Numerical prediction of irregular cyclic variation of temperature in an underground metro system
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ABSTRACT
In this paper, a first Law of Thermodynamics analysis is made to predict the daily variation of the average temperature in a subterranean railway tunnel system operating under irregular cyclic conditions. The results of the finite difference method used to determine the temperature history indicate that the conjugate heat transfer to and from the tunnel wall attenuates the temperature amplitude and causes a lag in the oscillations.

INTRODUCTION
In general, unsteady periodic heat transfer takes place with the temperature varying either harmonically or irregularly. Examples in these two categories are, respectively, an air conditioning plant with cosinusoidally varying ambient conditions and trains in transit through an underground tunnel according to a specified timetable. An exact harmonic analysis for the very low frequency and constant mass flow rate through a control volume has been made by Barrow [1]. That work was later extended by Barrow and Pope [2], who developed a numerical approach to allow for variable energy input and a wider frequency range. In both studies, the heat transfer at the wall takes place conjugately as described in detail by Eckert and Drake [3] for example. While the theoretical and numerical harmonic analyses have proved successful, a more universal method is needed to cater for the irregular conditions in a rail tunnel. In the following, a method is developed to predict the average temperature variation in an intensively operated metro system when the traffic and, hence, levels of energy release vary markedly through the day. Variation in traffic also gives rise to a related variation of air flow movement through the system. Further, there
is also a diurnal change in ambient temperature which behaves approximately cosinusoidally. Thus the work rate and the conjugate heat transfer at the tunnel wall both vary periodically and irregularly.

**THE ENERGY EQUATION**

For a stationary control volume shown in Figure 1, the energy equation may be written:

\[ Q - W = \sum_i \left( m_i h_0 - m_i h_i \right) + \frac{d}{dt} \left( E \right) \]

\[ \text{(1)} \]

In the notation shown, and introducing the appropriate specific heat, \( c_i \),

\[ Q - W = \sum_i \left( \dot{m}_i c T_0 - \dot{m}_i cT_i \right) + \sum_i \left[ M_i c \frac{dT}{dt} + T \frac{d}{dt} (Mc)_i \right] \]

\[ \text{(2)} \]

Most frequently, the mass streams are continuous when the differential continuity equation applies to each constituent which are assumed here to have the same temperatures. For the rail tunnel, this does not present any difficulty since the traffic stream may be assumed to be continuous over specified periods albeit that it is not strictly a continuum flow. While all parameters may be time dependent, it is important to recognise that the change in the time averaged temperature in the control volume (CV) in the cyclic transient problem, must equal the temperature change in the equivalent pseudo-steady problem, i.e. one with the same net energy transfer and same average mass flow rates. This will become more evident later.

**Solution Procedure**

The history of the CV outlet temperature \( T_0 \) may be determined numerically from equation (2) provided all the parameters are known functions of time. The heat transfer rate \( \dot{Q} \) at the boundary is of course conjugated and has to be evaluated interdependently by a method for a semi-infinite solid as outlined in Schneider [4], for example. The flow rates are considered to be equal over the time interval. Using a backward-difference finite-difference scheme, \( T_0 \) may then be determined for any prescribed CV geometry, properties and conditions.

**The Railway Tunnel example**

The details of the problem are incorporated in Figure 2. Using a ventilation mass flow rate of 920 (trains/h) \( \frac{1}{2}/6 \), the calculation was made in time steps
of 1800s being repeated until convergence was obtained. Only 32% of the train stream was considered to be effective, this "thermal factor" being determined using a lumped-capacity analysis or Newtonian heating model, [4]. The power input was calculated from the specified rate throughput of trains and the energy per train in transit. As outlined previously, the conjugate heat transfer rate was determined interdependently for a semi-infinite solid of appropriate properties. For this purpose, a constant tunnel wall heat transfer coefficient of 10 W/m²K, (2 W/m²K overnight), was assumed.

RESULTS

A typical set of results is shown in Figure 2 where the inlet and tunnel air temperatures are shown together with the variation of tunnel wall heat flux. The average temperatures are indicated, and for completeness, the data for an adiabatic tunnel are included. (Similar results were obtained for air ventilation only). The overall average temperature increase is 9°C when heat transfer to both the air and the trains is considered. For heat transfer to the air only, the temperature rise is 20°C. These changes compare well with the temperature increases calculated on the basis of steady-state conditions, i.e. the time-averaged power divided by the thermal capacity of the time-averaged streams. This pseudo-steady criterion serves to check the results of the more complex transient numerical analysis. In Figure 2(c), it is to be noted that the integrated wall heat transfer is approximately zero, it being tacitly assumed in the calculation that there is negligible net heat loss from the tunnel by conduction alone, as indicated by the imaginary adiabatic surface shown in Figure 1.

DISCUSSION

Despite the irregular nature of the curves of outlet temperature in Figure 2, it can be clearly seen that the amplitude of the temperature variation is less than that of the inlet temperature where there is wall heat transfer and that there is a lag of about 2 hours in real time. This demonstrates the damping influence of the wall. The corresponding heat flux distribution is in keeping with the characteristics of the $T_0$ curve. Not surprisingly, the curve of $T_0$ for the hypothetical adiabatic tunnel resembles that of the inlet temperature albeit that its average value is 9°C greater. There are significant fluctuations in the heat flux at 0 and 0600 in time as a consequence of the abrupt changes in the traffic density, despite the care taken to satisfy the convergence criterion in terms of the Fourier and Biot numbers for the conjugate convection-conduction heat transfer at the tunnel wall interface. However, the fluctuations in the q-curve (and also in the $T_0$ curve) do decay as anticipated.
Similar calculations for air ventilation alone lead to the conclusion that the temperatures in that case are prohibitive, the average value now being 37.5°C. It is clear then, that energy storage in the trains themselves is important in the cooling of the tunnel system and this is a matter which needs to be pursued further, particularly as far as assessment of "thermal effectiveness" of the train structure is concerned.

It should be pointed out that the cooling in the present study is effected by passive means only, but the calculations are easily extended to include additional cooling by piped water, for example, should this be a requirement to maintain temperatures within specified limits.

CONCLUSIONS

A numerical finite-difference procedure has been developed for the purpose of calculating the temperature history in stationary control volume analysis under irregular cyclic conditions. In particular, a realistic example of an underground railway tunnel system or metro has been chosen to study the temperature variation under conditions of passive cooling, viz, cooling produced by the induced air flow through the system and by heating of the train traffic in transit. Meaningful quantitative results have been obtained in terms of the thermodynamic criteria which must be satisfied. By comparing the results for a tunnel with heat transfer at the tunnel wall interface, and those for an adiabatic tunnel, the role of the tunnel wall and its environment in attenuating temperatures is clearly shown. Furthermore, the importance of convection of energy out of the system by energy storage in the trains is demonstrated by comparing the results for "air cooling" and for combined "air and train" cooling.

Finally, it needs to be emphasised that the present calculations have been made assuming the air to be dry when the perfect gas model pertains. In the real situation, mass transfer from wet surfaces within the tunnel void and from human occupancy of the trains and stations leads to a significant increase in the cooling efficiency of the induced air. Accordingly, the perfect gas analysis results in a conservative assessment of the temperature rise within the tunnel system.

ACKNOWLEDGEMENT

The authors wish to thank the Project Director, CrossRail Project Team, for permission to publish this paper.
REFERENCES


Figure 1. Stationary control-volume model for the analysis of periodic energy transfer. (negligible potential and kinetic energy changes)
The heat transfer rate at CV boundary takes place by conjugate heat conduction into a semi-infinite solid.
Figure 2. Variation of train traffic, air inlet temperature, outlet temperature (with and without wall heat transfer), and wall heat flux, $q$, when both air and trains are effective.