Laminar flow forced convective heat transfer in a helical square duct with a finite pitch
C.J. Bolinder & B. Sundén
Division of Heat Transfer, Lund Institute of Technology, Box 118, 221 00 Lund, Sweden

ABSTRACT
The effect of a finite pitch or torsion on the temperature field and the Nusselt number for laminar flow in a helical square duct is presented. Fully developed conditions are assumed. As thermal boundary condition is used constant axial wall heat flux with constant peripheral wall temperature. For ducts of small and moderate pitch, the flow and heat transfer characteristics are found to be very similar to those for a toroidal duct with the same dimensionless curvature ε.

INTRODUCTION
Curved ducts frequently appear in heat transfer applications, and are often advantageous. The mean heat transfer rate for laminar flow is increased compared to straight ducts. This is due to the so-called secondary flow, which is set up in the curved duct. By applying curved ducts one may also obtain a more compact equipment.

Theoretical studies of the flow in curved ducts most often concern the toroidally curved duct, for which the curvature κ of the centre-line is constant. Helically coiled ducts with a finite pitch are often used in practice, and they are characterized by the centre-line having a constant curvature and a constant torsion τ. Mathematically, a finite torsion or pitch calls for a non-orthogonal coordinate system, which makes the choice of velocity components ambiguous. This has led to controversy among the existing theoretical papers on helical duct flow, see Bolinder[^1] for a review.

The fluid particles in a curved duct describe helical paths. To obtain a picture of these, the flow is decomposed into an axial flow, which is the projection of the velocity vector onto the tangent of the centre-line, and a secondary flow, which is the projection of the velocity on the cross-plane of the duct. For example, for a toroidal duct, the maximum of the axial flow is shifted towards the outer wall, and the secondary flow consists of two symmetric counter-
rotating cells. A modified Reynolds number, \( Re = \frac{\bar{w}d_h}{v} \), called the Dean number, \( De = \frac{Re}{Kd_h} \), where \( d_h \) is the hydraulic diameter, is often used when curved ducts are considered. For a toroidal duct with small curvature, i.e. large radius of curvature, the Dean number is the single similarity parameter for the flow characteristics, see e.g. Berger et al.\(^\text{15}\).

Previous investigations on forced convection in curved ducts mainly concern toroidally curved ducts of circular cross-section, see the review articles by Nandakumar and Masliyah\(^\text{3}\) and Shah and Joshi\(^\text{4}\). The following thermal boundary conditions are most often considered: constant wall temperature axially and peripherally, designated T by Shah and London\(^\text{5}\), constant axial wall heat flux with constant peripheral wall temperature, designated H1, and constant wall heat flux both axially and peripherally, designated H2. The H1 condition, which is used in the present study, may be realized experimentally for example in electrically heated ducts with highly conductive walls.

Laminar forced convection in toroidally curved ducts of rectangular cross-section has been studied by relatively few authors. Fully developed conditions and the H1 boundary condition were considered by Cheng and Akiyama\(^\text{6}\) and by Mori et al.\(^\text{7}\), who also conducted experiments. Cheng et al.\(^\text{8}\) studied thermally developing flow with the T and H2 boundary conditions. The asymptotic Nusselt number was found to be similar for the two boundary conditions, and a single correlation was given, presumed to be valid for both conditions. Komiyama et al.\(^\text{9}\) investigated ducts with varying aspect ratio, utilizing the H1 boundary condition. More recently Hwang and Chao\(^\text{10}\) considered fully developed conditions and the T boundary condition, but without neglecting the axial conduction term in the energy equation.

The effect of a finite pitch on forced convective heat transfer has been studied in two recent papers, but only for a circular cross-section. Yang et al.\(^\text{11}\) considered fully developed conditions and the H1 boundary condition. Liu and Masliyah\(^\text{12}\) studied simultaneously developing flow and heat transfer, utilizing the T boundary condition. Interestingly enough, the asymptotic temperature profiles of Liu and Masliyah (their Figure 10) differ significantly from those of Yang et al. (their Figure 2), in spite of the fact that similar conditions prevail. Yang et al. predict a "rotated" profile for \( Pr=1 \), and a strongly skew profile for \( Pr=10 \). Liu and Masliyah predict a much weaker effect of a relatively small torsion. Considering the results of the present investigation, the predictions by Liu and Masliyah seem to be more reliable.

The present paper extends the flow field analysis presented in Bolinder\(^\text{1,2}\) to give a description of the influence of a finite pitch or torsion on forced convective heat transfer in a helical square duct. Fully developed conditions are assumed, and the H1 boundary condition is applied. The dimensionless curvature \( \varepsilon = \kappa d_h \) is kept fixed at 0.2, while the dimensionless torsion \( \eta = \tau d_h \) is varied from 0 to 1.
A helical duct is depicted in Figure 1. Indicated are the coordinates x and y in the cross-plane, which run in the directions of the normal \( n \) and binormal \( b \) of the centre-line, respectively. The coordinate \( s \) runs along the centre-line. \( 2\pi K \) is the pitch and \( R \) is the "radius" of the helical duct. The curvature \( \kappa \) and the torsion \( \tau \) of the centre-line can be computed from

\[
\kappa = \frac{R}{R^2 + K^2}, \quad \tau = \frac{K}{R^2 + K^2}.
\]  

(1)

The velocity vector is expanded in the orthonormal basis \( (t, n, b) \), where \( t \) is the tangent of the centre-line, thus

\[
v = wt + un + vb.
\]  

(2)

The axial flow is then described by \( w \) and the secondary flow by \( u \) and \( v \). The continuity equation and the Navier-Stokes equation in the present coordinate system are given in Bolinder\(^{1,2}\). For incompressible flow with constant fluid properties, and neglecting viscous dissipation, the energy equation reads

\[
\frac{\partial T}{\partial t} + \frac{w}{M} \frac{\partial T}{\partial s} + \left( u + \frac{\tau y}{M} w \right) \frac{\partial T}{\partial x} + \left( v - \frac{\tau x}{M} w \right) \frac{\partial T}{\partial y} = \frac{v}{Pr} \frac{1}{M} \left\{ \frac{\partial}{\partial s} \left( \frac{1}{M} \frac{\partial T}{\partial s} \right) + \frac{\tau y}{M} \frac{\partial T}{\partial x} - \frac{\tau x}{M} \frac{\partial T}{\partial y} \right\} + \frac{\partial}{\partial y} \left\{ - \frac{\tau x}{M} \frac{\partial T}{\partial s} - \frac{\tau^2 xy}{M} \frac{\partial T}{\partial x} + \left( M + \frac{\tau^2 x^2}{M} \right) \frac{\partial T}{\partial y} \right\},
\]  

(3)

where

\[
M = 1 - \kappa x.
\]
The present boundary condition of constant axial wall heat flux, with constant peripheral wall temperature, may be realized by assuming a temperature field of the form

$$T(s, x, y) = \gamma s + T_0(x, y),$$

with

$$T_0(x, y) = T_w = \text{constant at the wall}.$$

\(\gamma\) is then the constant axial temperature gradient. A dimensionless temperature may be defined by

$$\theta(x, y) = \frac{T_w - T_0(x, y)}{\gamma d_h}.$$ 

For constant fluid properties, the continuity and the Navier-Stokes equations may first be solved to obtain the velocity field. The procedure is described in Bolinder\(^1\).\(^2\). The energy equation is then solved for the temperature field, according to a similar procedure, i.e. by use of the finite-volume method, with a uniform staggered grid. Central differences are used for the diffusion terms and the source terms. While solving the Navier-Stokes equation, the hybrid difference scheme was applied for the convection terms. It was then verified that by using a 41×41 grid, mostly central differences appeared from the hybrid scheme, so that second-order accuracy was practically achieved. However, according to the energy equation, for Prandtl numbers greater than one, the magnitude of the convection terms will be enlarged compared to the diffusion terms. This means that an increased amount of upwind will appear from the hybrid difference scheme, and accuracy is lost. For Prandtl numbers greater than one, it was therefore found necessary to use the QUICK scheme by Leonard\(^13\), which in two dimensions should be second-order accurate. For Prandtl numbers less than one, the hybrid and the QUICK scheme gave identical results. A uniform 41×41 grid was employed. No convergence problems were encountered, although more heavy under-relaxation was required for simultaneously large Dean and Prandtl numbers. To check the accuracy, a few grid refinement studies were done. For example, for \(Pr=10\) and for the highest Dean number considered of 510, which was for the four-vortex branch of the toroidal duct, the \(h^2\)-extrapolated Nusselt number using the QUICK scheme and a 61×61 and a 81×81 grid was determined to 20.5. This is to be compared to 21.7 for the QUICK scheme on a 41×41 grid, and to 28.7 for the hybrid scheme also on a 41×41 grid. The above example is the "worst case" considered, and for smaller Dean and Prandtl numbers the error will also be smaller.

The mean heat transfer coefficient \(h_m\) is defined by

$$\bar{q}_n = h_m(T_w - T_b),$$

where \(\bar{q}_n\) is the peripheral mean heat flux normal to the duct wall, and \(T_b\) is the bulk mean temperature, which is defined by
By integrating the energy equation over a finite duct length, one may deduce the following expression for the mean Nusselt number

$$\text{Nu}_m = \frac{h_m d_h}{k} = \frac{\overline{q}_n d_h}{k(T_w - T_b)} = \frac{\gamma \text{Pr Re} d_h}{4(T_w - T_b)}.$$  

(8)

Alternatively, the mean Nusselt number may be computed by explicitly integrating the local wall heat flux around the periphery. The difference between the two methods of computation was generally less than 5 per cent. The last expression in Equation (8) is believed to give the most accurate result, and this is used below.

RESULTS AND DISCUSSION

Flow field

The non-linear nature of the governing equations leads to the existence of different solution branches. Figure 2 shows the three branches detected for a toroidal square duct. On the vertical axis is the dimensionless u-component at a point close to the outer wall. $S_1$ and $S_5$ are two-vortex branches, and $S_3$ is a four-vortex branch: the secondary flow has a pair of extra vortices at the outer wall. The $S_3$ branch is unstable to asymmetric perturbations, and could only be detected by imposing symmetry about the x-axis. $S_1$ and $S_5$ are genuinely stable. The end points of the branches are called limit points, their precise locations have been determined in Bolinder\textsuperscript{1,2}. Figure 4 shows the extent of the
S\textsubscript{1} and S\textsubscript{5} branches for increasing torsion. S\textsubscript{3} could not be detected for a finite torsion. For ducts of small torsion, both the flow field and the stability structure are similar to the conditions for a toroidal duct with the same dimensionless curvature $\varepsilon$. For higher torsion, the S\textsubscript{5} branch could not be detected, and the flow approaches a one-vortex structure.

### Temperature field

In Figures 5 and 6 are shown contours of the dimensionless temperature $\theta$ at the same states as in Figures 2 and 4. Note the similarity between the $\theta$-contours and the $w$-contours for $Pr=0.71$. For $Pr=10$, the convective terms dominate in the energy equation, and the temperature field differs from the case with $Pr=0.71$. Especially for ducts with small torsion, the difference is very significant. For higher torsion, however, the temperature field seems to become less dependent upon the Prandtl number. The effect of torsion is to distort the symmetry of the temperature field. For moderate torsion, the distortion is rather small, though slightly more prominent for higher Prandtl numbers.

### Nusselt number

Figure 7 shows the variation of $Nu^*_m/(Nu^*_s)$ with Dean number for a toroidal square duct and for $Pr=0.71$. The index $s$ refers to a straight duct, for which $Nu^*_m=3.608$ for the H1 boundary condition. The present results show good agreement with the experiments by Mori et al.\textsuperscript{7} and the correlations by Cheng et al.\textsuperscript{8} and Komiyama et al.\textsuperscript{9}. Note that the curvature in the above investigations was not the same. Thus, the curvature effect on the Nusselt number is well captured by the Dean number, at least for $\varepsilon \leq 0.2$. The correlation by Cheng et al.\textsuperscript{8} is recommended by Shah and Joshi\textsuperscript{4}, it reads

$$Nu_m = 0.152 + 0.627(De^{\frac{1}{2}}Pr^{\frac{1}{3}}), \quad 0.7 \leq Pr \leq 5, \quad 14 \leq De \leq 500.$$  \hspace{1cm} (9)
This correlation was originally presumed to be valid for the T and H2 boundary conditions, but obviously it works also for the H1 condition. For small Dean numbers it however underpredicts the Nusselt number.

The variation of Nusselt number with Prandtl number for a toroidal square duct is shown in Figure 8. The Nusselt number is increased for increasing Pr. Note also that the gap between the S3 and S5 branches increases with increasing Pr. The correlation by Cheng et al.\(^8\) gives reasonable values also for Pr=10, so its range of validity might be extended.

The effect of torsion on the Nusselt number is indicated in Figure 9. For moderate values of the torsion the effect is small: the Nusselt number is decreased, except close to the limit point L2, where a slight increase can be observed. The effect of torsion is more prominent for higher Prandtl numbers. For ducts of large torsion the Nusselt number approaches the value for a straight twisted duct, which is slightly less than that for a straight duct. Note that for a straight twisted duct, the Nusselt number is almost insensitive to vari-
ations in Prandtl and Reynolds number. This behaviour is in accordance with the results of Masliyah and Nandakumar\(^{14}\), for the T boundary condition.

An indication of the variation of the local Nusselt number around the periphery can be obtained by studying the temperature plots in Figures 5 and 6; large wall-gradients indicate a high Nusselt number. Finally, according to Bolinder\(^2\), the friction factor is much less affected by torsion than the Nusselt number.

REFERENCES


