Thermal stresses caused by the hydration reaction in massive concrete bodies

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1 Abstract

In massive concrete structure cracks may already develop at an early stage, due to the thermal stresses induced by the heat produced during the hydration reaction of the cement. A two-step numerical simulation using classical finite element procedures is presented, allowing for the prediction of these stresses and thus the assessment of the risk of cracking. A nonlinear transient thermal analysis, using a recently developed model for the internal heat production in the concrete, yields the temperature distribution history. The subsequent stress analysis is based on an elastic model with continuously developing material characteristics. Though simplified, this model already yields realistic results with practical usefulness, while more comprehensive mechanical models for the early age concrete are being developed. Results are presented concerning massive concrete blocks used for the protection of sea walls and breakwaters against waves.

2 Introduction

When casting massive concrete structures, thermal stresses arising from the exothermic hydration reaction of the cement may cause cracking of the early age concrete. The prediction of these cracks, and hence their prevention, is not straightforward.

Traditionally rules of thumb has been observed which limit the maximal temperature difference between an internal point and the concrete surface. Very often simplified models have been used for determining this temperature difference. A more reliable estimation may be found from numerical simulations of the changing temperature field in a model with realistic boundary conditions.
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However, it has been shown that temperature related criteria are not reliable in assessing the risk of thermal cracking [1]. Instead, the thermal stresses which develop during the hardening of the concrete should be evaluated. Therefore a mathematical model is needed for both the thermal and the mechanical behaviour of concrete at its early age.

The thermo-mechanical simulation can be dissolved in two separate phases: a thermal analysis yielding the temperature field at any desired time, followed by a stress analysis with the given temperature distribution. In both cases a complex non-linear computation results.

3 Nonlinear thermal analysis

The thermal analysis of a freshly cast concrete body is complicated by several factors:

- The thermal characteristics (conductivity, capacitance and notably the internal heat production) are temperature and time dependent.

- Convection on the part of the body’s surface in contact with open air is largely influenced by the (changing) atmospheric conditions. The presence of a formwork during the first stage of the lifetime is a further complication.

- Usually the bottom surface of the body is in contact with other solids (foundation, soil) which generally extend infinitely.

- The initial conditions are not well known. The temperature and thermal characteristics at the time of casting result from a very complex process of preparing and transporting the fresh concrete.

Because the boundary and initial conditions in practice are so complex and not well known, it is most convenient to perform a series of simulations with different (realistic) values for the intervening parameters. For the thermal characteristics of the concrete on the other hand a more comprehensive model can be used. Indeed, nowadays quality concrete is prepared in plants with full control over the composition, and notably the amount and type of cement. The internal heat production in the cement, and hence in the concrete, can be expressed in terms of some internal variable related to the progress of the hydration reaction.

3.1 Hydration reaction model

In our simulations we used the model proposed by Deschutter et al.[2], which uses as a measure for the degree of reaction the ratio of the heat produced thus far, over the total amount of heat that will be produced when the reaction has completed. As the hydration reaction proceeds, the degree of
reaction thus evolves from zero to unity. The instantaneous heat production \( q \) is an explicit function of the temperature \( \theta \) and the degree of reaction \( r \):

\[
q(r, \theta) = q_{\text{max}},20 \cdot f(r) \cdot g(\theta)
\]

with \( f(r) = c \cdot [\sin(\pi r)]^a \cdot e^{-br} \), \( g(\theta) = e \left[ \frac{E}{R} \left( \frac{1}{293} - \frac{1}{273 + \theta} \right) \right] \).

The material constants \( q_{\text{max}},20, a, b, c \) and \( E \) are obtained from experiments. The degree of reaction \( r \) is defined as

\[
r(t) = \frac{Q(t)}{Q(\infty)} = \frac{1}{Q_{\max}} \int_0^t q(t) \, dt,
\]

where the total heat production \( Q_{\max} \) is found from experiments and is always smaller than the total potential heat release of the cement. Eqn. (2) defines \( r(t) \) in an implicit way, since \( r \) appears in the expression (1) for \( q \).

Eqs. (1)–(2) hold for a portland cement. For a blast furnace slag cement there are two (or more) hydration reactions: the portland and the slag reaction. A separate internal variable is introduced for each reaction. From the combined heat production, which is merely a superposition of the individual contributions, an overall degree of reaction can be calculated.

The thermal characteristics (conductivity, capacity) of the hydrating concrete are made dependent upon this internal variable: \( \lambda = \lambda(r); c = c(r) \).

### 3.2 Numerical procedure for thermal analysis

The heat produced by the hydration reaction flows off by conduction and by convection to the ambient air. The remainder is accumulated in the concrete and increases its temperature. The heat transferred by moisture diffusion and the radiation are neglected. In order to establish the correct boundary conditions, a sufficiently large part of the foundation and the soil on which the concrete is cast should be included in the model.

The resulting differential heat equations are solved using classical finite element procedures for the spatial discretization:

\[
\theta = H(x, y, z) \cdot \Theta, \quad \theta' = B(x, y, z) \cdot \Theta,
\]

where \( \theta \) is the temperature, \( \theta' \) are its spatial derivatives, \( H \) is the interpolation matrix and \( \Theta \) is the vector with nodal temperatures.

For the time integration an unconditionally stable implicit method is preferred. If a backward Euler scheme is used, the following set of iterative equations result[4]:
In these equations a left superscript denotes the instant at which the variables hold; right superscripts stand for the subsequent iterations needed to solve the nonlinear equations at time \( t + \Delta t \). The matrices appearing in eqn.(4) are calculated from:

\[
K_k = \int_V B^T k B \, dV, \quad C = \int_V c H \, dV, \quad K_c = \int_{S_c} h H_{S_c} \, dS,
\]

\[
Q_V = \int_V q \, dV, \quad Q_k = \int_V k \theta' \, dV, \quad Q_c = \int_{S_c} h(\theta_c - \theta) \, dS.
\]  

In these equations \( k \) is a diagonal matrix with the conductivity for the three directions, \( c \) is the capacitance and \( q \) is the internally generated heat, calculated with the above mentioned hydration model. For the part \( S_c \) of the surface of \( V \) where convective boundary conditions apply, \( h \) and \( \theta_c \) are the convection coefficient and the external temperature, while \( H_{S_c} \) is the interpolation matrix on the surface.

The heat production, the degree(s) of reaction and all thermal characteristics are updated at each iteration. The stiffness matrix however is recalculated only rarely, since the parameters appearing in it are subject to small changes.

4 Stress analysis

4.1 Mechanical model

The mechanical behaviour of concrete at early ages is not very well understood yet, and experimental data are but scarcely available. While the development of a more sophisticated model is ongoing, we have chosen to implement a simplified model to give a rough idea of the residual stresses in a solidified concrete body.

It is assumed that for each (small) time step the stress increments are proportional to the strain increments using local and instantaneous elastic constants \( E \) and \( \nu \).

Suitable values were derived from experiments on concrete specimens at different age[3]. Using the hydration model described above, the test results could be correlated to the degree of reaction \( r \) attained in the specimen. It was found that the tangent modulus at zero stress can well be described by
This curve is shown in figure 1 for a particular concrete mix with $r_0 = 0.29$ and $b = 0.43$. Below the threshold value $r_0$, the fresh concrete has almost no stiffness and no stresses are built up in the material. This is in agreement with the fact that the solidified products are initially not contiguous. When the solid particles start touching each other, the stiffness increases very fast. The use of the tangent modulus at zero stress is justified by the consideration that the increase in stiffness should largely be attributed to the forming of new solid reaction products, initially unstrained. This process slows when approaching the end of the reaction.

The Poisson ratio may be calculated from

$$\nu = 0.18 - 0.16(1 - r)^{1.72}. \quad (9)$$

This expression is simpler than the one in [3], but diverges only for small values of $r$, where stresses are irrelevant for the remainder of the calculation.

For low values of $r$ this pseudo-elastic model is far from reality. Indeed, with a zero $E$-modulus the fresh concrete would not even be able to sustain a hydrostatic pressure. But since the stresses developed at low $r$ values are small, the results obtained with this model are already very useful. Nevertheless we are working on a more comprehensive material model, including visco-elastic effects.

### 4.2 Numerical procedure for stress analysis

In view of the simplified material model, it is not needed to use a very refined iteration strategy for the stress analysis. The thermal stresses are calculated incrementally with the same time steps as used in the thermal analysis. The stress increments at time $t$ are found from an elastic analysis
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with initial thermal strains corresponding to the temperature difference:

\[ \tau \sigma = t - \Delta t \sigma + C(\epsilon - \epsilon^{th}) \tag{10} \]

where \( \epsilon^{th} = \alpha (t - \Delta t) \left( \begin{array}{cccc} 1 & 1 & 1 & 0 & 0 \end{array} \right)^T \) and \( \epsilon \) are the incremental strains. Using the finite element discretization \( u = H \cdot U \) and \( \epsilon = B \cdot U \) for the incremental displacements and strains, the equations to solve for \( U \) are

\[ \begin{bmatrix} K \end{bmatrix} \cdot U = F, \quad \text{with} \quad K = \int \! B^T C B \, dV, \quad F = \int \! B^T C \epsilon^{th} \, dV. \tag{11} \]

5 Example results

For the protection of sea walls at the new port of Brugge, Belgium, massive concrete blocks weighing up to 30000 kg were used, the larger ones having the shape of a grooved cube with a side of 2.36 m. In our simulations a perfectly cubic geometry was used. The concrete blocks were cast on site on a 20 cm thick concrete foundation. The thin metal sheet formwork was left out of the model, as it hardly hinders the heat transfer from the concrete to the environment. Figure 2 shows the finite element model used in the thermal analysis. In view of the symmetry of the problem the model is restricted to one quarter. The model includes the foundation and part of the soil beneath it. For the stress calculation phase the model is restricted to the hydrating concrete part.

All results shown are for a slag cement of type CEMIII/C 32.5 (Euro-norm ENV 197). This cement is particularly suited for casting massive concrete volumes, since its reaction proceeds rather slowly, thus limiting

Figure 2: Finite element model for 1/4 of the concrete cube and the soil.
the thermal stresses to a minimum. Hydration model data for this cement are found in [4]. The initial temperature is 15°C for the fresh concrete and 10°C for the foundation and soil.

In the first example the ambient temperature is held constant at 15°C. Figure 3 shows the temperature history in a few points of the central axis: the top, center and bottom of the concrete cube, and the points at 10 cm distance from the top and bottom surface. A maximum temperature of 26.6°C is reached after 132 hours, in the center of the cube. At this point the hydration reaction has nearly completed. Afterwards, the concrete is cooling down slowly.

Figure 3: Temperature history for ambient temperature of 15°C.

Figure 4: Temperature history for ambient temperature of 25°C.
Figure 4 shows the results for an ambient temperature of 25°C. At higher temperature the hydration reaction proceeds faster. The maximum temperature is now reached after 110 hours and attains 37.4°C. Still the difference between the center and the outer face remains as low as 9.4°C.

Looking only at the temperature histories, the same practical conclusions would thus be drawn for both cases: hardly any risk of cracking. But things are quite different when a stress calculation is done. Figures 5 and 6 show the important stress components in three points of the cube: (1) the center of the cube, (2) the center of an upper edge of the cube and (3) the center of vertical edge. Both figures nicely illustrate the reversal of stresses which is
characteristic for this type of problem. Initially the internal heat production causes compressive stresses in the center of the cube and tensile stresses at the surfaces. When the reaction has finished and the concrete mass is cooling down, stresses tend to reverse. Although the final stresses are lower in the case of 25°C ambient temperature, the higher tensile stresses attained during the hydration reaction are more likely to produce cracks because of the smaller strength. Figure 7 shows the distribution of the maximum tensile stress at 100h.

![Figure 7: Maximum tensile stress after 100h for ambient temperature of 25°C. The figure shows 1/4 of the cube. The central axis is on the left.](image)

6 Conclusions

In order to reduce the risk of early cracking in concrete often rules of thumb are used that limit the maximal temperature difference between some points of the structure. A better assessment of the risk of cracking is obtained from numerical simulations of the changing stress fields in the concrete structure.
This should be based on accurate thermal and mechanical models for the hydrating concrete.

While thermal simulations based on recent hydration models may attain a high level of confidence, this is not the case for the mechanical part. No comprehensive mechanical models for hydrating concrete are available yet, due to the complexity of the problem and the scarcity of experimental data.

However, simplified mechanical models may already yield useful information for a better assessment of the risk of cracking. This should no longer be based on temperature data alone, but also on stress intensity as compared to the tensile strength. The simplified models may also contribute to the understanding of the thermal stress history in hydrating concrete, and thus add to the development of a more comprehensive model.

In this work we have used an incremental elastic model for the concrete. Results were presented for massive concrete armour units for the protection of sea walls. Both thermal and mechanical simulations were given.

References


