Natural convection heat transfer in cubical enclosures with partial heating of one of the vertical walls

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Abstract

Natural convection of air in a cubical enclosure of edge L with a cold wall covering one of the vertical surfaces and a hot square sector of side s centred on the opposite wall is considered. The overall heat transfer shows similar dependancies to the case of side heated cavities for $s/L \geq 0.5$. The main circulation cell is similar to that in the test problem, but becomes progressively distorted by expansions, contractions and secondary loops at high Ra and at low values of s/L. It is shown that the average fluid temperature can be used as a criterion for transition to a convective regime.

1 Introduction

Natural convection in enclosures is a very active area of research. Numerical work in this area continues to use of two-dimensional solutions, specially in high aspect ratio cavities. Since the works of Fusegi [1] Henkes et al. [2], Lee & Lin [3], 3D numerical solutions have increased in number. These authors studied flow structures, thermal fields, and stability conditions in vertical and inclined cubical enclosures with sidewall heating and cooling, describing three dimensional effects not considered in previous 2D studies. Three dimensional solutions give realistic description of natural convection phenomena in enclosed spaces, but are costly in computer time and difficult to display graphically. 3D solutions have made it possible to solve many problems in cavities with heaters on contiguous walls, top and bottom heaters, and partially heated walls with or without protrusions.

At variance with the sidewall heated case (test problem), miniaturisation of electronic components leads to the design of systems with hot and cold surfaces of different areas. This delays the transition from conduction to convection regime, in comparison to the test problem, where convective regime develops in the Rayleigh number range of 10^3-10^5, when Ra is defined in terms of cavity side and overall

temperature difference. As buoyancy forces on the smaller active surface are small owing to its reduced height and area, fluid mixing is made difficult. Well mixed fluid, characteristic of a fully convective regime, is attained at higher values of Ra than in the test case. A $Ra = 4X10^5$ has been proposed for the onset of the fully convective regime in some situations [4,5]. Because of an imbalance in the capabilities of the active sectors to generate buoyancy force, the average fluid temperature, which in the side heated cavity is equal to the average of the hot and cold wall temperatures, is displaced to a value nearer to the temperature of the biggest active surface.

As most 3D studies originate from applications to the cooling of electronic components, they use conditions of imposed heat flux at the hot surface. This condition, although realistic in that field, does not allow comparison of flow and heat transfer mechanisms and their evolution with Ra with the test case, for which the above issues are known. Geometries intended to model realistically these cooling systems tend to be complex. Less attention has been given to 3D cases, lacking a two dimensional counterpart, when active surfaces are isothermal. These cases are of great importance, specially from a fundamental point of view. In this paper we will study the natural convection regimes for active surfaces of unequal areas and imposed temperatures, located on opposite vertical sides of a cubical cavity. This case, fully three dimensional in nature, will be treated in the simplest way, with a square heater centred on a vertical wall. By means of this example, we will explore the transition regime. Also, the study of this problem allow the causes of the appearance of a full three dimensional flow pattern to be disclosed. One of the issues to be demonstrated here is the relationship between the stabilisation of an average fluid temperature and the achievement of a fully convective regime. This study would not be possible in a controlled heat flux situation, in which the average fluid temperature is unbounded.

2 Formulation

The physical situation is shown in Fig. 1. A cubical cavity of side L contains air (Pr=0.71). The wall at X=L acts as a cold surface at temperature T_C. The hot source is a non protruding square sector of side s at temperature T_H, centred on the wall at X=0. All other sectors of the cavity walls are adiabatic. The ratio s/L, an important geometrical parameter, will be given values of 0.1, 0.2, 0.5 and 0.7. The case s/L=1.0 is the sidewall heated cavity problem (test problem) whose solution has been reported [1,2]. It is assumed that steady state flow is attainable for the cases considered. This is supported by earlier evidence [2], according to which, time dependent flows occur at Rayleigh numbers slightly above 10^6 with air as the test fluid. The dimensionless equations of continuity, momentum and energy for the steady state laminar flow of an incompressible Boussinesq fluid inside the cavity are respectively:

$$\frac{\partial U_j}{\partial X_j} = 0$$

$$U_j \frac{\partial U_i}{\partial X_j} = -\frac{\partial P}{\partial X_i} + Pr \frac{\partial^2 U_i}{\partial X_j \partial X_j} + Ra_i Pr \Theta$$

$$U_j \frac{\partial \Theta}{\partial X_j} = \frac{\partial^2 \Theta}{\partial X_j \partial X_j}$$

where $Ra_2=g\beta\Delta TL^3/\nu\alpha$ is the Rayleigh number based on L, and $Ra_1 = Ra_3 = 0$. Reference quantities for adimensionalization are formed with L, α (thermal diffusivity), ρ (density) and $\Delta T=T_H-T_C$. U_i= U,V and W are dimensionless velocities along the coordinates X_i=X,Y,Z respectively, all velocities being zero at the walls. The dimensionless temperature Θ is defined as having values of 0.5 and -0.5 at the hot and cold walls respectively. The temperature boundary conditions are:
a) Θ=0.5 at X=0, $(1-s/L)/2 \le Y \le (1+s/L)/2$, $(1-s/L)/2 \le Z \le (1+s/L)/2$
b) Θ=-0.5 at X=1 c) $\partial\Theta/\partial X_i$ =0 elsewhere on the walls.

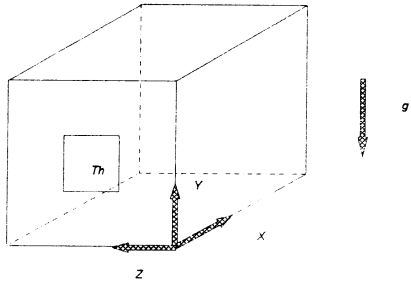

Figure 1: Physical situation and co-ordinates.

3 Numerical Method

The standard SIMPLER method is used. Uniform, staggered grids of 62 X 62 X 62 points were used. These grids were found to provide accurate solutions of the test problem for Rayleigh numbers up to 10^7. The solutions obtained for that problem were compared with results by Fusegi et al. [1], finding excellent agreement: at $Ra=10^6$, the maximum velocities U and V in the symmetry plane Z=0.5 differ from the ones reported in [1] by +0.13 and -1.59% respectively, while the overall Nusselt number differs by +0.84%. At lower Ra differences are smaller. No signs of numerical instability were found in any of the solutions in the whole range of Ra and the solutions converged to apparently steady state values in all variables.

4 Results, discussion and conclusions

Due to the difficulty of describing 3D flow patterns we examine first the heat transfer and average temperatures, and then we describe flow mechanisms.

4.1 The average temperature

An arithmetic average (Θ_{av}) of all temperatures in the enclosure, excluding the walls, is considered. This average does not represent the energy transported by the fluid (in the sense of the average mixing temperature used in channel flow). Fig. 2 shows the variation of Θ_{av} with Ra for different values of the dimensionless side of the heated sector. In the test problem (s/L=1), $\Theta_{av}=0$ at all Rayleigh numbers. For other ratios s/L, the Θ_{av} is negative and nearly constant at low Ra, increase from about $Ra=10^5$. Because the hot area is smaller, the average temperature at low Ra remains similar, but not equal, to the area-weighed average of the temperatures of the hot and cold sectors. For a given Ra, the value of Θ_{av} is higher for higher values of s/L. The figure shows that Θ_{av} increases to values similar to the arithmetic average of the imposed temperatures ($\Theta_{av}=0$), irrespective of the areas. It is interesting to note that Θ_{av} starts increasing at nearly the same Ra irrespective of the value of s/L. These considerations suggest that the levels of stratification reached at high Ra progressively resemble the levels attained in the sidewall heated cavity. Θ_{av} gives a measure of the intensity of the convective movement. The value of Ra at which Θ_{av} starts increasing can be therefore used as a tentative criterion for the onset of a fully convective regime.

4.2 Heat transfer

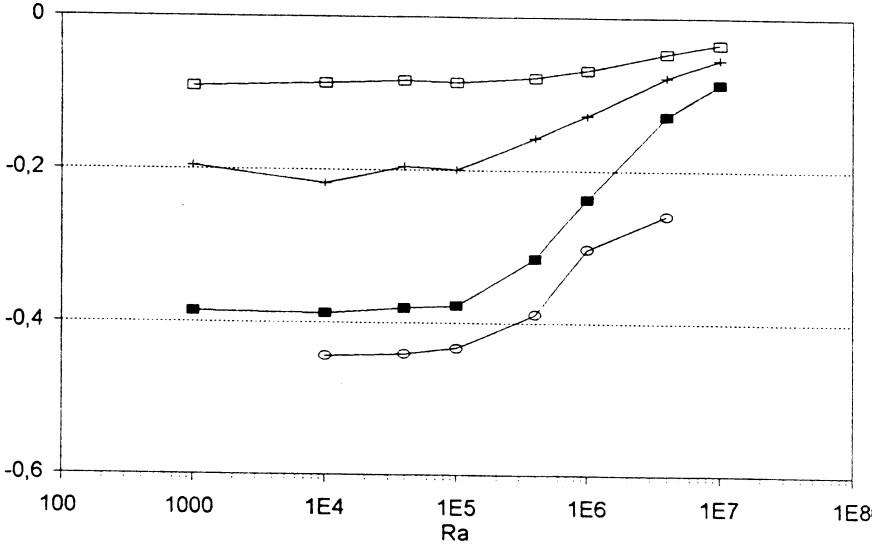

Figure 2. Average temperature in the enclosure. Curves, from top to bottom: s/L=0.7, 0.5, 0.2 and 0.1.

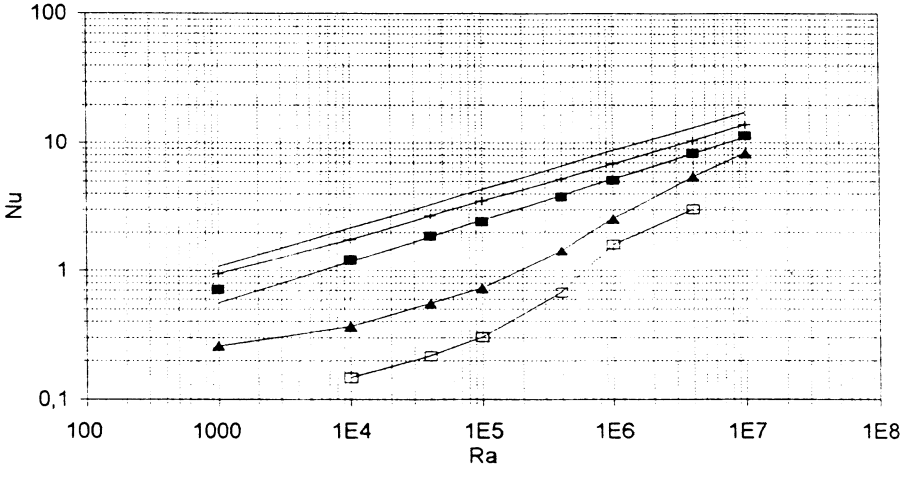

Figure 3. Overall Nusselt Number as a function of Ra: Curves, from top to bottom, show: Fusegi's results, s/L=0.7, s/L=0.5 and its regression line, s/L=0.2 and s/L=0.1.

The overall heat transfer characteristics (Nusselt number) for the cases analysed appear in Fig. 3. The curves for $s/L \geq 0.5$ are nearly parallel to the one for the side heated problem. Nusselt numbers are always lower at lower values of s/L at any given Ra. The curves for $s/L<0.5$, are not parallel to the others and show a slower increase of Nu with Ra, up to about $Ra = 4 \times 10^5$, but at higher Ra the increase in Nu is slightly faster that at higher values of s/L.

Therefore the value $s/L=0.5$ separates a Nu-Ra dependence which is similar (although giving lower Nu for the same Ra) to that for the test problem ($s/L \geq 0.5$) from one with different characteristics ($s/L<0.5$). In the upper s/L range, the curves, being essentially parallel to the one for the test problem, can be described by empirical equations of the type:

$$Nu = C\ Ra^{n}$$

where C grows with increases in s/L.

4.3 Temperature isograms

Sets of isotherms in several planes are shown in Figs. 4 and 5. For two values of s/L, at two different Rayleigh numbers, the isotherms show that the behaviour of the temperature fields in ZY planes is basically similar to that in the side heated cavity. At low Ra, the isotherms are nearly vertical, with low temperature gradients at the active walls, and the opposite situation appears at high Ra, with thermal stratification and boundary layers. The isotherms denote that a strong buoyancy is generated by the hot surface, but no buoyancy at the sides. This is observed in Figs. 4-5 where the hot source is delineated by the isotherms. Another characteristic of the stratified regime is that the level 0 of temperature is located well above the horizontal midplane, where it is found in the test problem, showing clear correspondence with the reduced values of the average temperature with respect to the test problem.

4.4 Velocity distributions

Figures 6 and 7 show velocity vectors at several planes at two different Ra, for a two values of s/L. They show that the flow pattern is symmetric with respect to the plane Z=0.5, and consists of a main circulation cell with secondary loops. The main cell is similar to the one observed in the test problem.

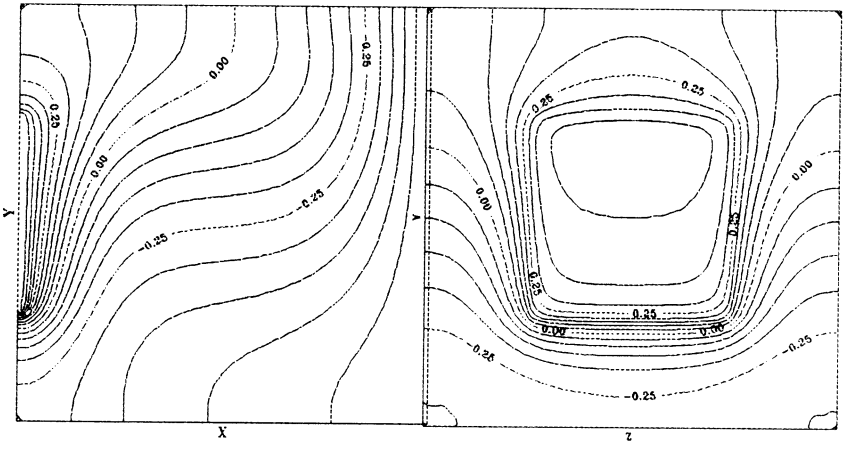

Figure 4. Isotherms for $Ra=10^4$, s/L=0.5. Planes Z=0.5 (left) and X=0.03.

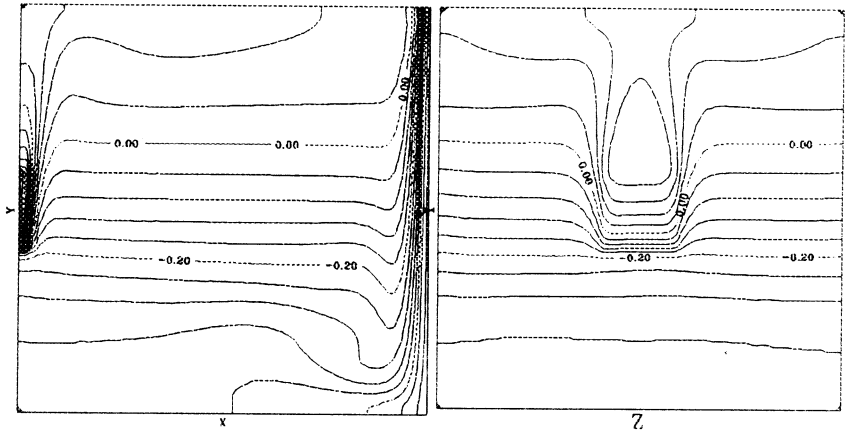

Figure 5. Isotherms for $Ra=4X10^5$, s/L=0.2. Planes Z=0.5 (left) and X=0.03.

At low Ra, the velocity is nearly uniform near the hot wall where the fluid ascends, except at the sides Z=0 and Z=1 where velocities are reduced by wall friction. This changes notoriously at high Ra, where the high buoyancy exerted by the hot wall sector at X=0, marks a very high velocity of ascension in front of this zone, whereas at the right and left of the hot sector where the wall is passive, the ascending velocities are much lower. The descending velocities at the cold wall (not shown) are more uniform near the cold wall, where there is negative buoyancy force all across the wall.

Also seen in Figure 7 is a vertical expansion of the stream which returns to the hot wall after sweeping the cold one. The lack of buoyancy below the hot sector allows this expansion. The general circulation pattern is deformed with respect to the test case, because at high Ra, the zone of ascending flow near the hot wall accommodates to the size and shape of the hot sector, and the zone of descending flow has essentially the dimension of the cold zone. Expansions and contractions of the stream connecting the active zones take place. This distortion of the main circulation is more noticeable at lower values of s/L and at high Ra. This behaviour is reflected in the extreme values of the velocities: for all values of s/L, at $Ra=10^5$ and 10^6, the absolute maximum velocities U and V are located in the symmetry plane Z=0.5, at the lines defined by this plane and the planes X=0.5 and Y=0.5 respectively, as in the test problem. For higher Ra, the maxima, while still located in the plane Z=0.5, are displaced to other positions within that plane, and show great increases with respect to the values at the more symmetric positions detailed above. These effects are caused by the expansion/contraction of the main circulation cell.

The comparison the absolute maxima with those of the test problem for $Ra=10^6$ is at follows: in the test problem the absolute maxima are U=127.602 and V= 214.608 (our results). Surprisingly, at this Ra, the maximum U is higher than that of the test problem for s/L=0.5 and 0.7, being lower for the lower Ra. Conversely, the maximum V in our cases is always lower than that of the test problem. It seems clear that the heater height determines the vertical velocity, so vertical velocities higher than those for the test case cannot appear in our problem at any given Ra. However, the values of horizontal velocity are determined by inertia forces, therefore as the stream is contracted at the heater, the accommodation of the flow for returning to the cold surface, will generate higher X-component velocities near the hot zone

In the test problem, for Ra up to nearly 10^5, the flow is symmetric with respect to the plane Z=0.5, with maximum values of U and V in that plane. At higher Ra, the maxima are displaced out of this midplane. This is a secondary effect, caused by friction at the lateral walls. At high velocities, the flow cannot behave as a single cell of uniform conditions along the Z axis: Zero velocities are forced to appear at Z=0 and Z=1, because of wall friction. Some fluid is therefore displaced towards

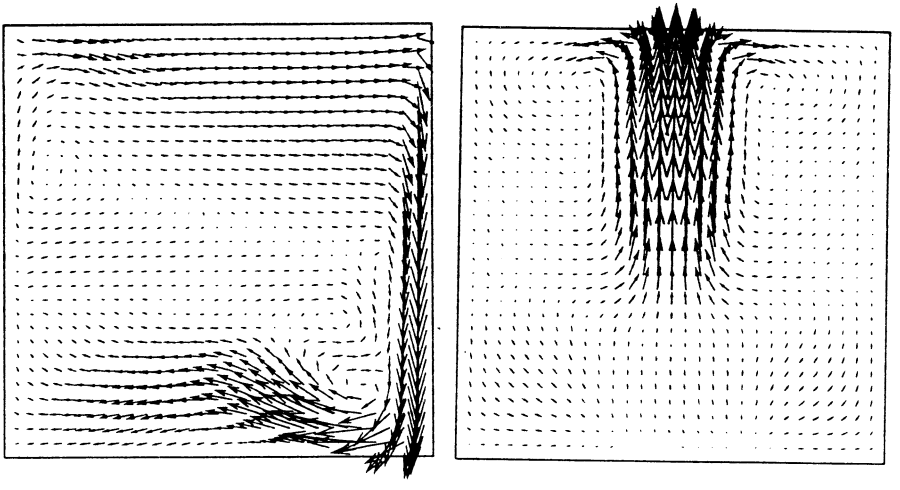

Figure 6. Velocities for Ra=4X10^5, s/L=0.2. Plane Z=0.5 (left) and plane X=0.03.

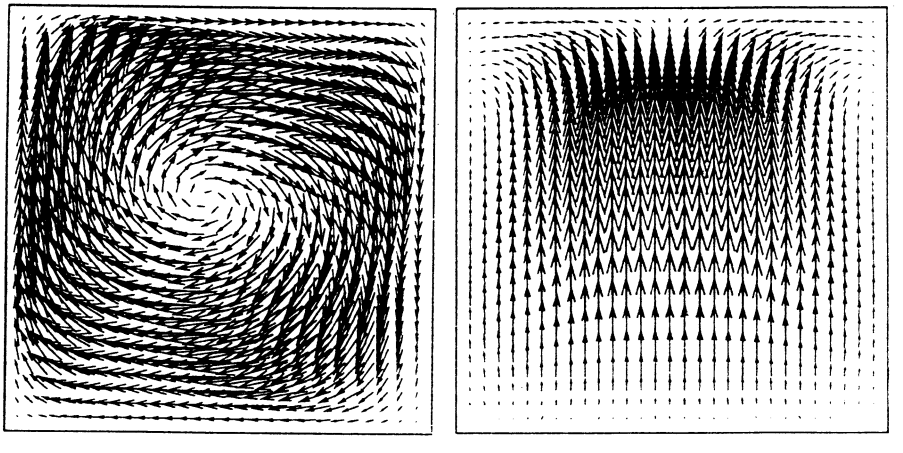

Figure 7. Velocities for Ra=10^4, s/L=0.5. Plane Z=0.5 (left) and plane X=0.03.

the core, being dragged by the main motion. This phenomenon does not propagate to the centre (Z=0.5), where velocities essentially keep the levels of the 2D problem. Conversely, in our problem, for all s/L values different from 1, there is no buoyancy at the sides of the heating section. This contributes to a reduction in the stress levels at the lateral walls, and the flow migrations caused by this stress are no longer present. Therefore for all cases analysed, maximum U and V velocities are always located at the symmetry plane Z=0.5.

The absence of buoyancy force at the sides of the heater causes also reduced velocities in part of the wall containing it. So the loop starts with a width similar to the width of the heater. The stream raised by this heater widens during its horizontal movement as it advances toward the cold wall, and it descends along it with the full width of the enclosure. The lateral movements caused by the enhancement of the recirculation loop causes strong W velocities, which point to the nearest lateral wall. The lateral flow generates secondary loops whose axes are parallel to the X-axis. These characteristics explain the heat transfer dependences described.

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