Sediment transport modeling by coupling technique

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Abstract

A coupled mobile-bed model which is capable of simulating the water movement and bed evolution in an alluvial channel was developed in this study. This model attempts to couple the water continuity equation, water momentum equation, sediment continuity equation, and hydraulic sorting equation. The governing equations are discretized by the use of the Preissmann's four-point scheme. This coupled model was examined through the comparison with some experimental data sets. From the results, one can conclude that this newly proposed coupled model can reasonably simulate the water and nonuniform sediment movements in a channel.

Introduction

The numerical model has been recognized to be an efficient tool for studying the sediment transport phenomenon in a natural streams. Many unsteady mobile bed models have been developed. Most of these models ignore the strong coupling between water and sediment movements, and solve governing equations sequentially and independently in one time step. Such models are referred as uncoupled models. The validity of the uncoupled technique rests on the assumption that the change in any one variable during a time step is small enough that its effect on the other variables during the time step can be ignored. Lyn (1987) indicated the drawbacks of uncoupled model. Lyn applied the perturbation methods to show that the uncoupled are not capable of satisfying either a general boundary condition or an arbitrary initial condition. Therefore, this kind of uncoupled model is limited to cases where the changes in the boundary conditions are negligible. For a long term simulation without any rapid
change of conditions, the use of uncoupled model may be suitable. For applications in which the discharge and sediment input are highly variable, a coupled model, capable of solving the governing equation simultaneously, is needed.

Recently, coupled models using finite-difference schemes for simulating the uniform sediment movement, e.g., Rahuel et al. (1989) and Correia et al. (1992), have been developed. Lai (1991) has developed a three-characteristics numerical model using a so-called multimode scheme with the capability of simulating coupled, uniform-sediment alluvial-channel flow. The above-mentioned coupled models can only suitable for the uniform sediment transport simulation. In order to meet the natural physical phenomenon, it is inevitable that the nonuniform sediment transport phenomenon needs to be considered in the model development. Most of the known uncoupled models have equipped with the capability of simulating the nonuniform sediment movement. Whereas, Holly and Rahuel (1990) recently have developed a coupled model which incorporates with nonequilibrium sediment transport theory to meet nowadays complicated alluvial-river practices.

This paper will introduce a coupled model which solves the water flow continuity, momentum, sediment continuity, and sorting equations simultaneously in one time step. Several sets of experimental data which were performed by Hydraulic Research Laboratory of National Taiwan University (Yen et al., 1988) are used here to examine the model applicability.

Governing Equations

The basic one-dimensional governing equations for unsteady water and nonuniform sediment transport in an open channel can be written as follows:

\[
\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0
\]  
(1)

\[
\frac{\partial A}{\partial t} + \frac{\partial}{\partial x} \left( \frac{Q^2}{A} \right) + gA \frac{\partial y}{\partial x} + gA \frac{Q}{K^2} = 0
\]  
(2)

\[
(1 - p) \frac{\partial A_b}{\partial t} + \frac{\partial Q_s}{\partial x} = 0
\]  
(3)

\[
\frac{\partial F_l}{\partial t} = -\frac{1}{\alpha(1 - p)} \left( \frac{\partial Q_s}{\partial x} - F_b \frac{\partial Q_s}{\partial x} \right)
\]  
(4)

in which \( x \) = distance along the reach, \( t \) = time, \( y \) = water surface elevation, \( A \) = flow area; \( Q \) = water discharge; \( K \) = conveyance, \( g \) = gravitational constant; \( p \) = porosity; \( A_b \) = scouring/deposition volume; \( Q_s \) = sediment discharge, \( \alpha \) = mixing
layer thickness; $F_j = \text{fraction of sediment in the } j\text{th size range}$, $d_{s_j}$; $F_b = F_j$ in case of bed aggradation, and $F_b = F_{j0}$ in case of bed degradation, in which $F_{j0} = \text{fraction of sediment in the } j\text{th size range of the parent bed}$. Eqs. (1) and (2) are the continuity and momentum equations for the water flow respectively. Eq. (3) is the equation for conservation of bed material, and Eq. (4) is the sorting equation for each size fraction of sediment. Sediment transport formula can be written as

$$Q_s = \sum_{j=1}^{N} Q_{s_j} = \sum_{j=1}^{N} F_j f_j (Q, y, d_{s_j}, \ldots)$$  \hspace{1cm} (5)

where $f_j$ denotes transport capacity of $d_{s_j}$, which is a function of flow intensity and sediment properties. Here, Engelund-Hansen formula (1967) for estimating bed material discharge capacity is used:

$$Q_s = 0.05 Bu^2 \left[ \frac{d_{s0}}{g (s_g - 1)} \right]^{1/2} \left[ \frac{\gamma_h s_f}{(\gamma_s - \gamma) d_{s0}} \right]^{3/2}$$ \hspace{1cm} (6)

where $s_g =$ specific gravity of the sediment, $d_{s0} =$ median particle size; and $\gamma_s$, $\gamma =$ specific weights of sediment and water, respectively. The transport capacity $f_j$ for each size class $d_{s_j}$ is estimated simply from Eq. (6), in which $d_{s0}$ is replaced by $d_{s_j}$ and with a correction factor, $C_f$, of the form:

$$C_f = \frac{1}{d_{s0}} \sum_{j=1}^{N} d_{s_j}$$ \hspace{1cm} (7)

**Discretization of Equations**

The Preissmann four-point finite difference scheme (Cunge et al., 1980) is used to discretize Eqs. (1) to (4). After use of the Preissmann's discretization, the algebraic forms of Eqs. (1) to (4) can be obtained. Then with the use of Taylor's Series expansions as described by Liggett & Cunge (1975), the discretized form of Eqs. (1) to (4) can be linearized into the following matrix from:

$$[a_i] \{W_i\} + [b_i] \{W_{i+1}\} + \{c_i\} = 0, \quad i = 1, 2, \ldots, \ M - 1$$ \hspace{1cm} (8)

in which,

$$\{W_i\} = \begin{bmatrix} \Delta y_i \\ \Delta Q_i \\ \Delta z_i \\ \Delta F_{i,j} \end{bmatrix}$$ \hspace{1cm} (9)
\[ \Delta y_i = \text{increment of water level at section } i; \quad \Delta Q_i = \text{increment of discharge at section } i; \quad \Delta z_i = \text{increment of bed level at section } i; \quad \Delta F_{ij} = \text{increment of size fraction } j \text{ at section } i; \quad [a], [b], \text{ and } [c] \text{ are matrix coefficients.} \]

**Solution Algorithm**

As described in the previous section, a discretized form of the basic governing equations, together with boundary conditions, forms a system of algebraic equations in terms of unknown variables at the time level \((N+1) \Delta t\). The large system of non-linear algebraic equations can be solved by a successive iteration scheme. However, when the simulated model contains a tremendous amount of computational points, the advantages of the implicit scheme used in this paper are lost. In order to solve the system equations more efficiently, a matrix double-sweep algorithm was used which is analogous with the conventional scalar double-sweep method for the St. Venant equations (Cunge et al., 1980).

**Model Test**

To demonstrate the capability of the proposed coupled model for simulation of nonuniform-sediment mobile-bed hydraulics, several tests are performed and presented here. Some data sets extracted from experiments carried out in the Hydraulic Laboratory of National Taiwan University (1988) are used for the test simulation. The experiments are performed in a 72m long and 1 m wide flume with 0.0035 slope. The water flow discharge was kept constant which is 0.12 cms. The mean particle size used is 1.8 mm and the geometric standard deviation is 3.2. Along the flume there are six sections being equipped with instruments to measure the variation of bed level and water surface level. This physical model was designed to investigate the bed evolution caused by the overloading and underloading conditions. Before the test cases are performed, the equilibrium condition at the initial stage is usually required. For performing the equilibrium condition, the sediment inflow is given as 3.3 kg/min from the upstream point. After the equilibrium condition is reached, the sediment load is increased to 9.9 kg/min to study the overloading effect on the channel bed. After a new equilibrium condition is reached, the sediment load is reduced to the original equilibrium value of 3.3 kg/min. From these processes so designed that one can observe the overloading and underloading effects on the bed evolution. The numerical model was also used to simulate the above-mentioned overloading and underloading conditions. The comparison results are described briefly as follows.

**Overloading Condition**

For the overloading case, the experimental and simulated results of the bed elevation variations at sections 1, 3, 4, and 6 (9.5m, 21.5m, 26.5m, 37.5m) are
shown in Fig.1. From this figure, one can tell that the simulated results almost match with the experimental data. Fig.2 shows the variation of geometric standard deviation along the channel. Again, the standard deviation increases at the beginning and then decreases as the equilibrium condition is reached. This phenomenon is similar to the results observed from the experiments. The simulated mean particle size and standard deviation at the downstream point is 1.832 mm and 3.4281 which are greater than those measured data, 1.65 mm and 2.622. From the testing experience, one may be able to attribute this deviation to the immature procedures for the sorting processes used in the model.

Underloading Condition
After the new equilibrium condition in the overloading case is reached, the sediment inflow is reduced to perform the underloading simulation. Right after the equilibrium condition in the underloading case is reached, the clear water condition is conducted. The final results for the variation of bed elevation are shown in Fig.3. At section 1 near the upstream boundary a large difference between the measured data and the simulated results exists. The severe degradation occurring in the simulation results may be caused by the neglect of the armoring effect in the model. The model would not be able to form the armor coat as the reality does to prevent the excessive scouring. Fig.4 shows the variation of geometric standard deviation along the channel. Fig. 5 shows the comparison of the measured and simulated size distribution at the downstream point at the end of underloading process (T=5100 minutes). The simulated mean particle size (2.4 mm) is almost the same as the measure result (2.354 mm). However, the simulated standard deviation (4.1466) is much greater than the measured and computed results as shown in Fig.4 can be caused by several reasons such as the unreal assumption of the constant mixed-layer thickness used during the simulation period and the neglect of the armoring processes in the model.

Conclusions
A coupled models which solves the water flow dynamic equations, sediment continuity equation, and sorting equation simultaneously in one time step has been introduced in this paper. This model is capable of simulating the nonuniform sediment movements under the unsteady flow condition. The governing equations are solved simultaneously to couple the interaction between the water flow and bed evolution. The results from the comparison study with certain set of experimental data indicate that this coupled model can satisfactorily simulate the nonuniform sediment transport processes in a flume channel. However, there are several other related topics, such as, armoring process, determination of mixed-layer thickness, and model verification under unsteady and natural river condition need to be further studied.
References


Fig. 1 Measured and simulated bed elevation in overloading period

Fig. 2 Variation of geometric standard deviation along channel in overloading period
Fig. 3 Measured and simulated bed elevation in underloading period

Fig. 4 Variation of geometric standard deviation along channel in underloading period

Fig. 5 Comparison of measured and simulated sediment composition for underloading case (time = 5100 min)