Inundation analysis by heavy rainfall in low-lying river basin

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Abstract

An inundation flow model including sewerage system for the low-lying river basin is developed in this study. This model is applied to the Neya River basin, Japan, a highly urbanized area. There are flood control facilities such as flood control zones and underground regulating ponds, which are also considered in the simulation. From the obtained results, it is concluded that this model can express the inundation process in the low-lying urban area by heavy rainfall and can support to estimate the effectiveness of those flood control facilities.

1 Introduction

In Japan, inundation by heavy rainfall occurs frequently in urban area. Japan has a tendency to suffer inundation because most of the big cities are located along the low-lying coastal area and the elevation of urban area is usually lower than high water level of the urban rivers, which makes it very difficult to drain the inundation water. Since 1960s, inundation by heavy rainfall has become relatively significant and still causes severe damage to urban areas. Therefore a lot of inundation flow models have been proposed so far, but many of them treat only a small catchment area or use simple models for sewerage system.

In this study, a comprehensive mathematical model for inundation analysis in low-lying river basin is developed, which can treat inundation process in detail. Then this model is applied to the Neya River basin, Japan, suffering from frequent inundation and flood hazard possibility due to heavy rainfall is discussed.


2 Computational method

As shown in Figure 1(a), the studied river basin consists of four parts: (1) mountainous area, (2) river network, (3) drainage basin and (4) sewerage system. In the mountainous area, runoff discharge is calculated by kinematic wave model. In the river network, characteristics method is applied to one-dimensional unsteady flow analysis. In the drainage basin, two-dimensional inundation flow analysis based on the unstructured meshes is conducted. In the sewerage system, discharge from the pump stations is calculated by the sewerage model considering drainage process and storage effect. The flow of rainwater is shown in Figure 1(b). The runoff discharge from the mountainous area is assumed as lateral inflow of the river network. All the rainfall given to the drainage basin is drained through the sewerage system, and the water left in the drainage basin is treated as inundation water. While, the direct overflow from the river network to the drainage basin or vice versa is not considered in this study.

![Diagram](image)

(a) Modeling of a river basin  
(b) Flow of rainwater

Figure 1: Framework of total model

2.1 Mountainous area

The runoff analysis in the mountainous area comprises slope flow and river flow. The governing equations based on the kinematic wave model are as follows.

\[ \frac{\partial h}{\partial t} + \frac{\partial q'}{\partial x} = r_e \]  
\[ q' = \alpha h^m \]

where \( x \): one-dimensional spatial coordinate, \( t \): time, \( q' \): discharge per unit width on the slope, \( r_e \): effective rainfall, \( h \): water depth and \( m, \alpha \):
coefficients \( m = 5/3, \alpha = \sqrt{\sin \theta_s/N}, \theta_s : \text{slope gradient, } N : \text{equivalent roughness, according to the Manning’s law}).

\[
\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = \frac{q_s}{B_s}
\]

\[
q = \alpha h^m
\]

where \( q \): discharge per unit width of the river, \( q_s \): lateral inflow per unit width from the slope, \( B_s \): river width, \( m = 5/3 \) and \( \alpha = \sqrt{\sin \theta/n} \), \( \theta \): river bed slope and \( n \): roughness coefficient.

In this study, the lateral inflow from the slope to the river is calculated by the characteristics method, and the runoff discharge along the river is calculated by the finite difference method using the Leap-Frog method [1].

2.2 River network

In the river network, the following continuity and St. Venant equation are used.

\[
\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = q
\]

\[
\frac{1}{g} \frac{\partial u}{\partial t} + \frac{u \partial u}{g \partial x} + \frac{\partial h}{\partial x} = s_0 - s_f
\]

where \( A \): cross sectional area, \( Q \): discharge, \( q \): lateral inflow per unit width of \( x \)-direction, \( u = Q/A \): averaged velocity over cross-section, \( s_0 = \sin \theta \): river bed slope, \( s_f = n^2 u |u|/R^{4/3} \): friction slope, \( R \): hydraulic radius and \( g \): gravitational acceleration. The detailed computational method follows that of Inoue et al.[2]

2.3 Sewerage system

The rainwater on the drainage basin is drained into the river network through the sewerage system. This system consists of pump stations, main sewers and sub sewers. The drainage process through the sewerage system is modeled as follows (see Figure 2).

2.3.1 Pump stations

Each pump station has its own maximum drainage capacity. The rainwater drained from the pump stations is immediately flowed into the river network, that is, the storage of the pump stations is neglected.

2.3.2 Main sewers

The main sewers have a tree-type connection. The rainwater flows down from a few (or one) upstream main sewers into one main sewer. At some junctions or downstream ends of downstream sewer pipes, the pump stations are located. The main sewer \( I \) has its design velocity \( v_I \) and design discharge \( q_I \), which are constant within the pipe. Assuming that the rain-
water in the main sewer $I$ flows down at the velocity $v_I$, this main sewer is longitudinally divided into small segments by the distance which water flows down during one time step ($2\Delta t$), and every time step, the water is conveyed to one segment downstream. Assuming that the discharge of the sewer pipe is supposed not to exceed $q_I$, the maximum water volume of each segment is defined as $q_I \cdot 2\Delta t$. In the actual computation, the discharged volume per $2\Delta t$ from an upstream segment to a downstream one is determined as the smaller volume of "water volume existing in the upstream segment" and "vacant volume of the downstream segment" so that the water volume of the downstream segment does not exceed $q_I \cdot 2\Delta t$ after flowing down.

2.3.3 Sub sewers
There is only one sub sewer pipe in each drainage basin mesh. The route of the sub sewer is assumed the shortest path to the nearest main sewer from the mesh centroid (see Figure 2). The design velocity $v_J$ and design discharge $q_J$ of the sub sewer $J$ are substituted by those of the main sewer $I$. The rainwater of the sub sewer is treated similarly as that of the main sewer.

2.3.4 Drainage channels
The rainwater drained from some pump stations is conveyed to the river network through the drainage channel as is shown in Figure 2. These drainage channels are assumed to be 1-D rectangular channels and the following equations are applied.

$$\frac{\partial h}{\partial t} + \frac{\partial M}{\partial x} = \frac{q_{in}}{B}$$  \hspace{1cm} (7)

$$\frac{\partial M}{\partial t} + \frac{\partial (uM)}{\partial x} = -gh \frac{\partial H}{\partial x} - \frac{gn^2|M|M}{h^{7/3}}$$  \hspace{1cm} (8)
where \( u, M \) : flow velocity and discharge flux in the \( x \) direction, respectively, \( H \) : water stage, \( q_{in} \) : lateral inflow per unit length from the pump station, \( B \) : channel width. The discharge at the downstream end is calculated by the drop formula[3].

### 2.4 Drainage basin

The 2-D inundation flow analysis based on the unstructured meshes is applied[4]. The governing equations are the following continuity and momentum equations neglecting the non-linear terms.

\[
\frac{\partial h}{\partial t} + \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} = r_e - q_{out} \tag{9}
\]

\[
\frac{\partial M}{\partial t} = -gh \frac{\partial H}{\partial x} - \frac{gn^2 M \sqrt{u^2 + v^2}}{h^{4/3}} \tag{10}
\]

\[
\frac{\partial N}{\partial t} = -gh \frac{\partial H}{\partial y} - \frac{gn^2 N \sqrt{u^2 + v^2}}{h^{4/3}} \tag{11}
\]

where \( v, N \) : flow velocity and discharge flux in the \( y \) direction, respectively, \( q_{out} \) : drainage discharge per unit area from computational mesh into sewage system. \( q_{out} \) is defined as follows.

The maximum drainage capacity \( Q_m \) into the sub sewer is assigned to each drainage basin mesh \( m \). \( Q_m \) is defined as distributed discharge of \( q_I \) corresponding to the mesh area.

\[
Q_m = \frac{A_m}{K_I} q_I \tag{12}
\]

where \( I \) : the main sewer which the rainwater of mesh \( m \) flows into, \( A_k, A_m \) : area of mesh \( k, m \), respectively, \( K_I \) : number of meshes from which rainwater flows into the main sewer \( I \). \( q_{out} \) is defined as the smaller value of “the maximum drainage capacity per unit area, \( Q_m/A_m \)” and “the discharge per unit area when all the inundation water of the mesh \( m \) is drained”.

### 3 Application to the Neya River basin

As shown in Figure 3, the Neya River basin is surrounded by hilly areas and river bankings. The studied mountainous area is 49km\(^2\), the drainage basin is 197km\(^2\), and the total length of the river network amounts to 89km.

#### 3.1 Mountainous area

There are 24 rivers flowing into the river network from the west slope of Ikoma mountainous area. Most of them seem to have the similar topographical conditions, so the runoff discharge from Oto River is only calculated and
it is assigned to the other rivers in proportion to their catchment area. But the runoff discharge from the three rivers, Sanra River, Kiyotaki River and Gongen River, which are located in the north part of the mountainous area is calculated individually because the topographical conditions may be different from those of Oto River. The hydrological parameters used here are as follows. The runoff percentage is 80%, equivalent roughness of slopes, $N$, is 1.0 and roughness coefficient of rivers, $n$, is 0.020.

### 3.2 River network

At the upstream end of the river network, a certain small constant discharge is given. At the downstream end, where water gates are located, the mean high water level is constantly given. Manning’s roughness coefficient is assumed 0.020. First, under the above mentioned conditions, the computations are executed until the water level and flow discharge become unchanged. Then the obtained results are used as the initial conditions.

As the effect of flood control facilities attached to the river network, the storage in the flood control zones (totally $3.59 \times 10^6 \text{m}^3$) and the drainage through Kema pump station (maximum capacity is $200 \text{m}^3/\text{s}$) are considered.

### 3.3 Sewerage system

The total pipe length of the main sewers considered here is 227km. The four drainage channels are also considered. The sewerage networks including the

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Figure 3: Studied area
drainage channels are shown in Figure 4. As the effect of flood control facilities attached to the sewerage system, the storage in the underground regulating ponds (totally $0.37 \times 10^6 \text{m}^3$) is considered.

3.4 Drainage basin
The drainage basin is divided into 2,915 unstructured computational meshes based on 35 catchment areas of the sewerage system. Figure 5 shows these computational meshes and surface elevation distribution. The runoff percentage is assumed 80% and roughness coefficient is set to be 0.067.

4 Results and discussions

4.1 Validity of the sewerage model
The computational results are compared with the actual drainage records of some pump stations in the studied area. The rainfall observed on June 25, 2000 at the Hirano-ichimachi pump station is uniformly given to Hirano-ichimachi catchment area in Figure 4. The computational time step is 0.5s.

The comparison between the computational results by this model and the actual records is shown in Figure 6. In spite of some unknown factors such as the manual pump operation and the flowing condition into regulating ponds, good agreement of the drainage tendency is obtained. Therefore the drainage process of the inundation water can be expressed to some ex-
4.2 Inundation by the design rainfall

The actual rainfall observed at Yao city in 1957, which is the design rainfall of the Neya River basin flood control plan, is used here. The maximum hourly rainfall is 63mm/hr and the total rainfall is 311mm. The computational time step is 0.1s for the mountainous area and 0.5s for the river network, drainage basin and sewerage system.

The discharge hydrographs of Oto River, Gongen River, Kiyotaki River and Sanra River obtained from the runoff analyses are shown in Figure 7.

The discharge hydrograph at Kyoubashi-guchi of Neya River, the reference point of the Neya River basin, and the flow discharge and direction of the river network when the maximum discharge at Kyoubashi-guchi appears, are shown in Figure 8 and Figure 9, respectively. The maximum discharge at Kyoubashi-guchi is 928m³/s, which exceeds design flood discharge 850m³/s.

Figure 10 and 11 show the temporal change of inundation water depth and maximum water depth, respectively. From these figures, it is seen that
Figure 9: Flow discharge and direction in the river network

The number denotes the discharge (m³/s)

Figure 10: Temporal change of inundation flow depth distribution
the inundation water left in the drainage basin is flowing toward the lower area. Especially along the rivers, such as 2nd Neya River, Onchi River and Hiranogawa diversion channel, the inundation water depth becomes larger due to blocking by the river bankings. Through the comparison between Hirano-ichimachi catchment area where the sewerage system is well developed and the other areas, such as Moriguchi and Kounoike catchment area in Figure 11, it is indicated that there is some difference of drainage rate among catchment areas.

Figure 11: Maximum water depth

5 Conclusions

The inundation flow model which can treat inundation by heavy rainfall in urban area was newly developed and applied to the Neya River basin, Japan. This model can express the inundation process in the low-lying river basin appropriately. The points to be improved are the modeling of sewerage system described by continuity and momentum equation and the more detailed validation by comparing computational results with more observed data.

References