A numerical evaluation of an inverse heat conduction procedure for determining contact resistance during metal forming
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ABSTRACT

In order to experimentally determine the contact resistance during metal forming processes a technique to measure the temperatures very close to the two sides of the interface has been developed. This technique has been initially applied to a situation in which a short cylinder made from aluminium is squeezed between two steel dies. In this experimental study, the material is unheated and the heat flux results purely from the work being done on the aluminium. Using the measured temperature-time variations on each side of the interface, an inverse heat conduction procedure is used to deduce the value of the contact resistance. In this technique the internal heat generation is assumed to uniform in the aluminium. In the present study, the effect of non-uniform internal heat generation and errors in the temperature measurements on the accuracy of this procedure have been numerically investigated.

NOMENCLATURE

\( c \) = specific heat
\( E \) = error function
\( G \) = parameter used in describing \( q_i \)
\( h \) = contact resistance coefficient
\( k \) = thermal conductivity
\( L_b \) = thickness of billet
\( q_i \) = internal heat generation rate per unit volume
\( q_{IM} \) = mean internal heat generation rate per unit volume
\( T \) = temperature
\( T_i \) = interfacial temperature
\( T_0 \) = initial temperature
\( t \) = time
During the forming of metals, heat is generated internally within the billet as a result of the work done on the billet. The temperature distribution within the billet that arises as a result of this can have an important influence on the quality of the product. The temperature distribution in a given situation is influenced by the contact resistance at the interface between the die and the billet. Although this contact resistance is usually relatively small, the heat fluxes across the interface can be very high due to the unsteady nature of the process and the relatively short time over which the process occurs. As a result of this, the contact resistance can have an important influence on the process. Several attempts have been made to experimentally determine the contact resistance coefficient, often termed the heat transfer coefficient, during metal forming. Several of these studies have used an inverse heat conduction procedure in which the temperature variation with time was measured at several points in the die and/or billet. Typical of these studies are those reported by Semiatin et al [1] and Malinowski et al [2]. A technique has recently been developed [3] to measure the temperatures very close to the interface in the die and the billet, i.e., essentially, to measure the temperature “jump” across the interface resulting from the contact resistance. Because these measurements are taken at the interface when they are used in conjunction with an inverse heat conduction procedure to obtain the contact resistance coefficient they should give much more accurate results than those obtained in previous studies where measurements far from the interface were used. In the first implementation of this method, measurements are being taken with a short aluminium cylinder as the billet being compressed between two steel dies. In these experiments, the material is initially unheated so the heat flux is purely the result of the work done on the aluminium. The situation considered is shown in Figure 1. In the present paper the accuracy of the procedure being used to find the contact resistance has been numerically evaluated. One of the factors that influences the accuracy of the result is the fact that the deformation in the billet is not uniform and, as a result, the internal heat generation within the billet is not uniform. It is believed that relatively accurate results can, in fact,
be obtained by assuming a uniform internal heat generation equal to the average value of the actual internal heat generation. The accuracy of this assumption has been assessed in the present work. It has also been suggested that when only the interfacial temperatures are measured, small errors in the temperature readings will cause relatively large errors in the predicted interfacial heat transfer coefficients and this has also been investigated here.

GOVERNING EQUATIONS

In the experimental arrangement that is being evaluated in the present work the temperature-time variations on each side of the interface are measured and these are used in conjunction with the solution for the heat conduction in the die and billet to determine the heat transfer coefficient. In the present study, experimental results have not been used in evaluating the adequacy of the method. Instead, a non-uniform internal heat distribution has been assumed and the variation of the interfacial temperatures with time have been calculated for an assumed value of the contact resistance coefficient. These calculated temperatures have then been used effectively as “experimental” results and the inverse heat conduction procedure based on the assumption that the internal heat generation rate is uniform has been used to derive the heat transfer coefficient. This predicted value has then been compared with the actual assumed heat transfer coefficient to determine whether the assumptions on which the inverse heat conduction procedure is based are adequate.

Now, in the actual experimental study, there is a significant reduction in the thickness of the billet. This has not been considered in the present work which is aimed purely at evaluating the effect of the assumptions of a uniform rate of internal heat generation and of small
errors in the temperature measurements on the results obtained using the procedure.

In the present work it has been assumed that the temperature distribution is one-dimensional, that the contact resistance coefficient is constant, that the thermal properties of the die and billet remain constant and that the die and billet are initially at the same uniform temperature. The temperature variation in the billet is given by:

\[ k_B \frac{\partial^2 T}{\partial x^2} + q_I = \rho_B c_B \frac{\partial T}{\partial t} \]  

(1)

\( q_I \) is a specified function of \( x \) in the billet. Similarly the temperature in the die is given by:

\[ k_D \frac{\partial^2 T}{\partial x^2} = \rho_D c_D \frac{\partial T}{\partial t} \]  

(2)

There is, thus, assumed to be no internal heat generation in the die. At the interface, the following applies:

\[-k_B \left( \frac{\partial T}{\partial x} \right)_B = h \left( T_{iB} - T_{iD} \right) = -k_D \left( \frac{\partial T}{\partial x} \right)_D \]  

(3)

The boundary conditions on the solution are, \( x \) being taken as zero at the interface:

- At \( x = -(L_B / 2) \) : \( \frac{\partial T}{\partial x} = 0 \)
- For large positive \( x \) : \( T = T_0 \)

i.e. the temperature distribution in the billet is symmetrical about the centre-line and far from the interface the die temperature is at the initial temperature \( T_0 \).

In the present work, the value of \( h \) and the distribution of \( q_I \) in the billet are first assumed. The above equations are then solved using a simple implicit finite-difference procedure to give the values of \( T_{iB} \) and \( T_{iD} \) at various values of time. These calculated results, obtained, in general, using a non-uniform distribution of \( q_I \), are then used as inputs to an inverse heat transfer procedure which calculates \( h \) assuming a uniform distribution of \( q_I \). In this procedure, the value of \( h \) is first guessed and the variation of the temperatures at the die and billet interfaces with time are calculated. The difference between these calculated values of the interface temperatures and values calculated using the specified value of \( h \) and the specified variation of \( q_I \) is expressed in terms of the following error function:

\[ E = \sum_{0}^{t_m} \left[ \left( T_{iBE} - T_{iBC} \right)^2 + \left( T_{iDE} - T_{iDC} \right)^2 \right] \]  

(4)

where \( t_m \) is the maximum dimensionless time to which the measurements and calculations are taken. The subscript \( E \) refers to the temperatures given using the specified \( h \) value and the specified \( q_I \).
distribution while the subscript $C$ refers to the temperatures given using the guessed value of $h$.

Now, to find the “best” value of $h$, its value must be adjusted to minimize the error function, $E$, i.e. the value of $h$ must be adjusted to give $\frac{dE}{dh} = 0$. But using equation (4):

$$\frac{dE}{dh} = 2 \sum_{0}^{t_m} \left[ (T_{iBE} - T_{iBC}) \frac{dT_{iBC}}{dh} + (T_{iDE} - T_{iDC}) \frac{dT_{iDC}}{dh} \right]$$

the $T_{iBE}$ and $T_{iDE}$ values not, of course, being dependent on $h$. Since $h$ is guessed, the value of this derivative will not generally be zero and the value of $h$ must be adjusted to make it zero. Now if $T_{iBC}$ and $T_{iDC}$ are the calculated interfacial temperatures obtained using a guessed value of $h$ then to make the derivative zero, the value of $h$ must be changed by $\Delta h$ which is such that:

$$\sum_{0}^{t_m} \left[ (T_{iBE} - T_{iBC} - \frac{dT_{iBC}}{dh} \Delta h) \frac{dT_{iBC}}{dh} + (T_{iDE} - T_{iDC} - \frac{dT_{iDC}}{dh} \Delta h) \frac{dT_{iDC}}{dh} \right] = 0$$

This can be used to find $\Delta h$ provided that the values of $\frac{dT_{iBC}}{dh}$ and $\frac{dT_{iDC}}{dh}$ as functions of time are determined. They are numerically obtained, in the present method, by calculating the $T_{iBC}$ and $T_{iDC}$ variations using an initial guessed value of $h$ and then incrementing $h$ by a small selected amount, $\delta h$, (say, equal to 1% of $h$) and then recalculating the $T_{iBC}$ and $T_{iDC}$ values using this new value of $h$.

Because of the approximate way in which the derivatives are obtained, an iterative approach has to be adopted. This involves the following steps.

1. Guess a value of $h$
2. Calculate the variations of the interface temperatures up to the maximum time being considered.
3. Increase $h$ by a chosen amount, $\delta h$, calculate the interface temperature variations and use these to find the derivatives. Then use these in equation (6) to find $\Delta h$ and use this to get a new values of $h$.
4. Beginning at step (2), repeat the process until the value of the error function, $E$, ceases to change to some selected degree of accuracy.
RESULTS AND DISCUSSION

As discussed above, the aim of the present study was to determine whether in applying the inverse heat conduction method, it is adequate to assume that the billet is at a uniform temperature and to determine whether if a small error exists in one of the interfacial measurements it will cause a relatively large error in the predicted value of $h$. To answer these questions, temperature variations were calculated using a prescribed $h$ value and a prescribed internal heat generation rate distribution and then using these as known inputs to the inverse heat conduction procedure which gave a predicted value of $h$ which could be compared with the original inputted value to assess the adequacy of the assumptions being used.

In carrying out this procedure, the actual internal heat generation rate was assumed to be given by:

- For $x > -GL_B/2$: $q_I = 0$
- For $x < -GL_B/2$: $q_I = q_{IM}/G$

where, $q_{IM}$ is the mean internal heat generation rate which in a real situation can be found from the measured force on the billet and the measured rate of deformation of the billet. Here, $q_{IM}$ will be specified.

Now in the actual experimental study, the billet is deformed at an approximately constant rate up until some time $t_1$ after which there is no further deformation, the force on the billet being kept constant. In this second phase there is, of course, no internal heat generation. Measurements are continued for times beyond $t_1$ up to some time $t_2$. It will therefore be assumed here that:
Fig. 3 Variation of derived value of $h$ with error in $T_{IB}$ for $q_{IM} = 6 \times 10^6 \text{ W/m}^2$ and $t_1 = 10 \text{ s}$ and $t_m = 20 \text{ s}$

- For $t < t_1$: $q_{IM} = \text{a specified value}$
- For $t > t_1$: $q_{IM} = 0$

In order to assess the effect of a small error in one of the interfacial temperature measurements on the predicted value of $h$, the inputted values for the temperature on one side of the interface were all altered by a selected value before applying the inverse heat conduction procedure.

Results have only been obtained for the situation being used in the experimental evaluation of the proposed procedure i.e. for dies made of steel and an aluminium billet with a thickness of 25.4 mm.

Figure 2 shows typical variations of predicted $h$ with $G$ for two typical values of the initially assumed $h$ for a fixed mean rate of internal heat generation. When $G$ is near one, the predicted values are very close to the inputted value. Only with the highest inputted value of $h$ (i.e. the lowest contact resistance) do significant errors arise at the lower values of $G$. Results have been obtained at other values of $q_{IM}$ and of $t_1$ and $t_2$ and these results all have the same basic form as those given in Figure 2. Thus it appears that ignoring the spatial variation of internal heat generation will only lead to significant errors in the derived value of $h$ when the contact resistance is low. The effect of an error in one of the interfacial temperatures on the derived value of $h$ is illustrated by the results given in Figure 3. It will be seen that even with errors of as large as 0.1°C the derived values of $h$ are in error by significantly less than 10%.

CONCLUSIONS

The results obtained in the present study indicate that:
1. Unless the contact resistance is very low (i.e. \( h \) very high) the experimental results can be analysed by assuming that the internal heat generation in the billet is uniform.

2. Small errors in the experimental interfacial temperature measurements will not cause large errors in the derived values of \( h \).

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REFERENCES


