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Numerical simulation of buoyant plumes using a spectral element technique

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1 Introduction

Highly convection-dominated thermally-driven problems appear frequently in the engineering practice. Their investigation often involves the necessity of direct simulation of transition to turbulence which requires high accuracy of spatial and time discretization. In the present study numerical simulation of unsteady plumes is presented performed with a spectral element algorithm. The spectral element method (SEM), proposed by Patera [1], is a high-order Galerkin type method which demonstrates excellent properties (small numerical diffusion and dispersion error) with respect to convection-dominated problems (see Timmermans & van de Vosse[2]). A major problem related to such high-order methods is that the resulting matrix is quite full in comparison to low-order methods. Because of this their application to fluid flow problems is often combined with some splitting procedure allowing the decomposition of the Navier-Stokes and energy equations into a set of positive definite symmetric algebraic systems which can be efficiently solved by means of iterative methods (see for example Karniadakis et al. [3]).

The direct numerical simulation of unsteady plumes induced by heated bodies is a relatively recent research topic. This problem is of practical importance in relation to, for example, the cooling of electronic devices and the heat transfer from pipes in heat exchange systems. A very extensive study of the buoyant plumes over a line heat source in the interior of a rectangular cavity is presented by Desrayaud and Lauriat [4]. The way to chaos of such flows is studied in details showing an intermittent scenario after a two-frequencies-locked state. Bastiaans et al. [5] present large-eddy simulations of transient plumes using different subgrid models.

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In the present study a modified splitting procedure combined with spectral element discretization is developed and some preliminary results of the direct numerical simulation of unsteady plumes in a square cavity induced by a finite-area heat source on its bottom are performed. From preliminary experiments (see van de Burgt [6]) and large-eddy simulations (see Basiaans et al. [5]) it is found that the flow deviates from laminar at Rayleigh number about $R = 10^8$. Therefore in this study calculation are performed at $10^8 \leq R \leq 10^9$. At $R = 10^8$ the flow is clearly laminar and then it develops undergoing the first and the second bifurcation toward a two-frequencies locked state at $R = 10^9$.

2 Governing equations

The equations for natural convection in a 2-D domain Ω (belonging to the x-y plane) presuming an incompressible flow with constant fluid properties except for the density in the buoyancy term, which is supposed to be linearly dependent on the temperature, read:

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}.\nabla)\mathbf{u} = -\nabla p + RPrT\mathbf{g} + Pr\nabla^2\mathbf{u} \tag{1}$$

$$\nabla.\mathbf{u} = 0 \tag{2}$$

$$\frac{\partial T}{\partial t} + (\mathbf{u}.\nabla)T = \nabla^2 T \tag{3}$$

with $\mathbf{u}$, p and T the dimensionless velocity, pressure and temperature respectively, $R = (g\beta q'' l^4)/(\lambda\kappa\nu)$, $Pr = \nu/\kappa$ the Rayleigh and Prandtl numbers, $\mathbf{g} = (0,1)^T$. Here g is the acceleration of gravity, l is the characteristic length, q'' is the specific heat flux, β is the thermal expansion coefficient, λ is the thermal conductivity, κ is the thermal diffusivity and ν is the kinematic viscosity of the fluid.

All the numerical experiments presented below are performed in a square 2-D domain with dimensionless length 1. No-slip boundary conditions for the velocity are imposed on all the walls of the cavity. The side and bottom walls are adiabatic except for an area centered at the midpoint of the bottom where the flux is prescribed. On the top wall zero temperature is prescribed.

3 Numerical algorithm

There are three main problems related to the numerical solution of the system (1) - (3). The first one concerns the treatment of the convection operators because of their non-linearity and asymmetry. The second problem is related to the coupling between the velocity and temperature via the source term in the momentum equation and the convection part of the

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energy equation. The third one is how to impose the incompressibility constraint and how to calculate the pressure.

Since the convection operator is suitable for an explicit treatment, in the present technique a so-called operator-splitting approach is adopted. It is discussed in details by Maday et al.[7]. If the convection part of (1) - (3) is splitted in this manner and the resulting diffusion problem is discretized in time with a second-order backward difference scheme the following semi-discrete system appears:

$$\frac{3\mathbf{u}^{n+1} - 4\tilde{\mathbf{u}}^n + \tilde{\mathbf{u}}^{n-1}}{2\Delta t} = -\nabla p^{n+1} + Pr\nabla^2\mathbf{u}^{n+1} + RPrT^{n+1}\mathbf{g} \tag{4}$$

$$\nabla.\mathbf{u}^{n+1} = 0 \tag{5}$$

$$\frac{3T^{n+1} - 4\tilde{T}^n + \tilde{T}^{n-1}}{2\Delta t} = \nabla^2 T^{n+1} \tag{6}$$

Here the quantities marked by $\tilde{}$ are the corresponding quantities at level n-i (i = 0, 1), convected according to the following equation:

$$\begin{aligned} \frac{\partial \tilde{Q}^{n-i}(s)}{\partial s} &= (\mathbf{u}(s).\nabla)\tilde{Q}^{n-i}, \qquad 0 \le s \le (i+1)\Delta t, \qquad i = 0, 1 \\ \tilde{Q}^{n-i}(0) &= Q^{n-i} \end{aligned} \tag{7}$$

where Q is either $\mathbf{u}$ or T.

Taking into account the stability regions of the multistep schemes a 3-step Runge-Kutta scheme is used to solve (7). For the convection of the temperature a second-order extrapolation for the velocity at the moments $t^n + \tau \quad (\tau = (m+\theta)\Delta s, \quad m = 0, M-1; \theta = 0, 1/3, 1/2, 1)$ is used:

$$\mathbf{u}^{n+\tau} = (1 + \tau/\Delta t)\mathbf{u}^n - \tau/\Delta t\mathbf{u}^{n-1} \tag{8}$$

The velocity/ pressure saddle-point problem (4)-(5) can be solved with an implicit source term. This is performed with a projection procedure including the following steps:

1. Calculation of an intermediate velocity field $\mathbf{u}^*$ by choosing the pressure at the previous time level:

$$\frac{3\mathbf{u}^* - 4\tilde{\mathbf{u}}^n + \tilde{\mathbf{u}}^{n-1}}{2\Delta t} = -\nabla p^n + Pr\nabla^2\mathbf{u}^* + RPrT^{n+1}\mathbf{g} \tag{9}$$

2. An equation for the pressure correction $p^c = p^{n+1} - p^n$ is obtained by subtracting (9) from the original equation (4):

$$3/2(\mathbf{u}^{n+1} - \mathbf{u}^*) - \Delta t Pr\nabla^2(\mathbf{u}^{n+1} - \mathbf{u}^*) = -\Delta t\nabla p^c \tag{10}$$

Further, applying divergence to both sides and taking into account (5) yields:

$$\nabla^2 q = \frac{3\nabla.\mathbf{u}^*}{2\Delta t} \tag{11}$$

which is a Poisson equation for the quantity: $q = p^c + Pr\nabla.\mathbf{u}^*$.

3. $\mathbf{u}^*$ is projected on a divergence-free subspace by subtracting of its rotation-free part:

$$\mathbf{u}^{n+1} = \mathbf{u}^* - 2/3\Delta t\nabla q \tag{12}$$

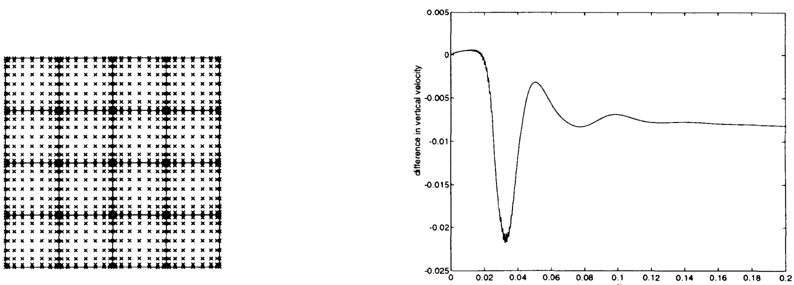
The corresponding pressure is calculated from:

$$p^{n+1} = p^n + q - Pr\nabla.\mathbf{u}^* \tag{13}$$

As it is proved by Timmermans et al.[8] these velocity and pressure satisfy the original saddle-point problem. The details concerning the boundary conditions for the equations above can be found also there.

As mentioned above, a spectral element discretization is applied to the semi-discrete system (9)-(13). The SEM is a high-order Galerkin technique which uses the Gauss-Lobatto points as collocation and integration points (resulting in a diagonal mass matrix). Similarly to the classical finite element methods it also allows decomposition of the physical domain into isoparametric quadrilaterals. More details can be found in Maday & Patera[9]. The scheme described above is extensively discussed and validated by Minev et al. [10].

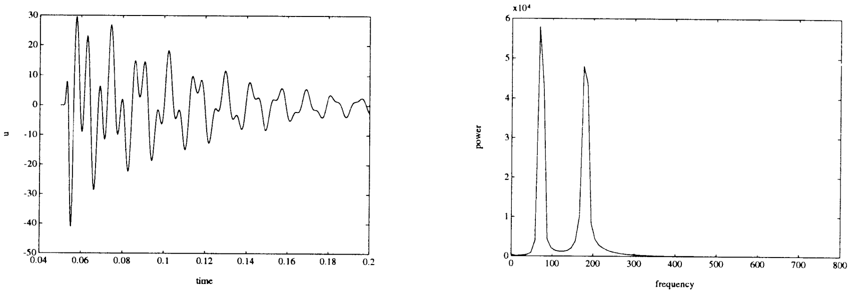
4 Results



a) b)

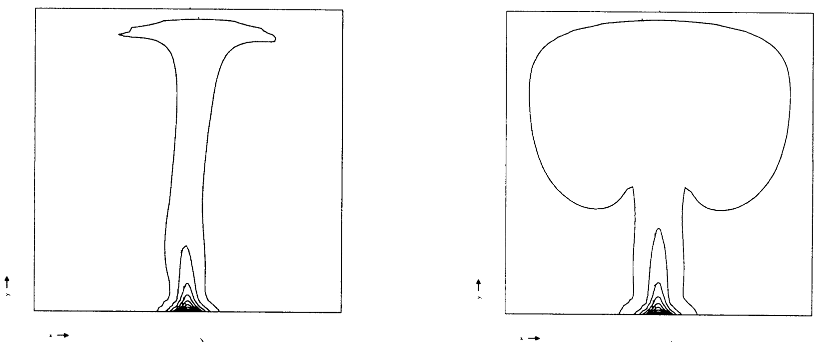
Figure 1: $R = 10^6$ a) the spectral element mesh; b) relative difference in the vertical velocity.

First, the result of the simulation of a laminar plume at $R = 10^6$ is compared with the numerical result obtained under the same conditions by Bastiaans et al . [5]. In the SEM simulation a mesh of 4×4 spectral elements of 8 order (33×33 points) is used (see fig 1a) while by Bastiaans a mesh of 77×77 points and an explicit finite-volume technique is used. For the sake of consistency in both studies the source on the bottom is taken as a smooth Gaussian hill: $\partial T/\partial n = e^{(-100(x-x_0)^2)}$ with x_0 the centre. The history of the relative difference in the vertical velocity components in the centre of the cavity, as found with both methods, is presented in fig. 1b. The result indicates that the velocity differs less than 2.5%; besides the temperature in sample points differs less then 2%.



a) b)

Figure 2: $R = 10^8$ a) history of the horizontal component of the velocity; b) corresponding power spectrum.



a) b)

Figure 3: $R = 10^8$; isotherms.

Next, the flow at $R = 10^8$ is simulated prescribing a heat source: $\partial T/\partial n = e^{(-400(x-x_0)^2)}$ and using a mesh of 8×8 spectral elements of 8 order and $\Delta t = 2.5 \times 10^{-5}$. After a relatively short transience it reaches a steady state. In order to study its stability a disturbance is introduced shifting smoothly the centre of the source for half a period with a physically relevant frequency 200 (see fig. 5b) and an amplitude 0.01. The flow responds with two frequencies oscillations: about 68 and 176 respectively, which are damped after a certain time. This can be clearly seen in fig. 2

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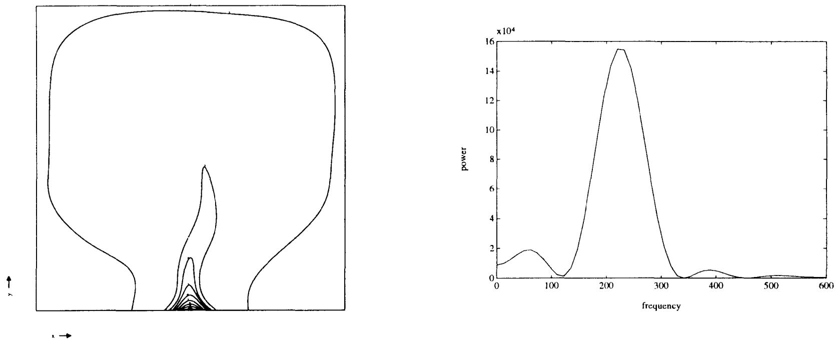
where the graphics of the horizontal component of the velocity in the centre of the cavity is presented together with the corresponding power spectrum. The temperature isolines at the beginning and the end of this development are presented in fig. 3.



a) b)

Figure 4: $R = 2 \times 10^8$ a) isotherms at t = 0.05; b) the power spectrum of the horizontal component of the velocity in the centre.

The result at $R = 10^8$ is used as an initial condition for the simulations at $R = 2 \times 10^8$ and $R = 10^9$. At the first value the flow clearly undergoes the first bifurcation showing a quasi-steady oscillatory behaviour - see fig. 4a. A single frequency of about 220 can be recognized on the power- spectrum diagram presented in fig. 4b. Further increase of the Rayleigh number to $R = 10^9$ gives rise of two incommensurate frequencies of about 161 and 234 (see fig. 5). The structure of the flow is more complicated now - see fig. 6 where the isotherms and the streamlines at the dimensionless time instant 0.05 are presented. The plume is stuck to the right wall and the flow resembles the flow in a differentially -heated cavity at low Rayleigh number. This is also observed in the preliminary experiments.

5 Conclusions

The application of high-order spectral element methods to natural convection problems is quite efficient if it is combined with a high-order splitting procedure for time integration. The present study illustrates this with direct numerical simulation of thermal plumes induced by a local heat source on the bottom of a square cavity. The results are in a good agreement with some preliminary experiments and large-eddy simulations. The numerical technique splits the continuous initial formulation in two Helmholtz equations for the velocity and temperature respectively and one Poisson equation for the pressure which can be efficiently solved with iterative solvers. More-

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over, it allows the usage of a non-staggered grid for the pressure and velocity (see [8]).

The further investigations will include simulations at larger Rayleigh numbers in order to study the full scenario of the transition. 3-D numerical simulations will also be performed in order to study the 3-D effects in this process.

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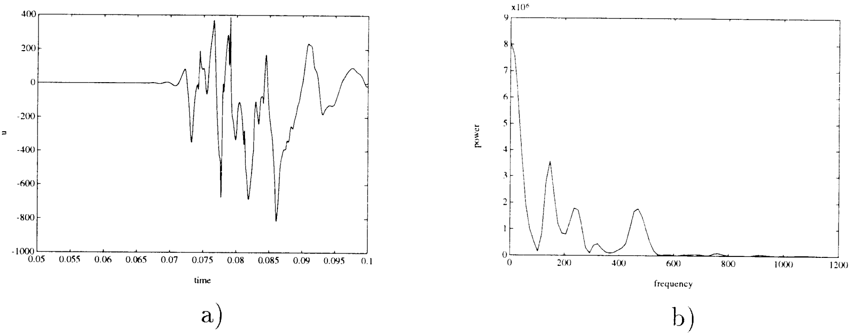

Figure 5: $R = 10^9$ a) history of the horizontal component of the velocity; b) corresponding power spectrum.

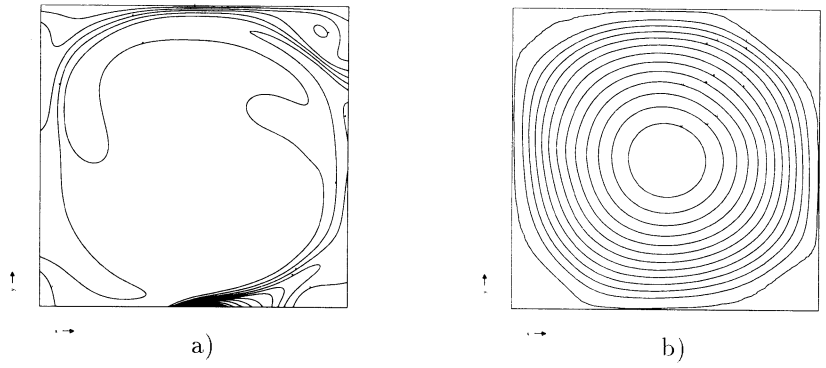

Figure 6: $R = 10^9$ a) Isotherms; b) streamlines at t = 0.05.

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