Numerical simulation of temperature distribution and seam forming in a narrow gap welding
D. Weiß, U. Franz, G. Lube & J. Schmidt
Otto von Guericke University of Magdeburg,
Graduate College MBI, P.O. Box 4120, D-39106 Magdeburg, Germany

ABSTRACT

The knowledge of the temperature distribution of the workpiece during arc welding is necessary in order to ascertain change in material structure as well as strength of the joint.

The simulation has not only to consider the heat effect of the arc but also the weld pool deformation influenced by arc pressure and gravitation.

A coupled non-linear system based on the description of energy transport and minimization of surface energy has been employed. The described system is solved by using the Galerkin/least-squares formulation of the finite element method. Numerical results for examples are given.

MODELING

The topic of this paper is the numerical simulation of temperature distribution and weld pool deformation during narrow gap arc welding. The knowledge of the temperature field and the seam geometry is the basis for process optimization. From these facts we can derive the quality of the joint, its strength and material structure.

The modeling of the process has to comprehend the most important effects of the arc – energy input and acting forces. Most of the known models take into consideration pool convection, surface deformation and different energetic parts of arc, radiation, passing drops and evaporation by assumption of Gauss distributed surface sources [3].

A strong surface deformation appears in the case of narrow gap welding. This is to consider by modeling of the energy transport.

We consider as an example, narrow gap welding in inclined or vertical
position, respectively. An intensity distribution \( \dot{q}_{\text{arc}} = \dot{q}(x^2 + y^2) \) according to Gauss independent of \( z \) takes insufficiently into account, that the energy input takes place where the transitional resistance is lowest. This resistance is determined primarily by the distance between electrode and pool surface as well by the conductivity of the plasma. The conductivity is influenced by evaporation of the metal on the pool surface. If the improved electrical conductivity is incorporated by a reduced transitional resistance \( R \), the distribution function \( \Phi \) results as follows:

\[
\Phi(x, y, z, T) = \exp \left( -\frac{R^2 - R_{\text{min}}^2}{R_{\text{max}}^2 - R_{\text{min}}^2} \right).
\]

(1)

For primary studies, a linear relation was formulated for the dependence of \( R \) from \( T \):

\[
R(x, y, z, T) \sim \begin{cases} 
L & : T < T_{\text{liq}} \\
L \cdot \left( 1 - \vartheta \frac{T - T_{\text{liq}}}{T_{\text{evap}} - T_{\text{liq}}} \right) & : \text{otherwise}
\end{cases}
\]

(2)

\( \vartheta \) is a constant weighting factor for incorporation of the ionization conditions \((0 < \vartheta < 1)\). \( L \) is the distance between the electrode and the pool surface.

The maximum values for pressure \( p_{\text{arc, max}} \) and heat flux density \( \dot{q}_{\text{arc, max}} \) in equation (3) and (4) are calculated in such a way that the corresponding surface integrals on the domain of the arc influence are equal to the total force or the total energy input, respectively:

\[
p_{\text{arc}}(x, y, z, T) = p_{\text{arc, max}} \Phi(x, y, z, T) \cdot (\vec{n}_{\text{arc}} \cdot \vec{n}_{\text{surf}})(x, y, z)
\]

(3)

\[
\dot{q}_{\text{arc}}(x, y, z, T) = \dot{q}_{\text{arc, max}} \Phi(x, y, z, T) \cdot (\vec{n}_{\text{arc}} \cdot \vec{n}_{\text{surf}})(x, y, z).
\]

(4)

The basis for the modeling is formed with the following additional assumptions:

- The domain is a plate of constant thickness stretching to infinity in the direction of welding,

- constant conditions for energy input and transport,

- the boundary of the pool underside is a pool support or solid material and

- electro-magnetic and chemical effects are disregarded.

The energy transport in the domain \( \Omega \) (figure 1) is described by the Fourier-Kirchhoff equation in the stationary form by means of an appropriate coordinate system. The problem which has to be solved can be described as follows:
Figure 1: Domain $\Omega$ for the energy transport problem with the free surface $\Gamma_{top}$

$$
\Omega = \Omega(T) := \{ P = (x, y, z) \in \mathbb{R}^3 : -L_{x_1} < x < +L_{x_2},
0 < y < L_y, 0 < z < Z(T) \}
$$

$$
\partial \Omega = \Gamma_{top}(T) \cup \Gamma_{front} \cup \Gamma_{side} \cup \Gamma_{symm} \cup \Gamma_{back} \cup \Gamma_{bottom}
$$

$$
-\nabla \cdot (\lambda(T) \nabla T) + c_p(T) \rho \bar{v} \cdot \nabla T = \dot{q}_{vol} := 0 \quad \forall \ P \in \Omega = \Omega(T)
$$

$$
T = T_{front} \quad \forall \ P \in \Gamma_{front}
$$

$$
-\lambda(T) \frac{\partial T}{\partial n} = 0 \quad \forall \ P \in \Gamma_{symm} \cup \Gamma_{back}
$$

$$
-\lambda(T) \frac{\partial T}{\partial n} = -\alpha_{eff}(P, T)(T - T_\infty(P)) \quad \forall \ P \in \Gamma_{bottom} \cup \Gamma_{side}
$$

$$
-\lambda(T) \frac{\partial T}{\partial n} = \begin{cases} 
\dot{q}_{\text{input}}(P, T) : r = \sqrt{x^2 + y^2} \leq \hat{L}_{arc} \\
-\alpha_{eff}(P, T)(T - T_\infty(P)) : r > \hat{L}_{arc} 
\end{cases} \quad \forall \ P \in \Gamma_{top} = \Gamma_{top}(T)
$$

with: $\dot{q}_{\text{input}}(x, y, z, T) = \dot{q}_{arc,max} \Phi(P, Z, T) \cdot (\vec{n}_{arc} \cdot \vec{n}_{surf})(P, Z)$

$$
-\alpha_{eff}(P, T)(T - T_\infty(P)) - \dot{q}_{\text{evap}}
$$
The forming of the pool surface \( \hat{z} = L_z - z \) is determined by minimization of the surface energy:

\[
\int_{\Omega} \left\{ \sigma(T) \left( \left( 1 + |\nabla \hat{z}|^2 \right)^{1/2} - 1 \right) - p_{arc} \hat{z} + \frac{1}{2} \rho g \hat{z}^2 \right\} \, dx \, dy \Rightarrow \text{MIN.} \quad (5)
\]

The problem is solved by using the Euler-Lagrange equation (cf. for details [4]).

\[
G := \{ Q = (x, y) \in \mathbb{R}^2 : -L_{x_1} < x < +L_{x_2}, \, 0 < y < L_y \}
\]

\[
\Gamma_{\text{top}} = \Gamma_{\text{top}}(T) : -\nabla \cdot \left( \frac{\sigma(T) \nabla \hat{z}}{\sqrt{1 + |\nabla \hat{z}|^2}} \right) + c \hat{z} = f(Q, Z, T) \quad \forall Q \in G
\]

with: \( c = \rho g \cos \theta_{\text{plate}} \)

\[
f = f(Q, Z, T) = -p_{arc,\text{max}}(Z) \Phi(Q, Z, T) \cdot (\vec{n}_{\text{arc}} \cdot \vec{n}_{\text{surf}})(Q, Z) + \rho g \left( L_z \cos \theta_{\text{plate}} + x \sin \theta_{\text{plate}} \right) + p_0(Z)
\]

\[
Z = L_z \quad \forall Q \in \partial G : y > 0, \, x < +L_{x_2}
\]

\[
\frac{\partial Z}{\partial \bar{n}} = 0 \quad \forall Q \in \partial G : y = 0
\]

\[
\frac{\partial Z}{\partial n} = 0 \quad \forall Q \in \partial G : x = +L_{x_2}
\]

During iteration, the free constant \( p_0 \) is calculated in such a way that the mass bilance of the additional material is carried out concerning to the reinforcement of the seam. The term for the static pressure was adapted for the case of the inclined plate.

**NUMERICAL SOLUTION**

The described coupled non-linear system is linearized by using the method of "frozen coefficients" and solved iteratively with underrelaxation.
\[
\begin{align*}
\text{Pool surface:} \quad & \text{find } Z^{m+1} = Z(Z^m, T^m) \text{ such that} \\
& -\nabla \cdot \left( \frac{\sigma(T^m) \nabla Z^{m+1}}{1 + |\nabla Z^m|^2} \right) + c Z^{m+1} = f(Z^{m+1}, T^m) \\
\text{Temperature field:} \quad & \text{find } T^{m+1} = T(Z^{m+1}, T^m) \text{ such that} \\
& -\nabla \cdot (\lambda(T^m) \nabla T^{m+1}) + c_p(T^m) \vartheta^2 \cdot \nabla T^{m+1} = 0 \quad \text{in } \Omega(T^m)
\end{align*}
\]

until \(|T^{m+1} - T^m| < \varepsilon_T\)

\text{Cooling conditions: } t_{8/5}(y, z)

\text{Seam shape: } Z^{m+1}(y)|_{x=L_{x_2}}

The resulting linear problems of diffusion-convection-reaction type are solved by the Galerkin/least-squares formulation of the finite element method on a tetrahedral mesh (test function \(\varphi\) is piecewise linear). Now \(u\) stands for the unknown functions \(T\) or \(Z\), respectively:

\[
Lu \equiv -\nabla \cdot (a \nabla u) + \bar{b} \cdot \nabla u + cu = f \quad \text{in } \Omega
\]

\[
u = g \quad \text{on } \Gamma_1, \quad \alpha \frac{\partial u}{\partial n} + \alpha(u - u_\infty) = 0 \quad \text{on } \Gamma_2.
\]

Find \(u \in V^g := \{v \in W^{1,2}(\Omega) : v|_{\Gamma_1} = g\} \) such that

\[
a(u, \varphi) = f(\varphi) \quad \forall \varphi \in V^0,
\]

with

\[
a(u, \varphi) = \int_\Omega (a \nabla u) \cdot \nabla \varphi \, d\Omega + \int_\Omega \bar{b} \cdot \nabla u \varphi \, d\Omega + \int_\Omega cu \varphi \, d\Omega + \\
\int_{\Gamma_2} \alpha u \varphi \, d\Gamma
\]

\[
f(\varphi) = \int_\Omega f \varphi \, d\Omega + \int_{\Gamma_2} \alpha u_\infty \varphi \, d\Gamma
\]

In the case of the Galerkin/least-squares formulation stabilizing terms are added to the standard Galerkin formulation in dependence of the residual \(Lu - f\). After extension of the discrete test function \(\varphi_h\) to the form:

\[
\varphi_h = \varphi_h + \delta L \varphi_h,
\]
the discrete problem can be described as follows:

\[
\text{Find } u_h \in V_h^0 := \{v \in V^g, \ v|_K \in P_1(K) \ \forall \ K \} \text{ such that }
\]

\[
a(u_h, \varphi_h) + \sum_K \delta_K \int_K (Lu_h - f) L\varphi_h \, dK = f(\varphi_h) \quad \forall \ \varphi_h \in V_h^0. \tag{12}
\]

The method is stable and high order accurate due to the residual form of equation (12). A good adaptability to locally changing diffusive, convective and reactive influences can be noted [1].

The approach in [2] allows a simple calculation of the upwind parameter \( \delta_K \). \( h_K \) is here the element length in the flow direction:

\[
\delta_K = \frac{h_K}{2|\tilde{b}_K|} \frac{P\epsilon_K}{\sqrt{1 + P\epsilon_K^2}}, \quad P\epsilon_K = \frac{h_K|\tilde{b}_K|}{a_K}, \quad h_K = \frac{2|\tilde{b}_K|}{\sum_{i=1}^4 |\tilde{b}_K \nabla \varphi_i|}. \tag{13}
\]

**RESULTS**

The test of the model performance took place in comparison with the experiment. The solution attitude has been tested with reference to the seam shape for build-up welding in horizontal position. The strong reinforcement represents here a special requirement to the algorithm. The figure 2a) shows the comparison of the calculated seam shape after the solidification \( (x = 40 \cdot 10^{-3} \text{m}) \) with the experimental result. In figure 2b) the result is shown at the position of the electrode \( (x = 0 \text{ m}) \). The temperature distribution and the weld pool deformation correspond with the experiment well.

The temperature distribution for the narrow gap welding in vertical position is shown in figure 3. The distribution has been measured by means of thermography on the back side of the plate in the \( x-y \)-plane. The theoretical and experimental 2D-field are opposed in figure 3a). The transient temperature field for \( y = z = 0 \) (figure 3b) confirms the quality of the solution.

With the presented model we have a description of the process, which is in accordance to the specific conditions of the energy transport in the case of the narrow gap welding. The chosen formulation of the arc effect allows the application to different deep depressed pool surfaces and different welding positions.

For practical application of the algorithm, future work is necessary to ascertain the dependence of the model parameters from the technological parameters as the result of experimental and theoretical investigations.
**Heat Transfer**

\[ x = 40 \cdot 10^{-3} \text{m} \]

**Figure 2:** Temperature distribution and surface formation for build-up welding in cross section to welding direction (temperature in \(^\circ\text{C}\))

**Figure 3:** Comparison between measured and calculated temperature distribution on the back side of the plate for narrow gap welding (temperature in \(^\circ\text{C}\))
NOMENCLATURE

Ambient temperature \( T_\infty \) \( [^\circ C] \)
Arc length (average) \( L_{arc} \) \( [m] \)
Coefficient of heat transfer (conv. & rad.) \( \alpha_{eff} \) \( [W/(m^2 K)] \)
Cooling time from 800\(^\circ\)C to 500\(^\circ\)C \( t_{8/5} \) \( [s] \)
Density \( \varrho \) \( [kg/m^3] \)
Finite element \( K \)
Gravitational acceleration \( g \) \( [m/s^2] \)
Heat capacity \( c_p \) \( [J/(kg K)] \)
Heat flux density \( \dot{q} \) \( [W/m^2] \)
Heat source density (per volume) \( \dot{q}_{vol} \) \( [W/m^3] \)
Inclination of the plate in \( x - z\)-plane \( \theta_{plate} \) \( [^\circ] \)
Normal unit vector \( \vec{n} \)
Plate lengths \( L_{x1}, L_{x2}, L_y \) \( [m] \)
Plate thickness \( L_z \) \( [m] \)
Temperature \( T \) \( [^\circ C] \)
Thermal conductivity \( \lambda \) \( [W/(m K)] \)
Surface deformation \( Z \) \( [m] \)
Surface tension \( \sigma \) \( [N/m] \)
Velocity \( \vec{v} \) \( [m/s] \)

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REFERENCES


