Double diffusive natural convection in binary gases with horizontal thermal and solutal gradients resulting in opposing buoyancy forces

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Abstract

The natural convection of air in a square cavity with horizontal temperature and concentration gradients which give opposing buoyancy forces near the vertical walls was studied numerically. The basic equations solved are the continuity equation, the Navier-Stokes equations, the energy equation and the concentration equation. Computations were carried out for Prandtl number \( Pr=0.71 \), aspect ratio \( A=1 \), Rayleigh number \( Ra=10^8 \), buoyancy ratio \( = \) solutal buoyancy / thermal buoyancy \( N= 2 \) and various Lewis numbers \( Le \). For \( Le=0.5 \) a strong main flow was formed near the walls and then a stratified layer in both the temperature and the concentration fields was formed between two convection layers. Secondary small roll cells in each convection layer migrated periodically. For \( Le=1 \) and 2 the flow gradually became steady. For \( Le=5 \) a complicated pattern of small roll cells appeared near the walls.

1 Introduction

Double diffusive natural convection occurs when density changes are caused by both concentration and temperature gradients. This phenomenon is encountered in various situations, such as underground coal gasification and air pollution...
problems. A numerical study of natural convection of binary gases in a square cavity with combined horizontal temperature and concentration gradients was carried out. One vertical side wall is kept at a high temperature and at a high concentration of a heavy solute, the other side wall is kept at a low temperature and a low concentration, so the thermal and solutal buoyancy forces near the walls are opposite. If the Lewis number is unity, the rate of thermal conductance is the same as that of molecular diffusion. If the Lewis number is larger than unity, the thickness of thermal boundary layer becomes larger than that of solutal boundary layer. Reversely, if the Le is smaller, the thickness of thermal boundary layer becomes smaller. Then the thermal and solutal flows may coexist in the system. If the Le is large, the flow near the walls is mainly induced by solutal buoyancy and the flow in the interior region far from the walls by thermal buoyancy. These two flows interact and form different flow patterns.

Oscillatory natural convections have been observed in various situations as follows. The characteristic oscillatory phenomena at low Prandtl number with its effect on crystal growth in the liquid metal were reported by Hurle [1]. The double-diffusive natural convection with lateral heating and cooling for the solution system having a concentration gradient along the gravitational direction was studied by many investigators. The oscillatory phenomenon of the flow was reported by Kamakura and Ozoe [2,3]. It seems to be caused by the periodic collapse of the balance of thermal and solutal buoyancies or by the coexistence of opposing and augmenting circulation of flow. The oscillation flow for an opposing case was found experimentally by Kamotani et al. [4] and numerically by Chang et. al [5,6]. Kamotani et al. presumed the oscillation to be caused by the before mentioned interaction between thermal and solutal effects for $Le >> 1$ in water. The numerical simulations of double diffusive convection in such system were published by Lee and Hyun [7], Beghein et. al. [8] and Chang et. al [5,6].

In the present paper, we employed a finite volume method for the numerical analyses of this double-diffusive convection of air for the opposing case. The detailed flow pattern was studied for various Lewis numbers. This paper reports on the investigation of the coexistence of the thermal and solutal buoyancy flows and the resulting oscillatory phenomenon of average Nusselt number.
2 Numerical analysis

2.1 Mathematical model

The model equations for the double-diffusive natural convection consist of the continuity equation, the Navier-Stokes equations, the energy equation and the concentration equation as follows in dimensionless forms:

\[
\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{1}
\]

\[
\frac{\partial U}{\partial \tau} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + Pr \nabla^2 U \tag{2}
\]

\[
\frac{\partial V}{\partial \tau} + U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + Pr \nabla^2 V + PrRa (\Theta + N C) \tag{3}
\]

\[
\frac{\partial \Theta}{\partial \tau} + U \frac{\partial \Theta}{\partial X} + V \frac{\partial \Theta}{\partial Y} = \nabla^2 \Theta \tag{4}
\]

\[
\frac{\partial C}{\partial \tau} + U \frac{\partial C}{\partial X} + V \frac{\partial C}{\partial Y} = \nabla^2 C/Lc \tag{5}
\]

Dimensionless parameters \( Pr, Ra, Le, N \) and \( A \) are defined as follows:

\[
Pr = \frac{\nu}{\kappa}, \quad Ra = \frac{g \alpha \Delta T_{\text{max}} H^3}{\kappa \nu}, \quad Le = \frac{\kappa}{D}, \quad N = \frac{B \Delta C_{\text{max}}}{\alpha \Delta T_{\text{max}}}, \quad A = \frac{H}{B} \tag{6}
\]

Here, the dimensionless variables are defined as follows.

\[
X = x / H, \quad Y = y / H, \quad U = u H / \kappa \\
V = \nu H / \kappa, \quad \tau = t \kappa / H^2, \quad P = \rho H^2 / (\rho \kappa^2) \\
\Theta = (T - T_0) / \Delta T_{\text{max}}, \quad C = (C - C_0) / \Delta C_{\text{max}}
\]

where, \( \nu \) : kinematic viscosity, \( \kappa \) : thermal diffusivity, \( D \) : diffusion coefficient, \( \alpha \) : volumetric coefficient of thermal expansion, \( \beta \) : volumetric coefficient of expansion with concentration, \( \rho \) : density, \( H \) : height of a solution in the
apparatus, \( B \): horizontal width of a solution in the apparatus, \( t \): time, \( T \): temperature, \( \Delta T_{\text{max}} = T_{\text{hot}} - T_{\text{cold}} \), \( T_{\text{hot}} \): temperature on the hot wall, \( T_{\text{cold}} \): temperature on the cold wall, \( T_0 = (T_{\text{hot}} + T_{\text{cold}})/2 \), \( c \): concentration, \( \Delta c_{\text{max}} = c_{\text{max}} - c_{\text{min}} \), \( c_{\text{max}} \): concentration on the left wall, \( c_{\text{min}} \): concentration on the right wall, \( c_0 = (c_{\text{max}} + c_{\text{min}})/2 \).

### 2.2 Boundary Conditions

The following boundary conditions in dimensionless form are illustrated in Fig. 1.

- At \( Y = 0 \): \( U = V = 0 \), \( \partial \theta / \partial Y = 0 \), \( \partial C / \partial Y = 0 \)
- At \( Y = 1 \): \( U = V = 0 \), \( \partial \theta / \partial Y = 0 \), \( \partial C / \partial Y = 0 \)
- At \( X = 0 \): \( U = V = 0 \), \( \theta = 1 \), \( C = 1 \)
- At \( X = 1 \): \( U = V = 0 \), \( \theta = 0 \), \( C = 0 \)

Both initial temperature and concentration in the system were assumed to be 0.5.

\[
\begin{align*}
U &= V = 0 \\
\partial \theta / \partial Y &= 0 \\
\partial C / \partial Y &= 0 \\
\theta &= 0.5 \\
C &= 0.5 \\
\end{align*}
\]

\[
\begin{align*}
1: \text{thermal buoyancy} \\
2: \text{solutal buoyancy}
\end{align*}
\]

![Fig. 1 Boundary conditions.](image)

### 2.3 Computation

A two-dimensional system was presumed. A finite volume method was employed to solve the system numerically. The calculation algorithm of finite volume method is the same as that used by Henkes et al. [9,10]. The local Nusselt number on the hot wall was calculated from a Taylor series for the temperature field and the average Nusselt number was computed by an integration along the hot wall. Computations were carried out by HP Apollo 9000 Model 710. The time step width \( \Delta \tau \) for the numerical simulation was \( 2.25 \times 10^{-7} \).
2.4 Computational meshes
The number of grid points may markedly affect the numerical results. The computational meshes were refined near the walls. The finite volume grid for \(Le=0.5, 1\) and 2 was 60x60 but that for \(Le=5\) was 80x80. The grid points in the \(X\)-direction are positioned according to

\[
\frac{X_i}{H} = \frac{i}{i_{\text{max}}} - \frac{\alpha_x}{2\pi} \sin \left(2\pi \frac{i}{i_{\text{max}}} \right) \quad i=0, 1, \ldots, i_{\text{max}}
\]

Here \(\alpha_x\) are 0.5 for \(Le=0.5, 1\) and 2, and 0.8 for \(Le=5\). The same spacing was used for the grid points in the \(Y\)-direction. The mesh of 80x80 grid was fine near the wall than that of 60x60. Each mesh provides stable results independently of mesh size and was adopted in the following computation.

3 Numerical results

3.1 Computed contour maps
Computations were carried out for \(Pr=0.71, A=1, Ra=10^8, N=2\) and \(Le=0.5, 1, 2\) or 5. The thickness of thermal or solutal boundary layer depends on the Lewis number, namely, thin thermal boundary layer for small \(Le\) and thick one for large \(Le\). Main flow will be caused by solutal buoyancy because of \(N=2\).

Figure 2(a) shows the instantaneous contours of stream function, temperature and concentration at \(Le=0.5\). A strong main flow is formed owing to solutal buoyancy at the external region (near the walls) and the secondary small roll cells are formed at the internal region. As the thermal buoyancy is weaker than the solutal one \((N=2)\) and as the thermal boundary layer near the vertical walls is thinner than the solutal one as shown at \(\tau=0.0009\) or later, the thermal flow is not apparent but the interaction between two buoyancies may exist near the walls. Since the solutal convection is dominant, the temperature profile is apparently controlled by solutal convection. However, at \(\tau=0.0009\) the hot fluid descended along the heated wall due to the solutal convection becomes to rise near the cold wall with its thermal potential. This tendency is still kept at \(\tau=0.0144\).

Figure 2(b) shows the instantaneous contours at \(Le=1\). As the diffusivity of heat is the same as that of solute, the temperature equation becomes the same as the concentration equation and the same profiles were obtained. This is still for \(N=2\) and the solutal convection is dominant just like as Fig. 2(a). A steady state of flow was reached.
Fig. 2 Instantaneous contours of stream function (top), temperature (middle) and concentration (bottom) for \( Pr=0.71, A=1, Ra=10^8 \) and \( N=2 \).
Advanced Computational Methods in Heat Transfer

Fig. 2 (Continued)

$\tau = 0.0009 \quad 0.0027 \quad 0.0072 \quad 0.0144$  steady state  
(c) $Le=2$

$\tau = 0.0009 \quad 0.0018 \quad 0.0027 \quad 0.0045 \quad 0.0063$  
(d) $Le=5$

Figure 2(c) shows the instantaneous contours at $Le=2$. The thickness of thermal boundary layer near the vertical walls becomes large and that of solutal one becomes small. Then the thermal flow can coexist with main solutal flow. The thermal roll cell in inner region appeared at $\tau=0.0009$ or $0.0027$ but becomes small with time. The temperature and the concentration in the core region are uniform owing to thermal convection even at $\tau=0.0144$. The thermal roll cell disappeared at steady state.

Figure 2(d) shows the instantaneous contours at $Le=5$. The thickness of solutal boundary layer becomes progressively smaller. As shown at $\tau=0.0009$ or $0.0018$ the thermal convection becomes dominant near the walls. However a complicated flow pattern was formed near the vertical walls. Several small roll cells due to solutal buoyancy were formed near the walls. The thermal boundary layer is thicker than the solutal boundary layer and the interaction between two flows make the complicated flows.

### 3.2 Oscillatory phenomena

Figure 3 shows the transient responses of the average Nusselt number $N_{\text{uave}}$ on the hot wall. In the case of $Le=0.5$, the $N_{\text{uave}}$ exhibited periodic oscillation. In the case of $Le=1$ and 2, the $N_{\text{uave}}$ converge to a constant value. In the case of $Le=5$ $N_{\text{uave}}$ showed complicated periodic oscillation probably due to the combination of $Le>1$ (thermal effect is dominant) and $N=2$ (concentration effect is dominant). Figure 4 shows the instantaneous contours of stream function for $Le=0.5$. Secondary small roll cells in each convection layer migrated periodically due to the hot fluid plume near the foot of the cooled wall and the cold plume near the top of the left wall.

### 4 Conclusions

The double diffusive natural convection of air for opposing case was studied by the numerical simulation. If Lewis number is different from unity, thermal and solutal flows may coexist in the system. Main flow will be caused by solutal buoyancy because of $N=2$ and the interaction between the thermal and solutal flows makes oscillatory flow or complicated secondary small roll cells.
Fig. 3 Variation of $N_u$ on the hot wall with time at $Pr=0.71$, $A=1$, $Ra=10^8$ and $N=2$.

Fig. 4 Instantaneous contours of stream function for $Le=0.5$.

References

1. Hurle, D.T.J. Temperature Oscillations in molten metals and their
134 Advanced Computational Methods in Heat Transfer


