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Optimization of the short term supply of irrigation water for a multicrop scheme when conflicts between supply and demand arises

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Abstract

An optimization approach has been developed to aid decision making in real time for deficit irrigation when conflict between water supply and demand arises in a multiple crop irrigation scheme. The result is the optimal allocation of short-term supply of irrigation water. The optimization approach is based on Dynamic Programming. In the optimization approach, the short-term supply is optimized in function of a specified strategy determined by the user. The strategies that the user can select from are: maximum revenue, equitable maximum yield and maintaining equity in the system. The potential of the approach has been assessed through application of the model to Perkerra irrigation scheme in Kenya. In the 680 ha scheme, maize, cotton, onions, chilies and watermelon are cultivated in the irrigation season. Analysis of the results for the 1991/1992 season, where the water supply was 15 percent smaller than the demand, indicates that improvements in crop production can be achieved through application of the optimization approach.

1 Introduction

The renewable amount of water, annually available on the Earth is fairly fixed. During the 20th century the rate of water withdrawals in the world increased much faster than population, owing mostly to irrigation and industrial development. Today, it is estimated that irrigated agriculture accounts for 73 percent of the world water consumption. The current irrigated area of the world covers about 17 percent of the total cultivated area but contributes about one third of the world food production [1]. In the next few decades the world's population is expected to grow from 6 billion today to at least 8 billion by the year 2025, with about 90 percent of the increase being added to the developing world. It is therefore clear that achieving food security will continue to pose major challenges to decision-makers in the next few decades. At a time when industrial and municipal users are in competition with agriculture for water, farmers have to grow more food crops with the same or smaller amount of water, to feed the increasing population. This can be achieved by increasing the irrigation efficiencies by using new irrigation methods or by better planning of the water application. If there is still a water shortage, the limited amount of water to the crops should be distributed optimally.

This paper briefly describe an optimization approach to aid decision making in real time for deficit irrigation when conflict between water supply and demand arises in a multiple crop irrigation scheme, followed by application of the optimization approach to an existing system.

2 Definition of the optimization problem

The problem may be considered to be one of maximizing the utilization of the available water supply when conflicts between supply and demand arises at a particular moment in the season. The optimization is done according to one or another irrigation strategy.

2.1 System

Let the number of field groups in the system be NF. A field group consists of an area of land with a particular crop, in a particular crop development stage, on a particular soil type and receiving irrigation water from a particular turnout. A field group could consist of only part of a field, or of one or more fields. Let the cropping season be divided into NE short-term decision intervals. The length of NE should be longer than the irrigation interval but short enough to avoid fluctuation of both water supply and crop water demand. It should also be short enough to assume rainfall is zero. The objective functions are derived for maximizing the net return/minimizing crop losses from the NF field groups irrigated by water supplies available at the field level during the j^{th} decision interval when conflicts between supply and demand arises.

2.2 Objective Functions

If the objective is to minimize crop losses due to water shortage, crop yield response functions can be used to formulate the objective functions. Doorenbos and Kassam [2] found that crop yield response to water can be expressed in the following form:

$$\left(1-\frac{Ya}{Ym}\right)=Ky\left(1-\frac{ETa}{ETc}\right) \tag{1}$$

where Ya is actual crop yield, Ym is maximum crop yield under the given management conditions, Ky is crop yield response factor to water, ETa is actual crop evapotranspiration and ETc is crop evapotranspiration without water stress.

If in eqn (1), the relative water supply available for the crop (FIS/FIR) is assumed to be identical to the relative evapotranspiration (ETa/ETc), eqn (1) can be written as:

$$\left(1-\frac{Ya}{Ym}\right)=Ky\left(1-\frac{FIS}{FIR}\right) \tag{2}$$

In other studies ([3] and [4]) a similar assumption has been made. In eqn (2), FIS and FIR are respectively the field irrigation water supply and requirement. Rainfall is assumed to be equal to zero during the short-term decision interval i.e. rainfall does not contribute to FIS. Using eqn (2) three objective functions can be formulated, representing three different irrigation strategies that can be specified by the user, namely: maximum revenue, equitable maximum yield and maintain equity in the system.

2.2.1 Irrigation strategy 1: Maximum revenue

The objective function for maximum revenue strategy is based on eqn (1). Using eqn (2), it can be shown that the objective function to be maximized during the j^{th} decision interval for the NF field groups can be expressed as:

$$R=\max\sum_{i=1}^{NF}\frac{P_i}{P_{ref}}\frac{Ym_i}{Ym_{ref}}\frac{A_{j,i}}{A_T}\beta_{j-1,i}\left[1-Ky_{j,i}\left(1-\frac{FIS_{j,i}}{FIR_{j,i}}\right)\right] \tag{3}$$

where for the crop in field group i during decision interval j: P_i is the market price, P_{ref} is the market price for the reference crop, Ym_i is the maximum crop yield under the given management conditions, Ym_{ref} is the maximum crop yield for the reference crop under the given management conditions, $A_{j,i}$ is the area under crop, A_T is the total cultivated area in the system, $\beta_{j-1,i}$ is crop production status at the end of decision interval j-1, $Ky_{j,i}$ is yield response factor, $FIS_{j,i}$ is field level irrigation water supply and $FIR_{j,i}$ is field level irrigation water requirements. Note that for j = 0, the crop production status β_i for all field groups, is equal to one. Eqn (3) is maximized during the j^{th} decision interval, subject to the constraints equations.

2.2.2 Irrigation strategy 2: Equitable Maximum yield

If the objective is to maximize crop yields, or minimize crop losses, with some measure of equity, Wardlaw and Barnes [4], showed that an objective function

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similar to that given in eqn (4) is an appropriate function to use. The function is based on eqn (1).

$$R = \min \sum_{i=1}^{NF} \frac{Ky_{j,i}}{FIR_{j,i}} \left(FIR_{j,i} - FIS_{j,i}\right)^2 \tag{4}$$

The objective function eqn (4) is minimized during the jth decision interval, subject to the constraints equations.

2.2.3 Irrigation strategy 3: Maintaining equity

If the crop yield response factor is set to 1.0, eqn (4) reverts to water allocation function eqn (5), in which the objective is to meet specified irrigation demands with equity throughout the system [4]. In this study eqn (5) is used for the equity strategy.

$$R = \min \sum_{i=1}^{NF} \frac{1}{FIR_{j,i}} \left(FIR_{j,i} - FIS_{j,i}\right)^2 \tag{5}$$

The objective function given by eqn (5) maintains equity in the system for the NF field groups during decision interval j. The minimization is also subject to the constraints equations.

2.3 Constraints equations

The field level irrigation water supply $FIS_{j,i}$ during decision interval j for field group i, is constrained by the water supply W_j available at the headworks during the decision interval j. This can be expressed as:

$$W_j = \sum_{i=1}^{NF} U_{j,i} \tag{6}$$

$$U_{j,i} = \frac{FIS_{j,i}}{Ec_i} \tag{7}$$

where Ec_i is the conveyance efficiency from the headwork to the field group inlet and $U_{j,i}$ is the decision variable.

Doorenbos and Kassam [2] assumed that eqn (1) is valid for most crops for water deficits in the range (1- ETa/ETc $\leq$ 0.5). Therefore, the priority of irrigation during decision interval j can be introduced by using a factor τ_i with a valid range of $(0.5 \leq \tau_{j,i} \leq 1.0)$. This can be expressed as:

$$\tau_{j,i} FIR_{j,i} \leq FIS_{j,i} \leq FIR_{j,i} \tag{8}$$

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2.4 Problem solution

The real time irrigation water management problem is one of providing the best distribution of scarce water resources, such that a specified objective is maximized. It has a special structure that can be exploited in attempting to solve the problem. This structure arises because decisions are often carried out in a sequential manner in space, or they can be conceptually viewed in this way. Dynamic programming (DP) is particularly well suited for these kinds of sequential decision problems. By this method, large operational problems, like the real time irrigation water management problem can be decomposed into a series of smaller decision problems, which are readily solvable. The degree of generalization and availability of generalized computer codes is limited for DP. Labadie [5] recognized this and developed a comprehensive generalized Dynamic programming package called CSUDP, which has been employed in research programs in water resource management worldwide. The most recent version of CSUDP [6] is used in this study.

In this study, a field group constitutes a DP stage. The decision interval is set to 14 days. The state variable X_i is defined as the amount of water left for the remaining field groups after field groups 1 through i-1 have received water. The state equation in the inverted form, during decision interval j is given as:

$$U_{j,i} = X_{j,i} - X_{j,i+1} \tag{9}$$

The backward recursive algorithm is used. Optimal water allocation to DP stage i during decision interval j, $U^*_{j,i}$, is the ouput from the DP. This is used to calculate the relative field level irrigation water supply available for the crop (FIS/FIR), which is used as an estimate for the relative evapotranspiration (ETa/ETa). Given the irrigation depth that is currently applied, the relative evapotranspiration is translated into an irrigation interval by using BUDGET soil water balance model, developed by Raes [7].

3 Example application

3.1 Study site

The Perkerra irrigation scheme (PIS) situated near Marigat Township in Baringo District in Kenya was used as an application example of the optimization approach. It is located on sandy clay soils in a semi-arid region where the average monthly crop reference evapotranspiration calculated using the FAO Penman-Monteith method [8], ranges between 4.0 and 5.4 mm/day. The annual dependable rainfall in a wet, normal and dry year is 781, 637 and 492 mm respectively. National Irrigation Board (NIB) which was formed by an act of parliament in 1966 provides the management and extension service to PIS.

The schematic layout of the irrigation system of PIS is illustrated in Figure 1. The irrigation water in PIS is drawn from Perkerra River by gravity and

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fed through a system of canals and feeders. The water fee paid by the farmers is fixed per unit area. The current water fee is Ksh 450 per ha (1 US$ = 70 Ksh in 1999). Furrow method of irrigation is adopted. Since the inception of PIS in the 1940s, Perkerra River has gradually decreased in flow. Critical water shortage in PIS started in 1987 with the launching of Greater Nakuru water project, upstream of Perkerra River and PIS. Other factors attributed to the river flow decrease are destruction of the forests in the catchment area and cultivation on the riverbanks. As a result of the decrease in flow and unreliability of the rains in the area, the irrigated area has decreased from the original developed irrigation area of 680 ha to the current irrigated area of about 450 ha. The crops that are currently cultivated in the irrigation season in PIS are maize, cotton, onions, chilies and watermelon.

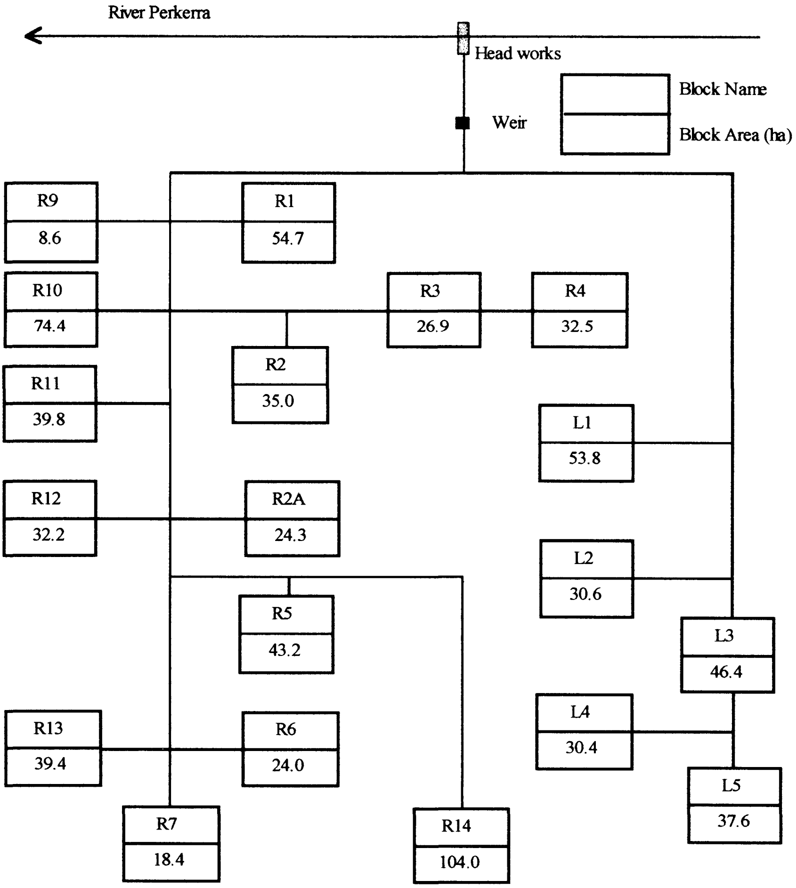

Figure 1: Schematic layout for the Perkerra Irrigation scheme.

3.2 Data

The cropping calendar for 1991/92 season (Figure 2) where the water supply was 15 % smaller than the demand is used in this study. The daily water supply to the system for1991/92 season, measured using the weir on the main canal (Figure 1) was aggregated into biweekly water supply. The areas under crop, the actual yield and market prices for each crop are presented in Table 1. The onion crop was considered as the reference crop because it has the highest returns per hectare. The crop yield response factors shown in Table 2 were transformed into 14 day yield response factors using graphical method [9]. The number of field groups NF considered in the system is 32 (Figure 2). In the 1991/92 season two water shortage periods were observed. The first water shortage period lasted 4 weeks (week 37-40) and the second one-lasted 10 weeks (week 53-62).

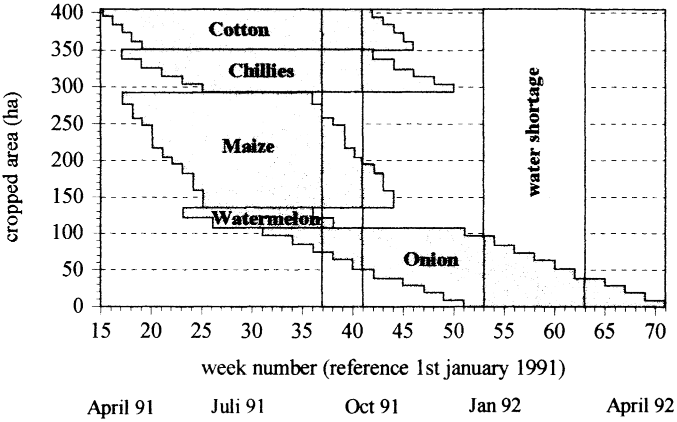

Figure 2: 1991/92 season cropping calendar with indication of the two water shortage periods.

When inputs such as fertilizers, pesticides, weeds control etc are at optimal levels, crops produce an absolute maximum possible yield (Ym_{abs}) when well watered. Often the above inputs are not at optimal levels in farmer's fields and crop yield is limited. In this study, the yield realized for no water stress conditions in the farmer's fields is referred to maximum yield under the given management conditions Ym. In order to determine Ym for the 1991/92 season, the BUDGET model was used. By running the model with the climatic data and irrigation calendar for the season, the relative crop evapotranspiration (ETa/ETc) for each crop cultivated in that season was determined. By using a multiplicative water production function based on eqn (1), the seasonal relative crop yield (Ya/Ym) for each crop was calculated. Finally the maximum yield under the given management conditions (Table 1, column 4) was determined from the observed actual yield (Table 1, column 3).

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Table 1. Area under crop, actual yield, maximum yield under the given management conditions and market prices for the 1991/92 season (Source: [10]).

Crop	Area under crop (ha)	Actual yield Ya (Tons/ha)	Maximum yield Ym (Tons/ha)	Market price (Ksh/Kg)
Maize	156.8	3.05	3.10	10
Onion	107.8	10.70	12.33	4.95
Chilies	57.3	2.73	2.82	20.5
Cotton	55.0	2.61	2.63	8
Watermelon	28.0	9.01	9.11	2.35

Table 2. Length (days) and yield response factors (Ky) for the physiological growth stages (Source: [2]).

Crop		Establishment stage	Vegetative stage	Flowering Stage	Yield formation Stage	Ripening stage
Maize	Length	21	42	21	42	14
	Ky		0.4	1.5	0.5	0.2
Onion	Length	28	35	70	42	21
	Ky		0.2	0.5	0.5	0.25
Chilies	Length	28	35	56	42	21
	Ky		1.1	1.1	1.1	1.1
Cotton	Length	49	28	-	49	21
	Ky		0.45	-	0.8	0.3
Watermelon	Length	14	21	21	21	14
	Ky		0.7	0.8	0.8	0.3

3.3 Set up for the optimization approach

At the present stage of the research, the soil water balance and crop water production models calculations are both independent of the optimization approach calculations. This requires that the crop production status for each field group β_i be derived from the simulated relative crop yield (Ya/Ym) in the previous decision intervals. Not yet included is the correction of Ya/Ym for the observed weather conditions instead of the assumed mean historic reference evapotranspiration and zero rainfall. This issue will be a subject of the on going research, where the possibility of using soil water balance model to keep track of the stress in the previous decision intervals will be considered. The field irrigation requirements (FIR) are computed using reference evapotranspiration and crop coefficients. The water supply available at the field level (FIS) in any biweekly interval, where conflicts between supply and demand arises, is allocated among competing crops using the optimization approach and the relevant data of crop yield response and other factors. The output of the optimization approach is the relative water supply (FIS/FIR) available to each field group, which is used as an estimate of the relative evapotranspiration (ETa/ETc). For surface irrigation systems, the irrigation depth applied in each

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irrigation event is constant for each field group throughout the growing season. BUDGET soil water balance model is used to adjust the irrigation interval corresponding to the calculated relative evapotranspiration.

Using the simulation results and a multiplicative water production function based on eqn (1), the relative crop yield for each field group is determined at the end of the season. The obtained relative yields plus the areas under crop and the market prices are used to calculate the returns in Millions Kenyan shillings (M Ksh) for each crop.

3.4 Results

The optimization approach has been applied to the Perkerra irrigation scheme in Kenya, and results compared to the observed revenue in the system for the 1991/92 season. The optimization approach calculations were done according to the three irrigation strategies. Crop revenue for the 1991/92 season, where the water supply was 15 % smaller than the demand, was increased from 15.440 million Kenyan shillings to 15.533 million Kenyan shillings for maintaining equity strategy, 15.588 million Kenyan shillings for equitable maximum yield strategy and 15.673 million Kenyan shillings for maximum revenue strategy. The revenue for each crop and for each strategy is shown in Table 3. From the results it can be noticed that there is no significant difference in the revenue for each of the three strategies. This is a result of the fact that the water shortage periods (Figure 2) occurred when most of the crops in the planted area had passed the most sensitive growth stages. As water shortage becomes more and more severe and might even occur when crops are in sensitive stages, difference between actual irrigation practices and optimal simulations are expected to be much large. This will be tested in the ongoing research, by using the PIS 1999/2000 season cropping calendar, where a very severe water shortage was observed. Maintaining equity strategy is expected to be more attractive to the managers of PIS, because the water fee paid by the farmers is fixed per unit area, while maximum benefit strategy might most like be attractive to a manager of a single owed system.

Table 3. Comparison of observed and optimization approach revenue.

Crop	Observed revenue 1991/92 season (M Ksh)	Maintaining Equity (M Ksh)	Equitable Maximum Yield (M Ksh)	Maximum Revenue (M Ksh)
Maize	4.782	4.785	4.787	4.829
Onion	5.710	5.748	5.779	5.804
Chilies	3.207	3.259	3.283	3.293
Cotton	1.148	1.148	1.145	1.147
Watermelon	0.593	0.593	0.593	0.599
Total	15.440	15.533	15.588	15.673

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4 Conclusions

The results indicate that application of the optimization approach for real-time irrigation water management can provide benefits in crop production. The optimization approach developed is relatively easy to apply, and has a potential as a decision tool for real-time irrigation management. Approaches to implementation are being considered under the ongoing Ph.D. research.

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