculations are repeated approximately 300-400 times, which however is controlled by a convergence error limit for total head potential equal to 0.001 m. In the cases where the pore pressure and the total stress effect is examined, the  $K$  value is determined at each iteration according to the hydraulic pressure at each mesh point as it was calculated at the previous iteration.

At this point, the specific explanation of how the stress dependent parameters are calculated and updated between iterations, to provide more insight for the reader to get a physical appreciation of how the values change is as follows: as soon as the new values of total head potential are established at each iteration, the permeability values at the nodes of the mesh are re-evaluated according to the pressure head.

The application of the square error is significant enough to make conclusions, however there are many factors not included in this study, in both recorded and calculated values. Factors not included in recorded values are the variation of

Table 1. Piezometer name, position, average value and standard deviation, as well as calculated total head potential and square error for the cases (a)  $K = 10 \exp(0)$ , (b)  $K = (\sigma + \theta) \exp(-0.5)$ , and (c)  $K = \theta \exp(-1)$ , where  $K = k_y/k_x$  and  $k_x = 10^{-4}$  m/s.

| name  | piezometer position |           | readings  |             |           |             | total head potential |         |         | square error         |                      |                      |
|-------|---------------------|-----------|-----------|-------------|-----------|-------------|----------------------|---------|---------|----------------------|----------------------|----------------------|
|       | u/s-d/s (m)         | elev. (m) | aver. (m) | st.dev. (m) | aver. (m) | st.dev. (m) | (a) (m)              | (b) (m) | (c) (m) | (a) (m) <sup>2</sup> | (b) (m) <sup>2</sup> | (c) (m) <sup>2</sup> |
| Pf-40 | -5.0                | 44.84     | 64.05     | 0.31        | 64.13     | 0.25        | 63.25                | 65.78   | 66.52   | 0.77                 | 2.72                 | 5.71                 |
| Pf-44 | -5.0                | 35.00     | 59.44     | 0.37        | 59.58     | 0.26        | 62.66                | 64.14   | 64.43   | 9.49                 | 20.79                | 23.52                |
| Pf-41 | 5.0                 | 44.84     | 53.69     | 0.47        | 53.96     | 0.27        | 58.80                | 56.39   | 55.42   | 23.43                | 5.90                 | 2.13                 |
| Pf-45 | 5.0                 | 35.00     | 55.45     | 0.45        | 55.71     | 0.30        | 59.30                | 57.57   | 56.62   | 12.89                | 3.46                 | 0.83                 |
| Pf-48 | 5.0                 | 20.00     | 57.13     | 0.43        | 57.37     | 0.29        | 60.33                | 60.32   | 59.92   | 8.76                 | 8.70                 | 6.50                 |
| Pf-43 | 15.0                | 40.06     | 55.08     | 0.51        | 55.36     | 0.29        | 58.44                | 56.62   | 55.68   | 9.49                 | 1.59                 | 0.10                 |
| Pf-46 | 15.0                | 30.00     | 56.61     | 0.51        | 56.91     | 0.29        | 58.79                | 58.46   | 58.14   | 3.53                 | 2.40                 | 1.51                 |
| Pf-64 | 15.0                | 19.32     | 57.00     | 0.44        | 57.27     | 0.26        | 59.10                | 59.56   | 59.41   | 3.35                 | 5.24                 | 4.58                 |
| Pf-42 | 30.0                | 44.84     | 54.28     | 0.48        | 54.51     | 0.35        | 56.64                | 55.60   | 54.98   | 4.54                 | 1.19                 | 0.22                 |
| Pf-49 | 30.0                | 20.00     | 56.24     | 0.47        | 56.48     | 0.33        | 57.10                | 58.11   | 58.39   | 0.38                 | 2.66                 | 3.65                 |

Note: u/s and d/s is upstream and downstream from the cutoff wall indicated with (-) or (+) signs respectively

Table 2. Square error (squared difference of piezometer reading minus total head potential at the position of the piezometer) and mean square error (MSE) in  $m^2$  of the 10 piezometers, for horizontal and vertical permeability in the alluvial foundation of the earth dam equal to  $k_x = 10^{-4}$  m/s and  $k_y = K k_x$  respectively.

| K (1)                              | square error for piezometers |           |           |           |           |           |           |           |            |            |       | MSE (12) |
|------------------------------------|------------------------------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|------------|------------|-------|----------|
|                                    | Pf-40 (2)                    | Pf-41 (3) | Pf-42 (4) | Pf-43 (5) | Pf-44 (6) | Pf-45 (7) | Pf-46 (8) | Pf-48 (9) | Pf-49 (10) | Pf-64 (11) |       |          |
| (10) $\exp(0)$                     | 0.77                         | 23.43     | 4.54      | 9.49      | 9.49      | 12.89     | 3.53      | 8.76      | 0.38       | 3.35       | 7.66  | (a)      |
| (10) $\exp(-0.5)$                  | 0.10                         | 14.98     | 3.46      | 6.00      | 14.44     | 8.88      | 3.42      | 9.55      | 1.49       | 4.67       | 6.70  |          |
| (10) $\exp(-1)$                    | 2.50                         | 7.08      | 1.66      | 2.64      | 19.54     | 4.93      | 3.57      | 9.92      | 3.96       | 6.40       | 6.20  | *        |
| (10) $\exp(-1.5)$                  | 6.00                         | 2.56      | 0.48      | 0.64      | 22.56     | 2.66      | 4.33      | 10.30     | 7.67       | 8.24       | 6.54  |          |
| ( $\sigma$ ) $\exp(0)$             | 0.77                         | 23.43     | 4.54      | 9.49      | 9.49      | 12.89     | 3.53      | 8.76      | 0.38       | 3.35       | 7.66  |          |
| ( $\sigma$ ) $\exp(-0.5)$          | 2.34                         | 6.60      | 1.37      | 1.90      | 20.61     | 3.88      | 2.46      | 9.00      | 2.34       | 5.15       | 5.57  | *        |
| ( $\sigma$ ) $\exp(-1)$            | 9.12                         | 0.66      | 0.01      | 0.04      | 29.48     | 0.23      | 2.34      | 8.53      | 6.66       | 6.86       | 6.39  |          |
| ( $\sigma$ ) $\exp(-1.5)$          | 11.90                        | 0.07      | 0.12      | 0.88      | 39.69     | 0.06      | 3.20      | 15.29     | 17.06      | 14.75      | 10.30 |          |
| ( $\theta$ ) $\exp(0)$             | 0.77                         | 23.43     | 4.54      | 9.49      | 9.49      | 12.89     | 3.53      | 8.76      | 0.38       | 3.35       | 7.66  |          |
| ( $\theta$ ) $\exp(-0.5)$          | 0.74                         | 10.18     | 2.13      | 3.39      | 17.06     | 5.52      | 2.37      | 8.41      | 1.54       | 4.33       | 5.57  |          |
| ( $\theta$ ) $\exp(-1)$            | 5.71                         | 2.13      | 0.22      | 0.10      | 23.52     | 0.83      | 1.51      | 6.50      | 3.65       | 4.58       | 4.88  | (c)*     |
| ( $\theta$ ) $\exp(-1.5)$          | 9.99                         | 0.32      | 0.01      | 0.31      | 29.81     | 0.00      | 1.77      | 8.29      | 8.29       | 7.40       | 6.62  |          |
| ( $\sigma + \theta$ ) $\exp(0)$    | 0.77                         | 23.43     | 4.54      | 9.49      | 9.49      | 12.89     | 3.53      | 8.76      | 0.38       | 3.35       | 7.66  |          |
| ( $\sigma + \theta$ ) $\exp(-0.5)$ | 2.72                         | 5.90      | 1.19      | 1.59      | 20.79     | 3.46      | 2.40      | 8.70      | 2.66       | 5.24       | 5.47  | (b)*     |
| ( $\sigma + \theta$ ) $\exp(-1)$   | 9.36                         | 0.50      | 0.00      | 0.09      | 28.94     | 0.17      | 2.50      | 8.94      | 8.07       | 7.67       | 6.62  |          |
| ( $\sigma + \theta$ ) $\exp(-1.5)$ | 12.11                        | 0.03      | 0.19      | 1.10      | 42.25     | 0.10      | 3.80      | 18.66     | 21.44      | 18.40      | 11.81 |          |

\* minimum value in each set of K-values



readings due to reservoir fluctuation history upstream and that of the re-regulating reservoir downstream, as well as the effect of high and low temperatures to piezometer readings.

The effect of the constant head boundaries at the left and right boundaries of the modelled domain is not accounted since these boundaries are at distances 300 m upstream and 300 m downstream from the dam axis, which is 5 times greater than the height of the dam plus the depth of the foundation. To justify the simplified field conditions and the use of the theory, winter recordings were used in the analysis, and total head potential was considered that of the winter levels of the reservoirs upstream and downstream of the dam.

Factors not included in calculated values are the unknown higher or lower permeability in certain areas in the alluvium, as well as the constant total head potential values at the boundaries of the finite differences model. The range of variability in vertical permeability with depth in these simulations is not known. The values of horizontal permeability were determined in the field by field tests or in the laboratory. The vertical permeability will be that which will give flownets close to those observed in the field.

Efforts to justify the assumptions regarding which factors have insignificant effect on the analysis, require the performance of a small sensitivity analysis, to show which of the factors can be ignored. The effect of the position of the boundaries can be easily investigated by performing a sensitivity analysis, which is usually a standard part of any modelling exercise.

The sensitivity analysis indicated that the left and the right constant head boundaries should be at a distance twice the height of the dam plus the depth of the permeable foundation. Also, the finite differences mesh should have nodes as close as possible to the positions of the piezometers, so that the comparison of the calculated to measured heads is more accurate.

## 7 Cases studied and results

Sixteen cases of variation of permeability in the vertical direction in the alluvial foundation were examined and the results are shown in Table 2. In these cases, the horizontal permeability is assumed equal to  $k_x = 1 \times 10^{-4}$  m/s, and the vertical permeability  $k_y$  is assumed equal to  $K k_x$ , where  $K$  is a factor equal to  $A^v$  shown in the first column of Table 2. The values of  $A$  are 10,  $\sigma$ ,  $\theta$ , and  $\sigma + \theta$ , and the values of  $v$  are 0, -0.5, -1 and -1.5. The values of square error for  $v = 0$  are the same for all  $A$ 's, since  $A^0 = 1$ .

Table 2 includes the values of the squared difference of the calculated total head potential at the position of each piezometer minus the winter average reading of the piezometer. The names of the 10 piezometers considered in the foundation of the dam are shown as titles of the columns 2 to 11. Column 12 indicates the mean square error, which is the mean value of the squared errors of the 10 piezometers and corresponds to the  $K$  value of column 1.

The mean square error in Table 2 is less or equal to 6.2 m<sup>2</sup> in the cases of  $K = 10 \exp(-1)$ ,  $\sigma \exp(-0.5)$ ,  $\theta \exp(-1)$  and  $(\sigma + \theta) \exp(-0.5)$ . The mean square error in the above cases annotated with an asterisk on Table 2, is 6.20, 5.57, 4.88 and 5.47 m<sup>2</sup> and corresponds to error plus or minus 2.49, 2.36, 2.21 and 2.34 m respectively. The mean square error value is greatly affected by the square error of piezometer Pf-44 which indicates extremely large square error compared to other piezometers. If Pf-44 is not considered in the calculation of the mean square error, then the previously calculated errors are reduced to 2.17, 1.97, 1.68 and 1.94 m (considering the square errors of 9 piezometers). These errors are

equal to 4.8 to 6.2 times greater than the highest standard deviation (0.35) of the piezometer readings in the winter period shown in Table 1.

The flownets of three specific cases (a), (b) and (c) shown in Table 1, with  $k_x = 1 \times 10^{-4}$  m/s and  $k_y = K k_x$ , where  $K = 10 \exp(0)$  in case (a),  $K = (\sigma + \theta) \exp(-0.5)$  in case (b) and  $K = \theta \exp(-1)$  in case (c), are shown in Figs 3, 4, and 5 respectively. The calculated total head potential and the square error of cases (a), (b) and (c) are shown in the six last columns of Table 1.

The distribution of the total head potential in Figs 3, 4, and 5 is shown as equal total head potential lines at 1 m head intervals ranging from 54 m to 68 m. The distribution of the total head potential between 49 m to 53 m is not shown on the figures as being further downstream from the cross section of the dam. The distribution of the flow in the foundation is indicated as equal value flow lines from bottom to top at every  $0.0001 \text{ m}^3/\text{s}$  flow intervals in Figs 3, 4, and 5.

In the case of  $\exp(0)$  in Table 2, the permeability in both x and y directions is constant equal to  $k_x = k_y = 10^{-4}$  m/s. In this case the mean square error is  $7.66 \text{ m}^2$ , and the mean error is  $2.77 \text{ m}$ . The flownet in this case is shown in Fig. 3 and the calculated flow is  $4.063 \times 10^{-4} \text{ m}^3/\text{s}$ .

The K values in Table 2 that give small mean square errors are  $K = (\sigma + \theta) \exp(-0.5)$  and  $K = \theta \exp(-1)$ . The flownets for these cases are shown in Figs 4 and 5 respectively, and the calculated flows are  $2.397 \times 10^{-4} \text{ m}^3/\text{s}$  and  $1.636 \times 10^{-4} \text{ m}^3/\text{s}$  respectively. These values of flow are almost half of the values of flow calculated in the case of  $K = 10 \exp(0)$ . The calculated values of the seepage flux vary by factors 2 to 3. The results are not compared to field observations. However, the simulation that gives realistic values is the one that gives the smallest mean square error, and that one is for  $K = \theta \exp(-1)$ .

## 8 Discussion

Homogeneous soil with constant permeability in the x and y gives exactly the same distribution of total head potential as in Fig. 3, but different flow distribution. The total flow will be larger if the value of permeability in the foundation is larger than  $10^{-4}$  m/s. Hence, homogeneity in the foundation of the dam with the same permeability in both x and y will give constant mean square error, the same as in the case of  $\exp(0)$  in Table 2.

The flow in the lower one third of the foundation of the two anisotropic cases in Figs 4 and 5, is essentially horizontal (from the left to the right boundary). The flow volume is controlled by the horizontal permeability and the heads specified at the left and right boundaries. The flow volumes reported may be considered reliable, if there is not a sensitivity analysis presented with respect to the type of boundary imposed or the effect of the size of the modelled domain. In this case, if the grid was from  $x = 220 \text{ m}$  to  $x = 380 \text{ m}$ , it would seem reasonable to enlarge the grid in the upstream and downstream directions and to see what effect moving the boundaries has on the results. However, the flownets presented are having grids with boundaries at  $x = \text{zero}$  and  $x = 600 \text{ m}$ , which results in a unique flownet for a larger or smaller range (plus or minus 100 m) by moving the boundary upstream or downstream from the existing boundaries.

The mean square error is greater than  $5 \text{ m}^2$  in most cases of Table 2. Small values of the mean square error less than  $5.57 \text{ m}^2$  (error plus or minus 2.5 m) are present in the cases of exponent  $v$  in the vicinity of -0.5 considering the effect of effective stress or total stress fields, and in the vicinity of -1 considering the effect of pore pressure field.

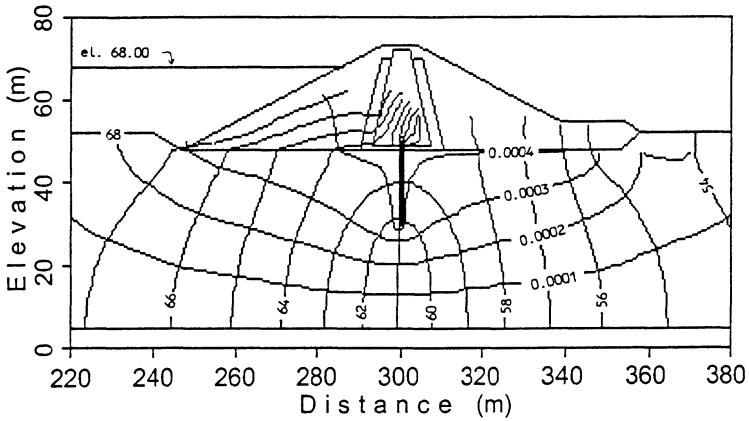


Fig. 3. Flownet for  $k_x = 10^{-4}$  m/s,  $k_y = 10 \exp(0) k_x$ .

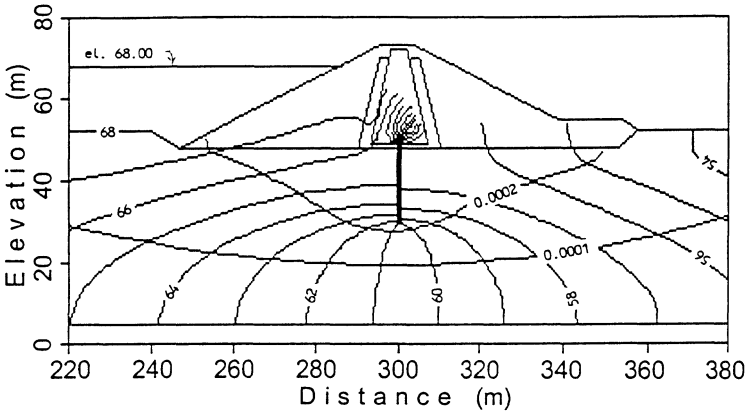


Fig. 4. Flownet for  $k_x = 10^{-4}$  m/s,  $k_y = (\sigma + \theta) \exp(-0.5) k_x$ .

In the homogeneous alluvial foundation of earth dams (such as the Stratos dam), it is stated as a conclusion, that the mean square error of the calculated total head potential at the position of the piezometers minus the average value of the piezometer readings remains small when the vertical permeability in the foundation is equal to the horizontal times a factor  $K = (\text{stress in m})^v$ , where  $v$  is in the vicinity of -0.5 for the effective or total stress fields and in the vicinity of -1 for the pore pressure field.

Assuming that the conclusion is accurate, the design engineer who is in charge with the calculations of the dimensions of the cutoff wall and the water losses of the reservoir in the foundation of the earth dam, will be on the safe side of assumptions, if  $k_y = K k_x$ , where  $K = (\text{pore pressure in m})^{-1}$ . The pore pressure value is calculated from a first estimate of the flownet in the foundation of the dam in steady state seepage condition.

The simulation indicates a specific value for the exponent in the different formulations that will provide the smallest error when compared to the measured

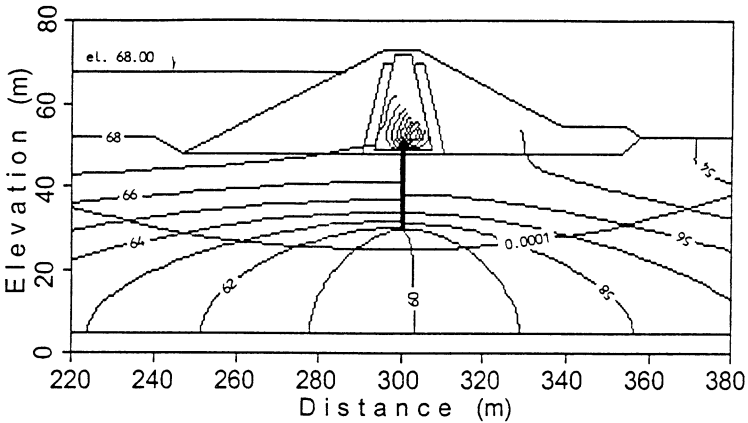


Fig. 5. Flownet for  $k_x = 10^{-4}$  m/s,  $k_y = \theta \exp(-1) k_x$ .

heads. The physical significance of these values is that the permeability in the  $y$  direction is inversely proportional to stress (in m) and especially to pore pressure. The insight into the behaviour of the porous medium drawn from this is that water seepage in the vertical direction in the alluvium is much more difficult as the pore pressure is greater.

What an engineer can draw from this study is that considering the horizontal permeability from field tests, and assuming variation of the horizontal and vertical permeability according to the conclusions of this paper, he can develop a flownet close to the one expected in the field and is indicated by piezometer readings.

The paper is focussing on confining stress as a key parameter in the permeability distribution at the foundation of the dam. Although, this may have a smaller effect than stratigraphy in case there are distinct layers of different materials, this is not the case here, because the alluvium indicated no separate layers. In order to better match the observed total head, the ratio of the vertical to the horizontal permeability was adjusted with depth so that the numerical model gave the best values for the permeabilities in the foundation of the dam.

## 9 Conclusions

The influence of anisotropy, which depends on the effective stress, pore pressure and total stress fields, on the permeability in the alluvial foundation of an earth dam is evaluated in the steady state seepage condition. The finite differences simulation is used to compare computed pressure heads with piezometer readings in the alluvial foundation of an earth dam.

It is concluded that the mean square error in evaluating the total head potential at the position of the piezometers remains small, assuming that the vertical permeability in the foundation is equal to the horizontal times a factor  $K = (\text{stress in m})^v$ , where  $v$  is approximately  $-0.5$  for the effective or total stress fields, and approximately  $-1$  for the pore pressure field.



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**Notations**

H = ground water potential, m

Q = flow potential, m<sup>3</sup>/s

x,y = coordinates in the (x,y) Cartesian system, m

x = distance, m

y = elevation, m

k, k<sub>x</sub>, k<sub>y</sub> = permeability, and in the x and y directions, m/s

v, v<sub>x</sub>, v<sub>y</sub> = velocity, and in the x and y directions, m/s

p = pressure equal to head of water column, m

ρ = density, gr/cm<sup>3</sup>

g = acceleration of gravity, 9.81 m/s<sup>2</sup>

σ = effective stress as head of water column, m

θ = pore pressure as head of water column, m

σ+θ = total stress as head of water column, m

v = exponent

grad =  $\partial/\partial x + \partial/\partial y + \partial/\partial z$

div(grad) =  $\partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$

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